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Bootstrap of the Vector and Tensor Mesons Using the Effective-Potential Method. III. The ϕ and f'

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We use the effective-potential method of Balázs, applied earlier to $\pi\pi$ and πK scattering, to calculate isospin-0 $K\bar{K}$ scattering, taking into account ρ, ω, f, and A_2 exchange, with parameters determined from experiment and SU(3), as well as ϕ exchange. We obtain self-consistent values of 1008 MeV and 0.14 for the mass and reduced width of the ϕ in the $K\bar{K}$ channel, compared with experimental values of 1020 MeV and 0.23. Using ϕ exchange with the self-consistent values of the parameters, together with the other exchanges, then yields an f' meson with mass and $K\bar{K}$ partial width of 1540 MeV and 31 MeV, compared with experimental values of 1514 MeV and 53 MeV. We also correct a numerical error in the earlier K^* bootstrap calculation, and make some general comments on the comparison of the pattern of vector and tensor masses and widths obtained using the method with experiment; the over-all results appear encouraging.

In two previous papers[1,2] we have reported reasonably successful "bootstrap" calculations of the ρ and K^* mesons which employed the method developed by Balázs[3] for constructing an effective energy-dependent potential. In each case we take the potential to correspond to the exchange of the vector and tensor mesons which are coupled to the crossed channels; the inclusion of tensor exchange is important in obtaining results in good agreement with experiment. Here we apply the same method to $K\bar{K}$ scattering in the isospin-0 P wave in order to obtain theoretical values for the ϕ meson mass and width.

Since the method neglects inelasticity, one must first ask whether it makes sense to apply it in a situation where the threshold and resonance masses are as large as in the case of $K\bar{K}$ scattering and the ϕ. However, there are several favorable factors which suggest that inelastic effects indeed may not be large. In the first place, conservation of G parity prevents the coupling of the isospin-0 P-wave $K\bar{K}$ system to any even number of pions. One must still worry about the three-pion channel. However, the experimental fact that the ϕ has a $K\bar{K}$ branching ratio of about 80% (Ref. 4) suggests that it may indeed be permissible to neglect the coupling to three pions and make the elastic approximation.

Before giving the results of the present calculation, we first wish to make a correction in the results of II; the correction in question also has a bearing on the results of the present paper. The values given in II for the ratios in the $\rho\pi\pi$ to $\rho K\bar{K}$ and $\rho\pi\pi$ to ρKK^* coupling constants predicted by SU(3) are too large by a factor of 2. [The error resulted from incorrectly relating the ratios of the decay widths to the values of SU(3) Clebsch-Gordan coefficients in a case where both decay products belong to the same SU(3) multiplet.] As a result, the numerical factors $\frac{8}{3}$ and 16 in Eqs. (4a) and (3′) of II should be replaced by $\frac{4}{3}$ and $16/\sqrt{2}$, respectively. This means that the difference between the SU(3) prediction for the value of the $\rho K\bar{K}$ reduced partial width [the parameter Γ'_ρ in the ρ-exchange potential as given by Eq. (3) in II] based on the experimental ρ width, and that based on the experimental K^* width, is only half as large as stated in II, and the corresponding range of self-consistent K^* parameters is thus reduced. In particular, the best solution obtained, namely, $m_{K^*}=930$ MeV and a K^* width of 50 MeV, now occurs with a ρ-exchange potential corresponding to a $\rho K\bar{K}$ coupling constant about 40% larger than that obtained from SU(3) and the experimental K^* width, and about 60% larger than that obtained from SU(3) and the experimental ρ width. In II it was suggested that the difference between the values of the $\rho K\bar{K}$ coupling, and hence of the ρ-exchange portion of the πK effective potential, obtained from the ρ, and from the K^* widths, together with SU(3) should give a reasonable estimate of the uncertainty in the results due to SU(3) breaking. If we confine the ρ-exchange potential to this range, we obtain a range of self-consistent K^* masses of about 1000 to 1030 MeV, so that the corrected theoretical values of the K^* mass are in somewhat worse agreement with the experimental value of 890 MeV, the discrepancy now being 10% to 15%. The predicted value of the K^* width is insensitive to this change, and remains in agreement with experiment, and the other qualitative results of II, e.g., the nature of the Regge trajectories, are unchanged.

Next we ask what exchange forces should be included in constructing our potential in the $K\bar{K}$ case. If we write the Schrödinger equation in the form

$$(\nabla^2 + k^2)\psi(r) = V(r, s)\psi(r), \tag{1}$$

then the contribution to the potential for isospin-0 $K\bar{K}$ scattering due to the exchange of a resonance of mass m_r, spin j, the isospin I in the t channel (we use the usual Mandelstam variables s, t, and u, with s being the c.m. energy in the direct channel) is given by[3] (in units with $\hbar = c = 1$)

$$V(r, s) = \frac{-8\beta_{0I} m_r \Gamma_r q_r^{2j}(2j+1)P_j(1+s/2q_r^2)e^{-m_r r}}{\sqrt{s}\, m_r r}. \tag{2}$$

In Eq. (2), β_{0I} is a crossing matrix element whose value is $\frac{3}{2}$ or $\frac{1}{2}$ for the cases $I=1$ or 0, respectively, $q_r = (m_r^2/4 - m_K^2)$, where m_K is the kaon mass, and Γ_r is the reduced width of the exchanged resonance in the $K\bar{K}$ channel, and is related to Γ_{rt}, the partial half-width in the energy-squared variable by

$$\Gamma_r = \Gamma_{rt} m_r/(8q_r^{2j+1}). \tag{3}$$

We note in passing that in this problem the u channel (KK) is exotic, so that, in our approximation where the potential is assumed dominated by resonances treated in the narrow-width approximation, there is no u-channel contribution to the potential.

There is first of all a contribution to the potential from ϕ exchange itself, so that we indeed have a bootstrap situation. From (2) we find the ϕ-exchange potential, V_ϕ, is given as

$$V_\phi = -6\Gamma_\phi m_\phi (2q_\phi^2 + s)e^{-m_\phi r}/(\sqrt{s}\, m_\phi r), \tag{4}$$

where m_ϕ and Γ_ϕ are the mass and reduced width [defined by Eq. (3)] of the ϕ.

In addition to the exchange of the ϕ itself, contributions to the potential binding the ϕ can come from the exchange of the ρ, ω, f, and A_2. (We neglect the f', whose effect would be small because of its large mass.) We will take the parameters of these particles from experiment, combined, where necessary, with SU(3), in constructing their contributions to the potential, though we note, as a matter of principle, that one could avoid experimental input by using the results of I together with SU(3) and would arrive at essentially the same results.

If we neglect, for simplicity, the small mass difference between the ρ and ω (this has a negligible effect), we may combine the ρ- and ω-exchange contributions into a single term, $V_{\rho\omega}$ in the potential. Putting numerical values into Eq. (2), one obtains

$$V_{\rho\omega} = -65(\tfrac{3}{2}\Gamma_\rho + \tfrac{1}{2}\Gamma_\omega)(-10.6 + s)e^{-m_\rho r}/(\sqrt{s}\, m_\rho r). \tag{5}$$

Assuming that the reduced widths, as defined by (3), are proportional to the squares of coupling constants which obey SU(3), one has for Γ_ρ the reduced partial width of the ρ in the $K\bar{K}$ channel

$$\Gamma_\rho = \tfrac{1}{2}\Gamma_{\rho\pi\pi} \tag{6a}$$

$$= \tfrac{2}{3}\Gamma_{K^*K\pi}, \tag{6b}$$

where $\Gamma_{\rho\pi\pi}$ and $\Gamma_{K^*K\pi}$ are, respectively, the reduced partial widths for the decays $\rho \to \pi\pi$ and $K^* \to K\pi$. Using the experimental values[4] of $\Gamma_{\rho\pi\pi}$ and $\Gamma_{K^*K\pi}$, Eqs. (6a) and (6b) yield $\Gamma_\rho = 0.105$ and $\Gamma_\rho = 0.14$, respectively, where, as discussed above and in II, the difference in these two numbers seems a reasonable estimate of the uncertainty due to SU(3) breaking. A similar ambiguity enters in determining Γ_ω. Let Γ_8 be the reduced partial width for the isosinglet member of the vector octet in the limit of exact SU(3). Then

$$\Gamma_8 = \tfrac{3}{2}\Gamma_{\rho\pi\pi} \tag{7a}$$

$$= 2\Gamma_{K^*K\pi}. \tag{7b}$$

The physical ω is related to the members of the vector octet and singlet by

$$|\omega\rangle = -\sin\theta|8\rangle + \cos\theta|1\rangle,$$

where θ is the ω-ϕ mixing angle. Since, in the limit of SU(3), the singlet particle cannot decay into two pseudoscalar mesons in a P state because the over-all [space-plus-SU(3)] wave function of the final state would be antisymmetric, Eqs. (7) yield

$$\Gamma_\omega = \tfrac{3}{2}\sin^2\theta\, \Gamma_{\rho\pi\pi} \tag{8a}$$

$$= 2\sin^2\theta\, \Gamma_{K^*K\pi}. \tag{8b}$$

Using the usual value of the mixing angle obtained by requiring the Gell-Mann–Okubo formula to hold for the vector octet, Eqs. (8a) and (8b) yield, respectively, $\Gamma_\omega = 0.14$ and $\Gamma_\omega = 0.22$. In determining $V_{\rho\omega}$, we take the value of Γ_ρ based on the K^* width, i.e., 0.14, since that gives the best result in the K^* bootstrap of II. We take the value of Γ_ω which, of course, does not enter in the K^* case, in our present calculations to lie at the lower end of its range, i.e., $\Gamma_\omega = 0.14$, since this turns out to yield the best agreement between the "bootstrapped" values of the ϕ parameters and experiment. However, as we shall see, the results are quite insensitive to variations of Γ_ω in the indicated range, partly because the importance of the ω-exchange force is suppressed by the small value of the crossing matrix element.

Lastly we write down the f- and A_2-exchange contributions to the potential. As in the $\rho\omega$ case, we neglect for simplicity the small f-A_2 mass difference, and combine the two contributions into a single term, V_{fA}, which is given by

$$V_{fA}=\frac{-37.2(\frac{1}{2}\Gamma_f+\frac{3}{2}\Gamma_A)[\frac{3}{2}(7.4+0.5s)^2-27]e^{-m_A r}}{\sqrt{s}\,m_A r}. \qquad (9)$$

The values of Γ_f and Γ_A, the reduced partial widths of the f and A_2 obtained from experiment,[4] are 0.15 and 0.12, respectively. There are rather large experimental uncertainties in the $K\bar{K}$ branching ratio of the f and hence in the value of Γ_f; however, as with the ω, the importance of f exchange is suppressed by the crossing matrix, and our results are very insensitive to even large variations in this branching ratio.

We now insert $V_{\rho\omega}$ and V_{fA} as determined above into the Schrödinger equation (1), together with V_ϕ, compute the output P-wave amplitude, and seek input values of m_ϕ and Γ_ϕ which generate an output resonance with the same parameters. We find we obtain self-consistency for the values $m_\phi = 1008$ MeV and $\Gamma_\phi = 0.14$, compared with the experimental values of 1020 MeV and 0.23 ± 0.02.[4] A reduced partial width of 0.14 corresponds to a partial decay width of 2.1 MeV for a ϕ of mass 1020 MeV in the $K\bar{K}$ channel, compared with the value of about 3.5 MeV based on the observed width and branching ratio of the ϕ. The actual width of our theoretical resonance at 1008 MeV is 0.7 MeV. However, because we are so close to threshold, the actual resonance width is very sensitive to the resonance mass, and it seems clear that the reduced mass is the significant quantity. We do indeed find that the actual resonance width in energy produced in the effective potential calculation does scale almost exactly with the quantity q_ϕ^3/m_ϕ^2, while the reduced width remains almost constant as the position of the resonance changes. For example, a change of about 30% in the input ϕ width moves the output resonance position to 1020 MeV and triples the output width, while leaving the output partial width essentially unchanged. Thus it seems clear that the small physical width of the resonance in our model reflects, not a factor-of-3 disagreement with experiment, but simply the small difference in the theoretical and experimental ϕ masses. Hence we conclude that we obtain a "bootstrapped" value for the ϕ mass in essentially exact agreement with experiment, and a value for the width which differs from the experimental value by about 40%. Interestingly, the width here turns out to be too small in contrast with the results of many bootstrap calculations.[5]

As we have mentioned, there is some ambiguity in the results, due to the uncertainty in Γ_ω. However, the sensitivity to variations in Γ_ω is not large. If we choose it at the high rather than the low end of what we have suggested as its reasonable range, i.e., we take it to have the value of 0.22 based on the K^* width, then the bootstrapped value of the ϕ mass is decreased by about 40 MeV (so that the ϕ becomes a bound state) while the reduced widths remains almost unchanged.

Before proceeding, we should, perhaps, note that there is no sign of a second bound state in the isospin-0 $K\bar{K}$ P wave. Such a state should appear if the ω were primarily a bound state in the $K\bar{K}$ channel. There is, however, no reason to suppose that is true; it seems much more likely that other channels, e.g., $\pi\rho$, are dominant in producing the ω, so its failure to appear in the current calculation is not surprising. In terms of SU(3), one might expect that, having successfully obtained the ρ and K^* in I and II as resonances of two pseudoscalars produced by vector and tensor exchange, one should similarly be able to generate the particle corresponding most closely to the remaining member of a vector octet, i.e., the ϕ. There is no reason why one should expect to obtain the ninth vector meson, an SU(3) singlet, which, in terms of physical particles, corresponds most closely to the ω, in a calculation of pseudoscalar-pseudoscalar scattering, in view of the decoupling of an SU(3)-singlet-vector particle from decay into two pseudoscalars.

Having obtained the self-consistent ϕ parameters, and hence determined V_ϕ, one can now calculate the $K\bar{K}$ isospin-0 D-wave amplitude. From the behavior of the ρ and K^* trajectories in I and II, we expect that, in the present calculation, the calculated Regge trajectory passing through the output ϕ resonance, which we have already obtained, will rise at least through $j=2$, so that we will find a D-wave $K\bar{K}$ resonance. In nature, the ϕ and f' Regge trajectories are approximately exchange-degenerate. Since, in our present calculation, we have exact exchange degeneracy, due to the absence of any contribution to the potential from u-channel exchanges, the D-wave resonance which we obtain theoretically should correspond to the f'. It is true that, in nature, the f will also appear as a $j=2$ resonance coupled to the $K\bar{K}$ channel. However, experimentally, the main coupling of the f is to the $\pi\pi$ channel. Moreover, from the theoretical point of view, we obtained in I an f in good agreement with experiment in a calculation involving $\pi\pi$ scattering. Therefore, we would not expect to obtain a resonance corresponding to the f in a calculation of $K\bar{K}$ scattering. Put another way, one would expect that the f would have to be inserted "by hand," i.e., that it would be a CDD pole,[6] in a $K\bar{K}$ calculation. Moreover, if there were a $j=2$ resonance in the theory at as low a mass as that of the f, then the Regge trajectory passing through it would give, by exchange degeneracy, a bound state in the P wave lying below the ϕ, i.e., the ω, which we have already seen is not present. A single-channel calculation of the f' is less justifiable than for the ϕ, both because of the rather high energy, and because the D wave can couple to two pions. One can only attempt to justify it in terms of the experimental dominance of the $K\bar{K}$ decay mode of the f', and, *a posteriori,* in terms of the fact that the calculation is relatively successful. Turning to the results of the calculation, we find a single output $j=2$ resonance at a mass of 1540 MeV with a width of 31 MeV. As in the case of the ϕ, the predicted mass of the f' is in good agreement with the experimental value of 1514 MeV, while the predicted $K\bar{K}$ partial width is roughly 60% of the experimental value of about 53 MeV, though the predicted value of the partial width essentially agrees with the experimental value to within the rather large uncertainty in the latter.[4] Strict self-consistency would imply that the f' also be included in the input potential. As mentioned previously, we have not done this. The combination of the large mass and rather small width of the f' and the small size of the relevant crossing matrix element mean that the inclusion of f' exchange in the input would have almost no effect on the output.

It seems appropriate here to make some general remarks concerning the results of I and II plus the present work. Using the effective potential approximation for the scattering of various combinations of pseudoscalar mesons, with the forces taken to be those due to the possible exchanges of vector and tensor mesons, we have obtained bootstrap values for the masses and widths of the eight members of the vector-meson octet by requiring that the input and output values of these parameters be self-consistent. We not only find theoretical resonances corresponding to each member of the octet, but the self-consistent values of the masses and widths are in fair to good agreement with experiment; the worst disagreement for the masses is about 11% for the K^*, and in widths about 40% for the ϕ. The calculation is free of adjustable parameters except for the limited uncertainty, due to SU(3) breaking, in the values of the $\rho K\bar{K}$ and $\omega K\bar{K}$ couplings, an uncertainty to which the general nature of the results is, in any event, insensitive. With this exception, all of the parameters entering the calculation are either determined self-consistently, or, in the case of some of the tensor masses and decay widths, and of course the pseudoscalar masses, taken from experiment. In the language of SU(3), given the average mass and mass splittings within the pseudoscalar octet, we have shown that the effective potential procedure allows one to account reasonably successfully for the average mass, mass splittings, and decay widths of the vector octet. This seems to us to offer rather encouraging support to the point of view that the vector mesons are composite particles, and that they may reasonably be regarded as composed primarily of two pseudoscalar mesons, with other channels not being of great importance in their formation.

The pattern of the results for the $j=2$ resonances is slightly more complicated, but appears also easily comprehensible. In I and the present work, we found the model gives good results for the parameters of the f and f'. We note that experimentally these are the two members of the tensor nonet whose decays are strongly dominated by modes involving two pseudoscalar mesons. In II, we found that, although the force in the πK isospin-1 D wave is attractive, no resonance, corresponding to the $K^{**}(1420)$, was generated, at least up to a mass of 2 BeV. This suggests that the effect of the symmetry breaking due to the mass differences within the pseudoscalar octet is so severe that, if channels involving two pseudoscalars were the only ones present, the tensor octet would be incomplete, or at least very badly split, and that at least one other type of reaction is crucial in forming the K^{**}: Presumably, this would be channels involving vector-pseudoscalar scattering, in agreement with the presence experimentally of important $K\rho$ and $K^*\pi$ decay modes of the K^{**}. We have made no attempt to calculate the A_2, since even if one considers only channels involving two pseudoscalars, one still has a coupled channel ($K\bar{K}$ isospin 1 and $\pi\eta$) problem to deal with. However, in view of the above remarks and the experimental dominance of the $\pi\rho$ decay mode of the A_2, we would expect (and hope) that even if one carried out a coupled channel $K\bar{K}$-$\pi\eta$ calculation, any $j=2$ resonances obtained would be at a much higher mass than the experimental A_2 mass, and that the inclusion of the $\pi\rho$ channel would be vital in a successful calculation of the A_2. If the A_2 is generated primarily in the $\pi\rho$ channel, exchange degeneracy in the reaction $\pi^+\rho^- \to \rho^-\pi^+$ will then give a $j=2$ state with isospin 0 (i.e., the ω) at the $j=1$ intercept of the A_2 trajectory, so that one would have a dynamical explanation of the exchange degeneracy of the ω and A_2 trajectories. The dynamical origin of the degeneracy of the ρ-f and ω-A_2 trajectories would remain obscure, and would appear as a numerical coincidence in this picture, since they would be generated primarily in different channels ($\pi\pi$ and $\pi\rho$, respectively) and the $K\bar{K}$ channel, in which all appear, would not be terribly important in determining their properties. In any event, the effective potential model considered in I, II, and the present work seems to correctly predict, apart from the A_2 for which the calculation has not been done, which of the tensor mesons are dominantly coupled to a decay channel containing two pseudoscalar mesons; and, for the two (f and f') which are so coupled, it gives good predictions for the masses and widths.

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