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π^0 Photoproduction in a Weak-Regge-Cut Model*

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A weak–Regge-absorption model using ω, ρ, B, and H exchanges is applied to neutral pseudoscalar meson photoproduction and related reactions. The theoretical constraints of duality, SU(3), and vector dominance are imposed in order to simplify the model as much as possible. Satisfactory agreement is found, with a small set of free parameters, for all of the reactions considered.

I. INTRODUCTION

In recent years, several models have been developed for generating Regge-cut corrections to Regge-pole–dominated high-energy scattering amplitudes.[1-3] When these models are applied to a particular process, the arbitrariness of the parametrization of the cuts often obscures the essential features and validity of the model. This arbitrariness can be minimized, however, by applying a given model to several processes, simultaneously, and imposing the theoretical constraints of SU(3) invariance, vector dominance, and duality on the parameters of the model. When the observables for the reactions involved have been carefully measured, severe constraints are then imposed on the remaining unspecified parameters of the model. If those remaining parameters can be chosen to fit the existing data, a much clearer picture of high-energy scattering will emerge.

In accordance with this philosophy, we have applied a weak Regge absorption model to the photoproduction of neutral pseudoscalar mesons and vector-dominance-related reactions. Specifically, we consider the reactions[4]

$$\gamma p \to \pi^0 p\,, \tag{1a}$$

$$\gamma n \to \pi^0 n\,, \tag{1b}$$

$$\gamma p \to \eta p\,, \tag{2a}$$

$$\gamma p \to X^0 p\,, \tag{2b}$$

$$\pi^0 p \to \omega p\,, \tag{3}$$

$$\pi^0 p \to \rho^0 p\,. \tag{4}$$

Reactions (1) and (2) are related by SU(3) invariance and the transverse part of the vector production reactions; (3) and (4) are related to (1) by vector dominance. That is, the isovector photon amplitude for reactions (1) is related to the transverse ρ amplitude for reaction (4), and the isoscalar photon amplitude for (1) is related to the transverse ω amplitude for (3). This set of reactions thus satisfies our criterion of imposing strong constraints on the model, since each exchange occurs in several reactions.

The exchanges that are included in the model are the ω, ρ, B, and H; the last being the lower-mass isoscalar member of the nonet to which the B is assigned. It is assumed that the residues of these Regge poles satisfy SU(3), vector dominance, duality, and exchange degeneracy. The remaining independent reduced residues are allowed a simple exponential dependence on momentum transfer. Regge cuts are generated from the lowest-order elastic rescattering correction, by a moving Pomeranchukon with strength approximately determined by πN elastic scattering. The remaining unconstrained parameters are then allowed to vary. We have thereby obtained a good over-all fit to the data.

The importance of this result is that it is possible to fit the existing data and satisfy all the theoretically desirable constraints on Regge poles – SU(3), vector dominance, duality – by included absorptive corrections. It is also evidence that the secondary trajectories, B and H, are essential in obtaining a correct description of the "fine structure" of the data – the filling of differential cross-section dips and various spin-polarization combinations.

In Sec. II we give arguments for the inclusion of the particular exchanges we have chosen, give evidence for absorptive corrections, and compare our description with that of other authors. The details of the calculation are presented in Sec. III. Results are presented and discussed in detail in Sec. IV. A summary of the results and the conclusions we have reached are contained in Sec. V. In the Appendix we derive various symmetry relations that are used in the paper.

II. REGGE CUTS IN MESON PHOTOPRODUCTION

Regge-pole exchange, when modified by absorptive corrections, is most conveniently described in terms of s-channel helicity amplitudes. We shall use the set

$$f_1 = f_{1+,0+},\quad f_2 = f_{1+,0-},\quad f_3 = f_{1-,0+},\quad f_4 = f_{1-,0-}, \tag{5}$$

where $f_{ab,cd}$ is the amplitude for a photon with helicity a striking the target nucleon of helicity b and producing a spin-zero meson and a nucleon with helicity d. Furthermore, in the limit of asymptotic energies we can define the following set of amplitudes of definite t-channel parity,[5]

$$f_1^\pm = f_1 \pm f_4,\quad f_2^\pm = f_2 \mp f_3, \tag{6}$$

where natural (unnatural) parity is indicated by superscript $\pm$.

The photoproduction observables which we shall consider may be written in terms of these amplitudes as follows.

Differential cross section:

$$\frac{d\sigma}{dt} = \frac{\pi}{2sk^2} \sum_{i=1}^{4} |f_i|^2 . \tag{7}$$

Polarized-photon asymmetry:

$$P = \frac{1}{2\Sigma} \sum_{i=1}^{2} (|f_i^+|^2 - |f_i^-|^2) = 2\,\mathrm{Re}(f_1^* f_4 - f_2^* f_3)/\Sigma , \tag{8}$$

where

$$\Sigma = \sum_{i=1}^{4} |f_i|^2 .$$

Recoil-nucleon polarization:

$$T = -\mathrm{Im}(f_1^{+*} f_2^+ + f_1^{-*} f_2^-)/\Sigma = 2\,\mathrm{Im}(f_4^* f_3 - f_1^* f_2)/\Sigma . \tag{9}$$

Polarized-target asymmetry:

$$A = -\mathrm{Im}(f_1^{+*} f_2^+ - f_1^{-*} f_2^-)/\Sigma = 2\,\mathrm{Im}(f_1^* f_3 - f_4^* f_2)/\Sigma . \tag{10}$$

Neutron/proton ratio:

$$R = \frac{\sum_{i=1}^{4} |f_i^S - f_i^V|^2}{\sum_{i=1}^{4} |f_i^S + f_i^V|^2} , \tag{11}$$

where f^S (f^V) is pure $I=0$ ($I=1$) in the t channel.

From vector dominance ω exchange is expected to be the dominant mechanism for π^0 photoproduction, the cross section for which is characterized by a pronounced dip at $t \simeq -0.5$ (GeV/c)^2. In a pure Regge model this is usually attributed to a nonsense wrong-signature zero (NWSZ) for the ω residue.[6] However, as this results in a zero in the cross section an additional exchange must be found which is nonzero at $t \simeq -0.5$ (GeV/c)^2. The exchange degeneracy argument which requires the NWSZ for the ω also requires one in the ρ residue for the same t value so that ρ exchange alone is unable to account for the nonzero cross section. Furthermore, the energy dependence in the dip region is compatible with that outside the dip region[7] so that secondary exchanges alone cannot be the source of the nonzero cross section. Thus, these arguments support the existence of Regge cuts as the mechanism for filling in the zeros of the Regge poles.

Regge cuts are in general smoothly varying functions of t and have a leading energy dependence of the form[6]

$$\frac{s^{\alpha_c}}{\ln s - i\pi/2} ,$$

$$\alpha_c = \alpha_R^0 + \frac{\alpha_R' \alpha_P'}{\alpha_R' + \alpha_P'}\, t .$$

With a slope for the Pomeranchukon of 0.4,[8] the cut trajectory becomes very shallow. The intercept of the cut trajectory is the same as that of the pole so the energy dependence of the cut is comparable to the pole.

Equation (8) for the polarized-photon asymmetry shows that at large energies this quantity is a measure of the relative amounts of natural- and unnatural-parity exchanges. The data[9] show that P is near unity except for a small dip near $t = -0.5$ (GeV/c)^2. This indicates the presence of an unnatural-parity contribution in addition to the natural-parity ω term. When Regge cuts are included, they will contribute to amplitudes of both parities and hence will cause P to be different from unity. However, if these cuts are generated from Regge poles with NWSZ they will be too small to account for the entire unnatural-parity amplitude. The most likely candidate for the necessary unnatural-parity exchange is the B meson.[6] B exchange will also help fill the dip in the cross section as well as account for the dip in P.

The ratio, R, of π^0 photoproduction from neutrons to that from protons is a measure of isoscalar-isovector interference since the isovector amplitude changes sign as shown in Eq. (11). ω exchange contributes to the isoscalar amplitude while both ρ and B exchange contribute to the isovector amplitude. The data[10] show some interference, especially in the region of the dip in the $\pi^0 p$ cross section thus indicating that some ρ exchange is needed. Vector dominance or the quark model suggest that the ω and ρ couple in the ratio of 9 to 1 in π^0 photoproduction which, with the assumption that the ω couples primarily to the nucleon helicity-nonflip vertex, leads to the simple approximation

$$R \simeq |f^\omega(1-\tfrac{1}{9})|^2/|f^\omega(1+\tfrac{1}{9})|^2 \simeq 0.64\,.$$

Comparison with the data shows that this value is too small except in the region of the dip.

An alternative estimate for R may be made using the value of the SU(3) d/f coupling ratio for the vector-meson–nucleon vertex obtained from $K^+\Lambda$ and $K^+\Sigma^0$ photoproduction.[11] For the helicity-nonflip nucleon vertex this value is $d/f = -\frac{1}{2}$ which leads to a ρ/ω ratio in π^0 photoproduction of $\frac{1}{21}$. Then R may be approximated by

$$R \simeq |f^\omega(1-\tfrac{1}{21})|^2/f^\omega(1+\tfrac{1}{21})|^2 \simeq 0.83\,.$$

This value is in closer agreement with the data, thus indicating that the vector-dominance coupling relations may have to be broken in order to obtain a good description of R.

η photoproduction is related to π^0 photoproduction by SU(3). Its cross section has no dip corresponding to that in π^0 photoproduction.[12,13] This is often cited as evidence against having NWSZ's in the ω and ρ residues. However, SU(3) predicts a stronger B exchange residue in η photoproduction than in π^0 photoproduction while the ω strength is reduced.[14] Thus, when B exchange is included there is a natural explanation for the absence of a dip in η photoproduction.

Further evidence for B exchange comes from vector-meson production. The combination of cross sections

$$X(s,t) \equiv \tfrac{1}{2}\left[\frac{d\sigma}{dt}(\pi^+p \to \rho^+p) + \frac{d\sigma}{dt}(\pi^-p\to\rho^-p) - \frac{d\sigma}{dt}(\pi^-p\to\rho^0 n)\right]$$

is pure $I=0$ in the t channel.[15,16] The transverse contribution to $X(s,t)$ is dominated by ω exchange and a dip is present at $t \simeq -0.5$ (GeV/c)^2 as expected. On the other hand, transverse ω production is dominated by ρ exchange and does not show a dip so some B exchange must be added to fill the dip.[6] A large B contribution would be evident by a nonzero value for ρ_{00} which only gets unnatural-parity exchange contributions.[5,17] Since large values of ρ_{00} are observed[18–22] this can be taken as evidence for B exchange. Furthermore, effective trajectories for ω production[18] show the effect of a lower-lying trajectory in addition to the ρ trajectory.

The importance of Regge cuts and/or lower-lying trajectories (collectively referred to here as background) in π^0 photoproduction can also be inferred by a comparison of the differential cross section with that for the πN charge-exchange reaction. The energy dependence of the latter reaction is well described by a ρ Regge pole with a trajectory $\alpha = 0.56 + t$.[6] However, the π^0 photoproduction differential cross-section energy dependence is characterized by an effective trajectory $\alpha_{\rm eff} = 0.18 + 0.26t$.[7] If only an ω Regge pole were present then the energy dependence would be the same as observed in the πN charge-exchange reaction. This indicates the need for additional background contributions. Furthermore, the ratio of the first maximum to the dip for πN charge exchange is approximately 1 to 100 while the same ratio is approximately fifteen for π^0 photoproduction. This is a further indication that the background terms are more sizeable in π^0 photoproduction than in πN exchange. Thus, the effective trajectory in πN charge exchange is altered only slightly from that given by the ρ pole alone. However, the relatively larger background terms present in π^0 photoproduction will alter the energy dependence from that given by an ω-pole term alone. As noted previously, the trajectory for a Regge cut will be flatter than that of its associated pole. Thus, the background will be characterized by an effective trajectory that is flatter and somewhat lower-lying than that of the dominant pole terms. This yields an over-all effective trajectory that is in qualitative agreement with the data.[7] This same behavior is seen in $\pi^\pm$ and K^+ photoproduction where again secondary trajectories and Regge cuts play important roles.[6]

The characteristics of the model outlined above result in part from the requirements of exchange degeneracy. If exchange degeneracy is not imposed on the pole residues, then the NWSZ's are no longer required and an alternate dip-generating method is needed. The mechanism most often used is that of pole-cut interference.[2,3] The resulting amplitudes have zeros due to destructive pole-cut interference whose position is dependent upon the relative pole and cut strengths. Furthermore, the cut strength is dependent upon the helicity structure of the particular amplitude[6] so this model allows correlations between net helicity flip and the presence of dips. This becomes useful if one can show that a certain exchange couples dominantly in a particular helicity-flip configuration. This model is then equivalent to the geometrical model of Harari.[23] The addition of a phenomenological "λ" factor which takes into account the effects of inelastic scattering corrections allows the position of each zero to be varied so as to best fit the data. Thus, there is no need for secondary exchanges in such a picture. However, it has been noted[24] that such a simple picture breaks down in ω production and that some B exchange will probably be necessary in this type of model as well. A fit to reactions (1)–(4) using a strong-cut model is given in Ref. 25.

The preceding discussion shows that Regge cuts in some form are needed in order to fit the data for reactions (1)–(4). We now wish to investigate the constraints imposed by duality and exchange degeneracy for the pole residues. Duality and the lack of exotic resonances for $PP \to PP$ and $PV \to PV$ scattering[26,27] predict exchange degenerate nonets with $J^{PC} = 1^{--}$ and 2^{++}. We choose to impose strong exchange degeneracy which requires approximate equality of the vector and tensor trajectories as well as definite relations between the residue functions. With this requirement the presence of NWSZ's in a residue is seen to be the result of ghost-eliminating factors in the opposite signature exchange-degenerate residue. Specifically, the tensor-meson residues have zeros at $\alpha = 0, -2, -4, \ldots$ in order to avoid poles (ghosts or tachyon poles) in the physical scattering region. Strong exchange degeneracy then places these same zeros in the vector-meson residues. However, there are no poles in the vector-meson amplitudes for these values of α so the result is a series of zeros at $\alpha = 0, -2, -4, \ldots$. Similar arguments lead to NWSZ's at $\alpha = -1, -3, -5, \ldots$ for tensor-meson residues.

In a similar fashion $PV \to PV$ and $VV \to VV$ scattering imply the exchange degeneracy of the 0^{-+} and 1^{+-} nonets. Now, however, the point $\alpha = 0$ is no longer a ghost pole but is instead the pseudoscalar meson pole. Thus, neither the pseudoscalar nor pseudovector residues get a factor of α. This implies that the first NWSZ for the B is at $\alpha = -2$ while that for the pion is at $\alpha = -1$.

We have now given arguments for the existence of ρ, ω, and B exchange and their associated cuts. These are expected to be the dominant exchanges in photoproduction but there can in principle be others as well. For the sake of completeness, it is of interest to consider what the orders of magnitude may be for these other exchanges.

The photon has $C = -1$ and the pseudoscalar mesons have $C = +1$ so all the possible exchanges will have $C = -1$. Using the quark model as a guide, we rule out 0^{--}, 0^{+-}, and 2^{+-} exchanges since they do not result from $q\bar{q}$ combinations. Thus, we are left with 1^{+-}, 1^{--}, 2^{--} exchanges and the higher-spin recurrences. Unfortunately, no firm candidates for the 2^{--} nonet have been found[28] so that estimates of their couplings cannot be given. We note that the 2^{--} exchanges would couple to the f_1^- amplitude, the result of which would be to break the equality between the polarized-target asymmetry and the recoil nucleon polarization.[29] Without such data it is not possible to establish the presence of the 2^{--} nonet exchanges so we shall assume their absence. We emphasize that this is done for simplicity so as not to have any unconstrained couplings in the fit. It is interesting to note, however, that exchange degeneracy will relate the 2^{--} couplings to those of the 1^{++} nonet. A reasonable estimate of the A_1 coupling, for example,[30] would allow predictions for the 2^{--} couplings to be made.

The role of ρ and ω exchange has already been discussed so that the remaining vector meson which can contribute is the ϕ. In the Appendix we show that a reasonable upper limit for ϕ exchange is 2% of the ω amplitude and hence it too may be safely neglected.

The question now remains as to the 1^{+-} nonet structure. If it is indeed a full nonet then there will be two isoscalar members, the H and H'.[31] If one solves the constraint equations resulting from $PV \to PV$ and $VV \to VV$ scattering,[26] one obtains ideally mixed pseudoscalar and pseudovector nonets. Experimentally, the pseudoscalar mixing angle is[28] $-11°$ so this is evidence for a breaking of duality.[26] However, exchange degeneracy is at best an approximation and the degree to which it holds may be related to the thresholds of the reactions which require the degeneracy.[26] Hence, $\pi\pi$ scattering yields ρ-f exchange degeneracy and it is well satisfied.[26] The breaking of the pseudoscalar-nonet structure can thus be viewed as a consequence of the higher threshold of the $VV \to VV$ reaction. However, the mixing in the pseudovector nonet is required by $PV \to PV$ scattering which has a threshold below that of VV scattering. Thus, the nonet structure will most likely be closer to the "ideally mixed" structure with a mixing angle close to the Okubo value.[26] Since there is no estimate for the H mass, we shall assume an ideally mixed structure for the B nonet. This will effectively decouple the H' from the nucleon vertex.[32] We are thus lead to consider H exchange in addition to B exchange by the constraints of approximate exchange degeneracy.

The preceding considerations do not completely specify a model for reactions (1)–(4). Several authors have incorporated some or all of these considerations in their fits to these processes. However, there are some significant differences between this fit and its predecessors.

Gault, Martin, and Kane[25] have fit these reactions with both strong- and weak-cut models. For their weak-cut model, they used B exchange but left a factor of α in the residue. As mentioned above, this is inconsistent with the exchange degeneracy with the pseudoscalar nonet. Their resulting B amplitude was suppressed by the factor of α and even with a generous upper limit on the B strength they had dips in η photoproduction and in transverse ω production. Removal of the superfluous factor leads to a B residue that more closely matches that expected from comparison with known pion couplings.

Tran Thanh Van has proposed a model for B exchange[33] which is capable of explaining the observed dip structure in the ω density matrix elements in $\pi^+n \to \omega p$. The essential feature is a NWSZ at $t \simeq -0.2$ (GeV/c)^2 which causes a dip in ρ_{00}. This NWSZ is not compatible with the requirements of exchange degeneracy and thus seems to us to be an unattractive feature of the model. We further note that there are two independent B couplings, both of which couple to nucleon helicity-flip amplitudes in the s channel. β_1 contributes to f_2 and f_3 while β_1 and β_2 both contribute to $f_{0+,0-}$ [see Eqs. (14) and (15)]. On the other hand, the pion only couples like β_1, so that exchange degeneracy implies the existence of a factor of α multiplying β_2. Alternatively, this zero structure will result when a Veneziano model is constructed for the invariant amplitudes due to the different s dependence of the invariant amplitudes involved.[34] The resulting suppression of the β_2 residue indicates that there is only one dominant B residue. This approach is verified in a π-B degenerate model for $\pi^+p \to \rho^0\Delta^{++}$ and $\pi^+p \to \omega\Delta^{++}$.[35] Now, Tran Thanh Van *et al.*, have a sizeable value for β_2 which is not suppressed by a zero at $t = m_\pi^2$. Thus, they underestimate β_1 and hence the s-channel transverse amplitudes so that a vector-dominance-breaking factor of 2.5 is required in order to fit both ω production and η production simultaneously.[36]

One final objection to this form of B exchange comes from some recent ω-production data[18] that show a broad minimum in ρ_{00} in the t channel for $t \simeq -0.35$ (GeV/c)^2 not $t \simeq -0.2$ (GeV/c)^2. This would require a very flat trajectory and/or a high intercept, both of which are ruled out by the observed energy dependence which gives $-0.3 \lesssim \alpha_0 \lesssim 0$.[18] An alternative suggestion for the structure in the ω density matrix elements is ρ-ω interference.[35] Some preliminary calculations show that such a mechanism can have an appreciable effect on the density matrix elements.

Worden[37] has recently proposed a model for pion and η photoproduction which utilizes a modified form of absorption in which a sum of Gaussians is used to approximate a sharp cutoff in impact-parameter space so as to prevent the overabsorption of the lower partial-wave amplitudes. The absorption we have used is based on conventional Pomeranchukon exchange with a nonzero value for the Pomeranchukon trajectory slope as suggested by recent proton-proton scattering results.[8] This form resulted from a fit to the $I = 0$ πN scattering amplitude using Pomeranchukon and f exchange and their respective first-order cuts.

Worden has included B and H exchange as suggested by duality and SU(3) and all the amplitudes have the NWSZ structure required by duality and exchange degeneracy. Each residue is given an arbitrary t dependence which is then constrained to fit the available cross section and polarization data as well as finite-energy sum rules. We have adopted an alternative approach wherein a smaller set of parameters is used to fit a larger set of reactions. We feel that this yields more information concerning the relative importance of each exchange even though the uncertain t dependence of each residue is not taken into account.

III. FORMALISM

We now proceed to parametrize the ω, ρ, B, and H exchanges as well as to give the expressions used in calculating the various Regge cuts. For vector exchanges we have

$$\begin{aligned} f_1 &= f_4 \\ &= \frac{\beta_1^V}{\Gamma(\alpha_V)}\,\Delta\,\frac{1-e^{-i\pi\alpha_V}}{\sin\pi\alpha_V}\,\nu^{\alpha_V} e^{-\Delta^2 c_V/2}, \\ f_2 &= -f_3 \\ &= \frac{-\beta_2^V\Delta^2}{2m\Gamma(\alpha_V)}\,\frac{1-e^{-i\pi\alpha_V}}{\sin\pi\alpha_V}\,\nu^{\alpha_V} e^{-\Delta^2 c_V/2}, \end{aligned} \tag{12}$$

where $\nu = (s-u)/2$ and $\Delta^2 = -t + t_{\min}$. β_1 and β_2 may be related to the couplings at the pole, whereby we obtain

$$\begin{aligned} \beta_1^V &= \frac{g_{\gamma V\pi} g_{VN\bar N}^V}{4\pi m_V}\,\frac{\pi\alpha_V'}{4\sqrt{2}}\,e^{-c_V m_V^2/2}, \\ \beta_2^V &= \frac{g_{VN\bar N}^T}{g_{VN\bar N}^V}\,\beta_1^V, \quad V = \rho, \omega. \end{aligned} \tag{13}$$

As mentioned previously, there are two pseudovector couplings, only one of which is of relevance to photoproduction processes. For the transverse amplitudes we have

$$\begin{aligned} f_1 &= f_4 = 0, \\ f_2 &= f_3 \\ &= \frac{\beta_1^A\Delta^2}{2m\Gamma(\alpha_A+1)}\,\frac{1-e^{-i\pi\alpha_A}}{\sin\pi\alpha_A}\,\nu^{\alpha_A} e^{-\Delta^2 c_A/2}, \end{aligned}$$

where

$$\beta_1^A = \frac{g_{\gamma A\pi}^1 g_{AN\bar N}}{4\pi m_A}\,\frac{\pi\alpha_A'}{4\sqrt{2}}\,e^{-m_A^2 c_A/2}, \quad A = B, H. \tag{14}$$

The additional coupling enters in the longitudinal amplitude for ω production which we include for completeness:

$$f_5 \equiv f_{0+,0-} = \frac{1}{\sqrt{2}\, m m_\omega}\left[\beta_2^B m_B{}^2 \alpha_B - \beta_1^B(\Delta^2 + m_\pi{}^2 - m_\omega{}^2)\right] \times \frac{1}{\Gamma(\alpha_B+1)} \frac{1-e^{-i\pi\alpha_B}}{\sin\pi\alpha_B} \nu^{\alpha_B} e^{-\Delta^2 c_B/2} . \tag{15}$$

In addition to these pole terms we also have their associated cuts generated by elastic rescattering in the final state. Vector dominance also allows one to include initial-state corrections as well.[38] The Pomeranchukon amplitude and each Regge amplitude gives rise to an eikonal of the form[1]

$$\chi(s,b) = \frac{1}{kW}\int \Delta d\Delta\, J_n(b\Delta) f(s,\Delta^2) , \tag{16}$$

where $n = \lambda_\gamma - \lambda_p + \lambda_p'$ is the net helicity flip for the amplitude. The eikonal for each pole and that of the Pomeranchukon are then combined to give the cut using

$$f(s,\Delta^2) = ik\sqrt{s}\int b db\, J_n(b\Delta)\chi_R(s,b)\chi_p(s,b) . \tag{17}$$

This expression agrees to first order with the various rescattering schemes such as the eikonal[1] model, the K-matrix model,[39] etc.

The elastic rescattering is described in terms of a moving Pomeranchukon of the form

$$f_P = ik\sqrt{s}\,\Gamma e^{-\Delta^2 R_P/2} , \tag{18}$$

where

$$R_P = 2\alpha_P'(\ln\nu - i\pi/2) + c_P .$$

c_P is the Pomeranchukon residue slope and α_P' is the slope of the Pomeranchukon trajectory which we take to be 0.4 as suggested by recent proton-proton scattering data.[8] We assume for simplicity that the elastic rescattering does not flip helicity.

Using Eqs. (12)–(18) the poles and their associated cuts may be calculated. However, the Γ function residues would necessitate numerical integrations so we make the following approximations which deviate less than 10% from the actual expressions in the region where the integrands of the cut integrals are sizable

$$\frac{1}{\Gamma(\alpha)\cos(\pi\alpha/2)} \simeq 0.64\alpha(\alpha+2) ,$$

$$\frac{1}{\Gamma(\alpha+1)\cos(\pi\alpha/2)} \simeq 0.54(\alpha+2) .$$

With the above expressions we are now able to discuss the actual fit to the data. The first step in such a fit is to fix as many of the parameters as possible which can be done by making use of previous fits of this type. The ρ trajectory is taken to be the same as in previous fits to πN,[40] KN,[41] and NN (Ref. 42) scattering and is $\alpha_\rho = 0.53 + 0.8t$. These same fits also determined the ω trajectory to be $0.45 + 1.0t$. However, in π^0 photoproduction the trajectory is very precisely determined by the location of the dip in the cross section and the polarized photon asymmetry. We found it necessary to fit the slope of the ω trajectory with the intercept being determined by $\alpha(m_\omega{}^2) = 1$. The B and H trajectories are taken to be equal to each other and to the π trajectory determined in ρ production,[43] $\alpha_\pi = 0.8(t - m_\pi{}^2)$.

In order to investigate the Pomeranchukon couplings we performed a fit to the isospin-zero πN scattering amplitude using Pomeranchukon and f exchange as well as the appropriate first-order cuts. The results were $c_P = 1.75$ GeV^{-2} and $\Gamma = 5.25$ GeV^{-2}. We chose to fix c_P at this value and to let Γ vary somewhat due to the fact that the initial-state scattering corrections may deviate somewhat from πN scattering.

Additional constraints resulting from the application of vector dominance are discussed in the Appendix. These constraints are used to fix the tensor to vector ratios for the ρ- and ω-nucleon couplings. In addition, we have found from fits to $\pi^- p \to \pi^0 n$, $K^- p \to \bar{K}^0 n$, and $K^+ n \to K^0 p$ that when $\beta_2^\rho/\beta_1^\rho$ is constrained to have the value indicated by vector dominance, $c_\rho \simeq 1.3$ GeV^{-2}. We have used this result and the vector-dominance constraints to fix these three parameters.

IV. RESULTS

Table I lists the resulting Regge-pole parameters. The underlined parameters were fixed prior to the fitting, leaving eight pole parameters to be determined by the data. In addition, the cut strength was found to be $\Gamma = 7.16$ GeV^{-2}, an increase of about 30% from the πN value. This is a common feature of weak-cut models for pion photoproduction.[38] Before analyzing the individual reactions, there are some general comments that may be made about the couplings.

TABLE I. Pole parameters for the fit to reactions 1–4. The underlined parameters were fixed prior to fitting the data.

	Exchange			
Parameter	ω	ρ	B	H
β_1 (GeV^{-1})	0.331	0.0339	0.316	0.318
β_2/β_1	<u>−0.21</u>	<u>3.71</u>	<u>0.0</u>	<u>0.0</u>
c (GeV^{-2})	0.2	<u>1.3</u>	1.5	1.5
α_0	<u>0.49</u>	<u>0.53</u>	<u>−0.016</u>	<u>−0.016</u>
α' (GeV^{-2})	0.83	<u>0.80</u>	<u>0.80</u>	<u>0.80</u>

Using the observed $\omega \to \gamma\pi$ width[28] and the vector mixing angle, $\theta_V = 40°$, we can calculate[44] $g_{\gamma\rho\pi} = 0.232\ \text{GeV}^{-1}$. Then using Eq. (13) we obtain $g_{\rho p\bar{p}} = 4.16$. This value may be compared with that expected by ρ universality[45] which, using $\Gamma(\rho \to \pi\pi) = 125$ MeV, is $g_{\rho p\bar{p}} = \frac{1}{2} g_{\rho\pi\pi} = 2.81$. This then is evidence either for a breaking of universality or some t dependence which we have not taken into account. We note, however, that $g_{\omega p\bar{p}}/g_{\rho p\bar{p}} = 2.53$ which is in agreement with SU(3) expectations if the ϕ decouples from the nucleon vertex.

Exchange degeneracy relates the B and π residues in charged photoproduction by $\text{Im} f^B = \frac{1}{3} \text{Im} f^\pi$. Using the known pion couplings,[45] we calculate $\beta_1^B \simeq 0.15\ \text{GeV}^{-1}$. Thus, our value represents a breaking of simple exchange degeneracy. A similar breaking effect has been noted in the ratio of $\pi^+ p \to \rho^0\Delta^{++}/\pi^+ p \to \omega\Delta^{++}$.[35]

The ratio of H to B coupling at the nucleon vertex is found to be $g_{Hp\bar{p}}/g_{Bp\bar{p}} = 0.459$. Assuming the ideal nonet structure this yields a d/f ratio of 1.74. Using Kim's values for the kaon couplings,[46] $g_{KN\Lambda}{}^2/4\pi = 13.5 \pm 2.2$ and $g_{KN\Sigma}{}^2/4\pi = 0.2 \pm 0.4$, as well as $g_{\pi NN}{}^2/4\pi = 14.5$ (Ref. 47) a d/f ratio of 1.6 ± 0.5 is obtained for the pseudoscalar meson-nucleon vertex. Thus, the value of the H/B ratio obtained here is consistent with that expected on the basis of exchange degeneracy. We now turn to a detailed analysis of the individual reactions.

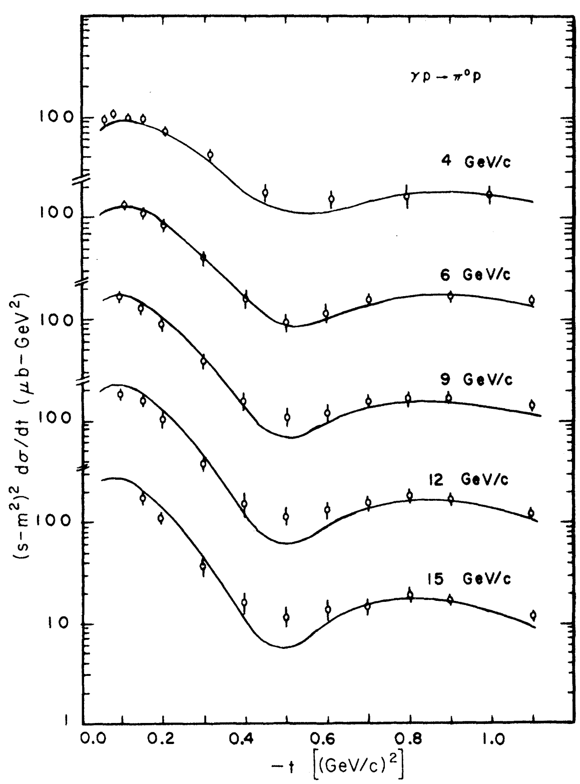

FIG. 1. Differential cross section for $\gamma p \to \pi^0 p$. The 4-GeV data are from Ref. 13 while the rest are from Ref. 7.

π^0 Photoproduction

Figure 1 shows the cross section for $\gamma p \to \pi^0 p$ over a range of energies. The over-all agreement is seen to be good with the exception of a systematic underestimation of the center of the dip. This energy dependence is due in part to our choice of a moving Pomeranchukon, since fits with $\alpha_P{}' = 0$ show less of this effect.[14] We note that at energies above 15 GeV this trend reverses and the dip fills somewhat due to the dominance of the ρ-P and ω-P cuts.

Figure 2 shows the polarized photon asymmetry which is well described by this fit. The dip due to the unnatural parity B and H exchanges is of the correct magnitude which supports our values for their residues. We note that in the energy region up to 15 GeV the depth of the dip is slowly varying with some tendency for it to become more sharply defined.

It is interesting to note that when the effective trajectories are determined using values of the photon laboratory momentum from 6 GeV/c to 15 GeV/c there is a negligible difference between the over-all effective trajectory and that part due only to natural-parity contributions. This means that the deviation from pure pole energy dependence is due almost entirely to the natural-parity parts of the ω-, ρ-, and B-Pomeranchukon cuts. Therefore, as is the case in any model incorporating absorptively generated cuts, we would find the difference between the effective trajectories for πN charge exchange and π^0 photoproduction to be due to the presence of larger cut effects in the

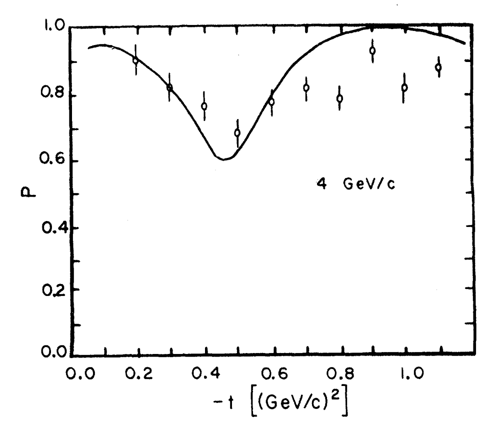

FIG. 2. Polarized-photon asymmetry for $\gamma p \to \pi^0 p$. The data are from Ref. 9.

latter reaction.[37] However, for very high energies, the cut terms dominate the pole terms except at $t=0$. So, if ρ-Pomeranchukon cuts are present in either the helicity-nonflip or helicity-flip amplitude for πN charge exchange, at high enough energies the photonic and hadronic reactions will have similar effective trajectories. With our model it is not possible to say at what energy this effect may be seen since our fits were not overly sensitive to changes in c_P, i.e., the energy scale of the absorption.

Figure 3 shows the data for R at[10] 4 GeV/c together with a typical curve from our fit. We find that our predictions for R are essentially energy-independent in the region from 4 to 15 GeV/c. In general our curve is too low indicating too much isoscalar-isovector interference. As discussed in Sec. II this indicates a need to reduce the ρ coupling in the nucleon nonflip amplitudes, f_1 and f_4. This would result in a value for the ρ tensor/vector ratio larger than predicted by vector dominance. Also, the ω tensor/vector ratio should be near zero in order to minimize the interference in the nucleon helicity-flip amplitudes, f_2 and f_3. If this is not done R will again be too low as shown, for example, in Ref. 25.

Figure 4 shows preliminary polarized-target data at[48] 4 GeV/c with our curves at 4 and 15 GeV/c. The curves are characterized by large positive values for $\Delta^2 \lesssim 0.45$ and a negative maximum near $\Delta^2 = 0.6$. The new data[48] are consistent with the preliminary data and do not show this effect. Detailed analysis of the amplitudes shows that the positive peak is due to interference between the $(\omega+\rho)$ contribution in f_1 and the $(B+H)$ cut in f_2. The particular term is that portion of the $(B+H)$ cut which does not vanish at $\Delta^2=0$. This term has an extra $\ln\nu$ factor in the denominator and will be referred to as the nonleading cut. The rapid energy dependence due to this nonleading cut is shown by the reduction in the peak in going from 4 to 15 GeV/c.

We have investigated ways of removing this peak and we find that it is a necessary consequence of having sizeable B and H contributions in this model. The peak may be trivially reduced by reducing or removing the B and H cuts altogether. This, however, reduces the depth of the negative maximum which is equally undesirable. Increasing the ρ tensor/vector ratio will also help the situation somewhat but this leads to problems in the ω-production curves. Our conclusion is that the B and H nonleading cut phase must more nearly match the phase of the ρ cut so that the positive peak will be removed and the depth of the negative maximum will be increased.

η and X^0 Photoproduction

Figure 5 shows the η photoproduction cross section over a range of energies. The general features are adequately described for small t values while a suggestion of a shoulder remains for $t \simeq -0.5$ (GeV/c)^2. The data at the larger t values are adequately fit as well in contrast to the results from Ref. 49 where a t-dependent SU(3)-breaking effect was suggested in order to explain the discrepancies in their fit. We note that our curves systematically underestimate the Cornell data at 4.00 and 8.00 GeV/c possibly indicating the need for a slightly larger value for θ_P.

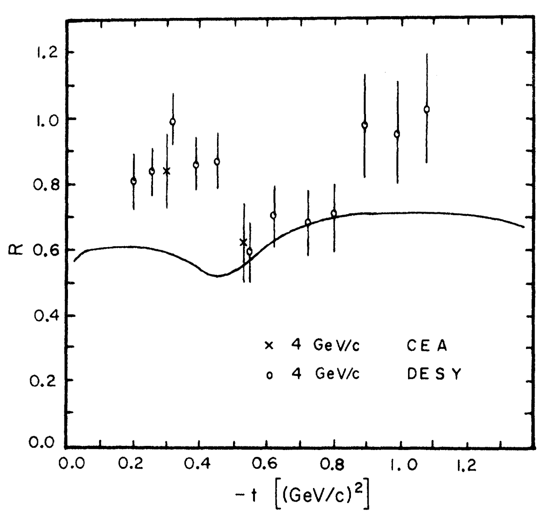

FIG. 3. The neutron/proton ratio for π^0 photoproduction. The data are from Ref. 10.

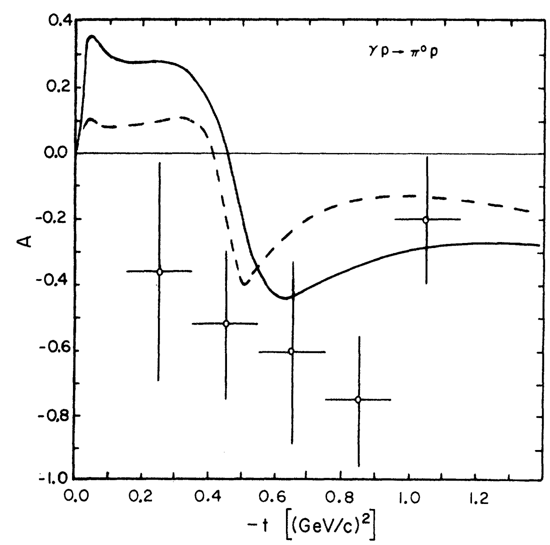

FIG. 4. Polarized target asymmetry for $\gamma p \rightarrow \pi^0 p$. The data are from Ref. 48.

The shoulder that appears near 9.00 GeV/c is characteristic of any model which uses lower-lying exchanges to fill in dips. Furthermore, the energy dependence of the absorption introduced by the nonzero Pomeranchukon slope enhances this effect. Variations in the details of the model allow the shoulder to be moved to higher energies as has been shown in a fit with a flat Pomeranchukon and a flatter B trajectory.[14] Further measurements of η photoproduction are necessary in order to determine whether or not such a shoulder appears and if so, at what energy.

The SU(3) relations needed for X^0 photoproduction are given in the Appendix. With the mixing angles we have used, they are nearly identical to those for photoproduction so that the results for both reactions are essentially identical. There is thus no need for a separate discussion or set of curves for X^0 photoproduction.

Figure 6 shows our predictions for the polarized-photon asymmetry in η (and X^0) photoproduction. The deep dip at $t=-0.5$ is due to the larger B strength and is a natural prediction of any model which uses secondary meson exchanges for dip-filling purposes. Such a deep dip is not present in models which generate dips via pole-cut interference.[25]

Figure 6 also shows the polarized-target asymmetry predictions for η photoproduction. The forward peak is now much larger and the dip at $t=-0.6$ (GeV/c)^2 is somewhat deeper. Both of these effects result from the enhanced B contribution. As in the π^0 case, the small-t region is strongly energy-dependent and the positive peak is greatly reduced at 15 GeV/c.

ρ Production

As noted previously in Sec. II, the $I=0$ contribution to ρ production may be isolated by considering[16,50]

$$X(s,t)=\frac{1}{2}\left[\frac{d\sigma}{dt}(\pi^+p\to\rho^+p)+\frac{d\sigma}{dt}(\pi^-p\to\rho^-p)-\frac{d\sigma}{dt}(\pi^-p\to\rho^0n)\right]. \tag{19}$$

In this fit $X(s,t)$ will receive contributions from both the ω and H exchanges and hence can in principle

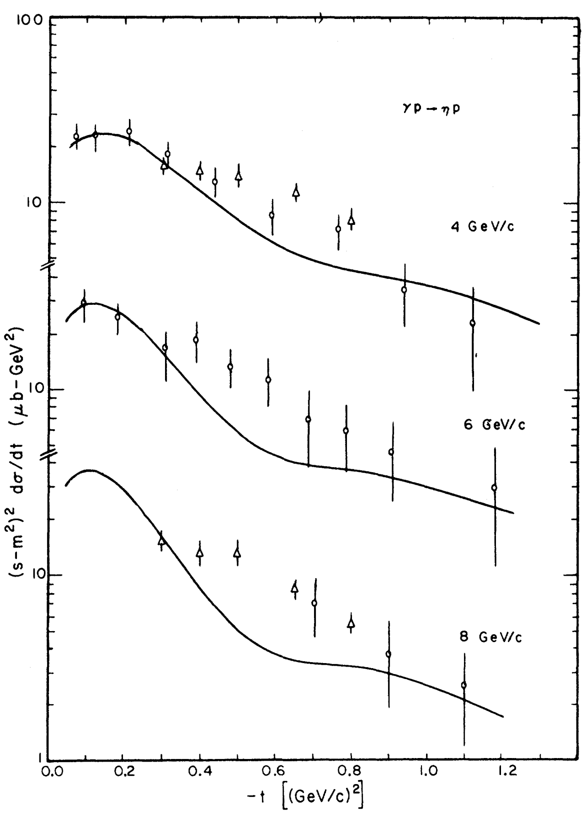

FIG. 5. Differential cross section for $\gamma p\to\eta p$. The data points denoted by Δ are from Ref. 12 and those denoted by O are from Ref. 13.

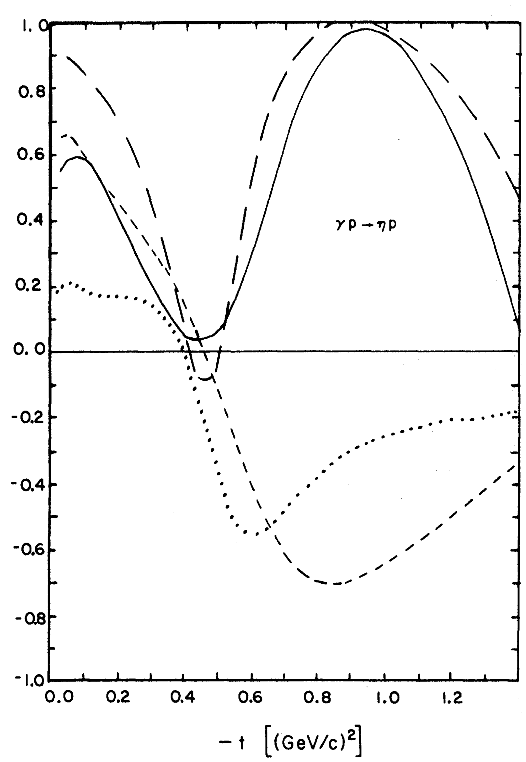

FIG. 6. Predictions for the polarized-photon asymmetry for $\gamma p\to\eta p$ at 4.00 GeV/c (solid line), and 15 GeV/c (broken line) and for the polarized-target asymmetry at 4.0 GeV/c (dashed line) and 15.00 GeV/c (dotted line).

have a longitudinal component. Since the photoproduction processes contain only the transverse amplitudes for each exchange, it would be desirable to isolate the transverse $I=0$ component by calculating

$$\rho_{11}^{S} X(s,t)=\frac{1}{2}\left[\rho_{11}^{S}\frac{d\sigma}{dt}(\pi^+p\to\rho^+p)+\rho_{11}^{S}\frac{d\sigma}{dt}(\pi^-p\to\rho^-p)-\rho_{11}^{S}\frac{d\sigma}{dt}(\pi^-p\to\rho^0n)\right] \tag{20}$$

where the density matrix $\rho_{mm'}^{S}$ is defined by

$$\rho_{mm'}^{S}=\sum_{bcd} f_{mb,cd}f^{*}_{m'b,cd}\Big/2\sum_{i=1}^{5}|f_i|^2 . \tag{21}$$

In practice this is not necessary because the ω couples only to the transverse amplitudes leaving H as the only contributor to the longitudinal part of $X(s,t)$. A recent analysis at 6.00 GeV/c (Ref. 16) is consistent with ignoring the longitudinal H component and hence assuming $X(s,t)\simeq\rho_{11}^{S}X(s,t)$. However, this may not be possible at lower energies where the H contribution may be larger. A preliminary analysis of $X(s,t)$ at 2.67 GeV/c (Ref. 51) shows that $\rho_{00}^{S}X(s,t)\neq 0$ so that there is a longitudinal component, presumably from the H.

If we assume for the moment that the H contributes primarily to the transverse amplitudes then we may use vector dominance to relate the photoproduction amplitudes to $X(s,t)$. We have

$$X(s,t)=\frac{1}{g_{\gamma\rho}{}^{2}}\frac{\pi}{sk^2}\sum_{i=1}^{4}|f_i|^2, \qquad \omega \text{ and } H \text{ only.} \tag{22}$$

Using Eq. (22) we have calculated $X(s,t)$, the results of which are shown in Fig. 7. The ω with its NWSZ is in good agreement with the data thus lending support to the simple vector-dominance picture used here. The H tends to fill the dip somewhat and at 6 GeV/c it contributes about 20% of the cross section at $t=-0.5$ (GeV/c)^2. Because the data are actually negative and hence unphysical at the location of the dip, further measurements will be necessary in order to determine whether or not this amount of H exchange is permissible. A more precise determination of the cross section at the dip is also of interest for better determining the form for Regge cuts. If, as has been suggested,[6] single helicity-flip amplitudes have no cuts but do have NWSZ's, then one would expect $X(s,t)$ to have a zero provided that $\beta_2^{\omega}/\beta_1^{\omega}=0$. If, on the other hand, weak cuts and/or secondary exchanges are present, then one would expect a small background to remain as shown in Fig. 7.

ω Production

Transverse ω production may be related to the ρ and B amplitudes in photoproduction using the vector-dominance model. We have

$$\frac{d\sigma}{dt}(\pi^0p\to\omega_{\rm tr}\,p)=\rho_{11}^{S}\frac{d\sigma}{dt}(\pi^+n\to\omega p)=\frac{1}{g_{\gamma\omega}{}^{2}}\frac{\pi}{sk^2}\sum_{i=1}^{4}|f_i|^2\times(\sin^2\theta_V+\sqrt{2}\,\cos\theta_V\,\sin\theta_V)^2, \qquad \rho \text{ and } B \text{ only.} \tag{23}$$

We have fitted $\rho_{11}^{S}d\sigma/dt(\pi^+n\to\omega p)$ using Eq. (23) with the results shown in Fig. 8. The data show no dip at $t\simeq-0.5$ (GeV/c)^2 and our fit accounts for this by including B exchange. The data are well described by our fit with at most a small deviation for $-t\gtrsim 0.9$ (GeV/c)^2 where the curve does not fall off rapidly enough. This can in fact be remedied by increasing the exponential slopes for ρ and B but the couplings must then be increased. Since we want to retain approximate π-B exchange degeneracy for small t values we have not done this. The minor deviations shown can easily be removed in a model with more complicated residue structure.

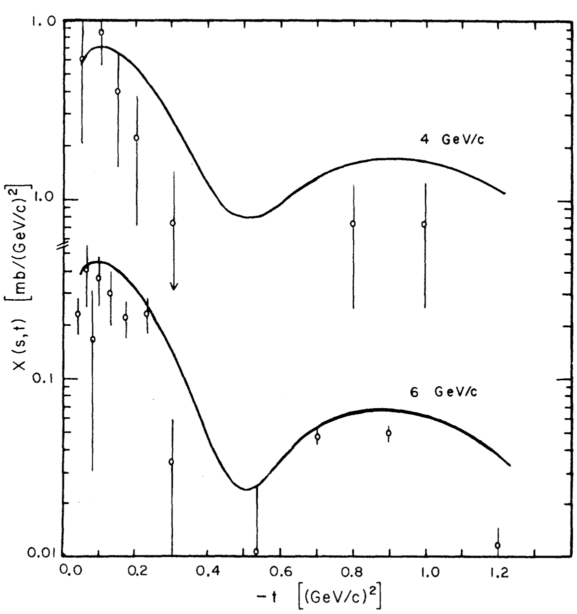

FIG. 7. The fit to $X(s,t)$ for 4.00 GeV/c (Ref. 15) and 6.00 GeV/c (Ref. 16).

Once the transverse amplitudes are determined, we have a value for β_1^B, and hence we can predict the remaining longitudinal amplitude [see Eq. (15)] if we assume $\beta_2^B = 0$. In fact, this will be true if the $B \to \omega\pi$ decay is predominantly a d-wave decay. Even if β_2^B is not zero, it will be suppressed by a factor of α as discussed previously.

Figure 9 shows our fit to the full $\pi^+ n \to \omega p$ cross section with $\beta_2^B = 0$. The data are well described by our curves thus lending support for the amplitude structure suggested by duality.

In Fig. 10 we have plotted ρ_{00}^S, ρ_{1-1}^S, and $\mathrm{Re}\rho_{10}^S$ at 5.0 GeV/c together with a representative sampling of the data.[18,19] The curves give an average description of the data but do not account for the details, which is not surprising considering the simplicity of our approach. Specifically, the apparent peaking for small t in ρ_{00}^S is not accounted for nor is the apparent structure near $t \simeq -0.2$. We emphasize the use of apparent since these features are not seen in all of the experiments,[20] and it has been suggested that some or all of the dip effects in ρ_{00}^S may be due to statistical fluctuations.[20] We note that due to angular momentum conservation $\rho_{00}^S = 0$ for $\Delta^2 = 0$ in this model. If $\rho_{00}^S \neq 0$ for $\Delta^2 = 0$, then there must be a contribution in $f_{0+,0+}$, an $n = 0$ amplitude. Just such a term appears in a model with ρ-ω interference because some π exchange is introduced and the nonleading π contri-

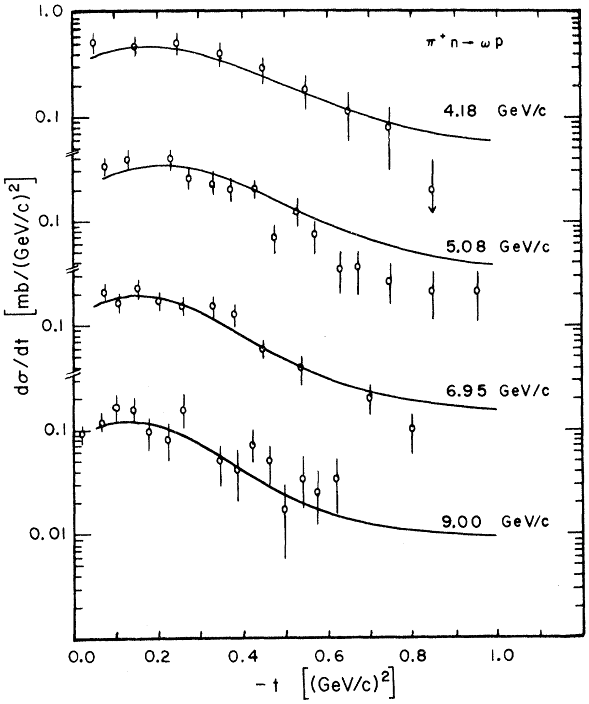

FIG. 9. The fit to the full $\pi^+ n \to \omega p$ cross section at 4.00 GeV/c (Ref. 33), 5.08 GeV/c (Ref. 20), 6.95 GeV/c (Ref. 19), and 9.00 GeV/c (Ref. 33).

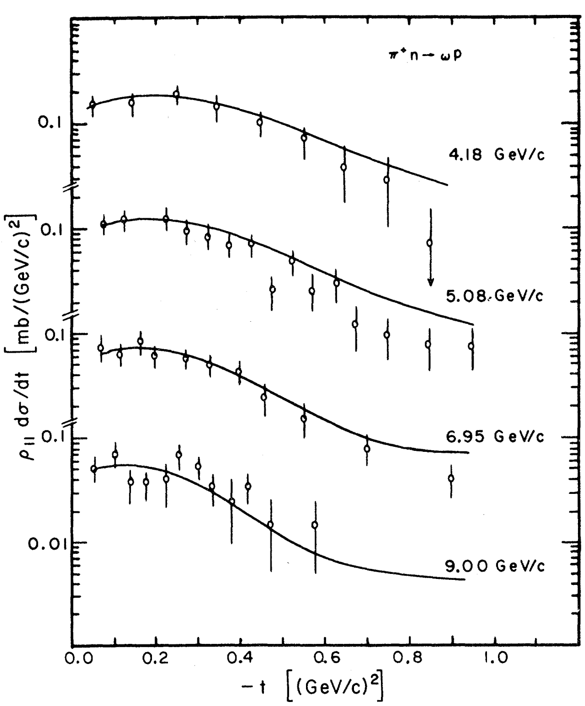

FIG. 8. The fit to $\rho_{11}^S d\sigma/dt$ ($\pi^+ n \to \omega p$). The data are from Ref. 25 and references cited therein.

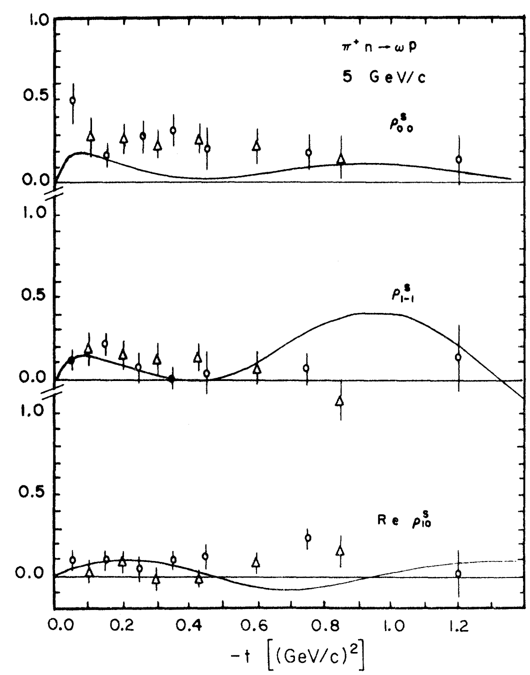

FIG. 10. Density-matrix-element predictions for $\pi^+ n \to \omega p$ at 5.0 GeV/c. The data denoted by Δ and $\bigcirc$ are at 4.5 GeV/c (Ref. 18) and 6.95 GeV/c (Ref. 19), respectively.

bution to $f_{0+,0+}$ is still important at these energies due to the proximity of the pion pole.[43] Thus, we cannot give a complete description of the density matrices with our simple model. However, we are encouraged by the fit of the model to the general features of the density matrices and we feel that it can serve as the basis for a more detailed calculation.

Figure 11 shows σ_+^S and σ_-^S at 5.00 GeV/c where

$$\sigma_\pm^S = \rho_{11}^S \pm \rho_{1\,-1}^S\,. \tag{24}$$

Asymptotically σ_+^S (σ_-^S) receives contributions only from natural- (unnatural-) parity exchanges.[5] Both σ_+^S and σ_-^S are adequately described by the fit which indicates that the unnatural- and natural-parity contributions have the correct ratio. The dip shown in σ_-^S for small t is another result of pole-cut interference in f_2 where the nonleading portion of the cut which does not vanish at $\Delta^2 = 0$ is interfering destructively with the pole. In this case, however, we have $\sigma_-^S \equiv \frac{1}{2}$ at $\Delta^2 = 0$ since from Eqs. (21) and (24) the same cut term appears in the numerator and denominator while all other terms vanish. The same effect would occur in σ_+^S except that the ρ NWSZ makes the ρ-P cut smaller and hence reduces the interference.

V. COMMENTS AND CONCLUSIONS

On the basis of this analysis we can now suggest some experiments which would help clear up some of the remaining questions.

1. Polarized target and recoil-nucleon asymmetries. These two quantities will be identical if $f_1^- = 0$. Members of the 2^{--} nonet will contribute to f_1^- and if an inequality is observed the 2^{--} residues may be estimated. An alternative explanation for an inequality would be the existence of some helicity-flip absorptive corrections. If these are present then the two $n = 1$ amplitudes will no longer be equal and hence $f_1^- \neq 0$ even if the 2^{--} exchanges are absent.

2. Polarized-photon asymmetry for $\gamma n \to \pi^0 n$. Because the B is an isovector it will change sign in $\gamma n \to \pi^0 n$ while the H will not since it is an isoscalar. Thus, the unnatural-parity term is altered if an H is present. This will show up in the asymmetry since this measures the opposite-parity interference.

3. η energy dependence. Because of the role of B exchange in η photoproduction there is a pronounced energy dependence in the region $0.4 < \Delta^2 < 0.7$. The B exchange term used here is compatible with the energy dependence observed in $\pi^+ n \to \omega p$ and its strength in η photoproduction is fixed via vector dominance and SU(3). If the energy dependence is incorrect then there must be some doubt as to the validity of the relations used.

4. ω density matrices. The present data come from experiments with 300 to 500 events each. Thus, statistical fluctuations may be the cause of some of the discrepancies concerning the dip structure. This dip is particularly important for determining the nature of the B amplitude since this is the only exchange expected in the longitudinal amplitude at high energy.

5. $X(s,t)$. Further refinements of the data in the region of the dip will help determine the cut structure of ω exchange in f_1 and f_4.

6. Polarization correlations. The use of polarized targets in conjunction with polarized beams allows a new set of measurements to be made which can often be instrumental in differentiating between various models. Figure 12 gives our predictions for 4 of these measurements. Using circularly polarized photons one can measure the asymmetry with the proton target polarized along the beam (E) or perpendicular to the beam and in the reaction plane (F). G and H are similarly defined but with photons polarized at 45° to the reaction plane. A more complete discussion of these measurements can be found in Ref. 37.

The set of predictions shown is very model-dependent with a dominant role being played by the pole-cut phase relationship. As such, these measurements may provide sensitive tests for different cut models. Without making any detailed comparisons we can note the following feature. Any NWSZ type model will have a peak in each curve at or near $t \simeq -0.5$ while a strong-cut model will give a zero for each measurement near $t \simeq -0.5$. This latter point may be understood by considering the sample strong-cut-model curves given in Ref. 25.

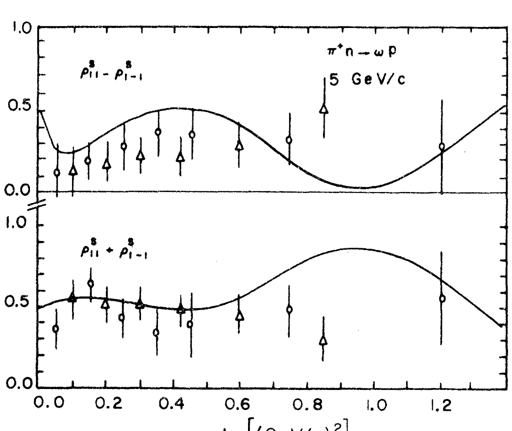

FIG. 11. The natural- and unnatural-parity cross sections for $\pi^+ n \to \omega p$ at 5.0 GeV/c. The data are as in Fig. 10.

In conclusion, we have presented a detailed calculation of neutral nonstrange pseudoscalar photoproduction and related processes which utilizes the theoretical constraints of duality, SU(3), and vector dominance. The role of secondary exchanges has been investigated and we have found that they are important in the energy region considered. A simple parametrization was used and the number of fitted parameters was kept to a minimum. We note that some improvement in detail could result from introducing separate cut strengths for each amplitude and exchange[36,25] and by introducing t-dependent residues.[37] However, the basic description and the conclusions would remain unchanged by such considerations.

The two major difficulties encountered were in fitting R and A for π^0 photoproduction. The data indicate less isovector isoscalar interference than is present in our fit to R and hence the ρ coupling will have to be altered to accomodate this feature. Our fit to the polarized-target asymmetry shows a large positive value for Δ^2 near zero. This can be attributed to the relative phase between the B and ρ nonleading cut terms. This feature appears to be a characteristic of any absorptively generated Regge-cut model which uses appreciable B and/or H exchange[37] and hence is another indication that the cut phases predicted by such a model are not necessarily correct at presently available energies.[49]

While recent work has shown the simple absorption model used here to be incorrect in detail for πN scattering,[52] we feel that the present fit provides a good approximation to the proper description of the reactions considered.

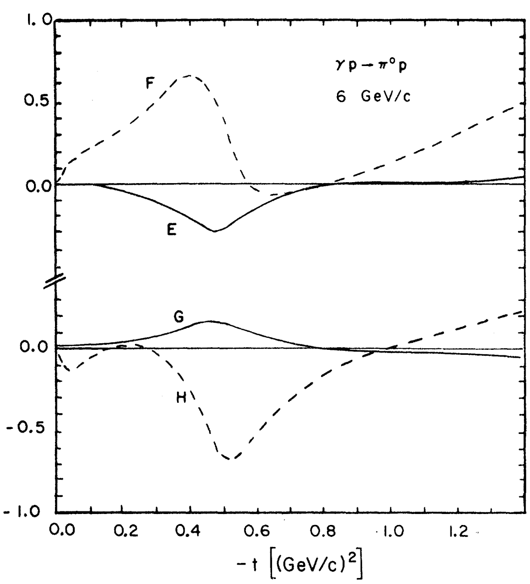

FIG. 12. Polarization correlation predictions for $\gamma p \to \pi^0 p$ at 6.00 GeV/c.

APPENDIX

In this section we give the SU(3) relations used as well as some estimates of relevant vector-meson couplings. For convenience we define the following functions of the nonet mixing angle.

$$A_X = \cos\theta_X - \sqrt{2}\,\sin\theta_X\,,$$

$$B_X = \sin\theta_X + \sqrt{2}\,\cos\theta_X\,,$$

where $X = P, V, A$ for the pseudoscalar, vector, and axial-vector nonets, respectively. For the coupling of a nonet to two octets there are three types of couplings so that we can describe the nucleon vertex by

$$\beta^f\langle[B,\overline{B}],\,V\rangle + \beta^d\langle(\{B,\overline{B}\} - \tfrac{2}{3}\langle B\overline{B}\rangle)V\rangle + \beta^s\langle B\overline{B}\rangle\langle V\rangle\,,$$

where $\langle\;\rangle$ indicates a trace. Working out the appropriate matrix multiplication gives

$$G_{\rho p\bar{p}} = -G_{\rho n\bar{n}} = (\beta^f + \beta^d)/\sqrt{2}\,,$$

$$G_{\omega p\bar{p}} = G_{\omega n\bar{n}} = (\tfrac{3}{2})^{1/2}(\beta^f - \tfrac{1}{3}\beta^d)\sin\theta_V + \sqrt{3}\;\beta^s\cos\theta_V\,,$$

$$G_{\phi p\bar{p}} = G_{\phi n\bar{n}} = (\tfrac{3}{2})^{1/2}(\beta^f - \tfrac{1}{3}\beta^d)\cos\theta_V - \sqrt{3}\;\beta^s\sin\theta_V\,.$$

The helicity-nonflip couplings have $\beta^d \simeq 0$ by vector dominance. The quark model gives $\sin\theta_V = 1/\sqrt{3}$, $\cos\theta_V = (\frac{2}{3})^{1/2}$ and $\beta_s = \beta_f$ so that the helicity-nonflip couplings become

$$G_{\rho p\bar{p}} = -G_{\rho n\bar{n}} = \beta^f/\sqrt{2}\,,$$

$$G_{\omega p\bar{p}} = G_{\omega n\bar{n}} = 3\beta^f/\sqrt{2}\,,$$

$$G_{\phi p\bar{p}} = G_{\phi n\bar{n}} = 0\,.$$

The observation of $\phi \to \rho\pi$ and $\phi \to \gamma\pi$ indicates that the mixing angle for the vector nonet is different from the quark-model value. The $\phi \to \gamma\pi$ decay rate can be described by[44]

$$\Gamma(\phi \to \gamma\pi) = \frac{m_\phi}{24}\,\frac{g_{\phi\gamma\pi}{}^2}{4\pi}\left(1 - \frac{m_\pi{}^2}{m_\phi{}^2}\right)^3.$$

With $\Gamma(\phi\to\gamma\pi)/\Gamma(\phi\to\text{all}) \leq 0.35\%$,[28] we have $|g_{\phi\gamma\pi}| \leq 0.0662$. Now, vector dominance and SU(3) give

$$R \equiv \frac{g_{\phi\gamma\pi}}{g_{\omega\gamma\pi}} = \frac{g_{\phi\rho\pi}}{g_{\omega\rho\pi}} = \frac{A_V}{B_V}\,.$$

The mixing angle is then given by

$$\tan\theta_V = \frac{1-\sqrt{2}R}{R+\sqrt{2}}\ .$$

From $\Gamma(\omega\to\gamma\pi)/\Gamma(\omega\to\text{all}) = 9.3\%$ we can get $g_{\omega\gamma\pi} = 0.695$. With the value of R thus calculated, θ_V is found to have the limits

$$35.3° \leq \theta_V \leq 40.7°,\quad R\leq 0$$

$$29.7° \leq \theta_V \leq 35.3°,\quad R\geq 0\ .$$

The (mass)^2 formula gives $\theta_V = 40°$ so we choose $g_{\phi\gamma\pi} < 0$ and $\theta_V = 40°$.

We now have a nonzero $\phi p\bar{p}$ coupling which is a function of β^s/β^f. The ratio of the ϕ and ω residues is proportional to

$$\frac{\gamma^\phi}{\gamma^\omega} \equiv \frac{g_{\gamma\phi\pi}}{g_{\omega\gamma\pi}}\frac{g_{\phi p\bar{p}}}{g_{\omega p\bar{p}}}$$

$$\leq R\,\frac{\beta^f\cos\theta_V - \sqrt{2}\,\beta^s\sin\theta_V}{\beta^f\sin\theta_V + \sqrt{2}\,\beta^s\cos\theta_V}\ .$$

If we set an upper limit on β^s/β^f by

$$3 \lesssim \frac{G_{\omega p\bar{p}}}{G_{\rho p\bar{p}}} \lesssim 5\,,$$

then

$$1 \lesssim \frac{\beta^s}{\beta^f} \lesssim 2\ .$$

Therefore,

$$\frac{\gamma^\phi}{\gamma^\omega} \lesssim 0.036\ .$$

Using ϕ-f' exchange degeneracy the ϕ trajectory is estimated to be $\alpha_\phi = 0.17 + 0.8t$. Thus, ϕ exchange is further suppressed by a factor of $\nu^{-0.3}$. For the energies under consideration the ϕ contribution will be less than 2% of the ω contribution.

As a result of the nonzero $\phi p\bar{p}$ coupling the isoscalar form factor will have ω and ϕ contributions. The helicity-flip/helicity-nonflip ratio may then be different from the usual vector-dominance prediction. However, since the ϕ coupling is expected to be small we have chosen to neglect its possible effects on β_2^ω. Motivated by a desire for simplicity we have chosen to use

$$\beta_2^\omega/\beta_1^\omega = -0.12$$

and

$$\beta_2^\rho/\beta_1^\rho = 3.71\ .$$

Vector dominance relates the photon coupling by

$$g_{\gamma\rho} = \frac{\sqrt{3}}{\sin\theta_V}\,g_{\gamma\omega} = \frac{\sqrt{3}}{\cos\theta_V}\,g_{\gamma\phi}\ .$$

Furthermore, $g_{\gamma\rho}$ is given by $g_{\gamma\rho} = e/g_{\rho\pi\pi}$. Recent estimates give $g_{\rho\pi\pi}^2/4\pi = 2.13$,[47] so that $g_{\gamma\rho} = 0.0585$. Therefore, $g_{\gamma\omega} = 0.0217$.

SU(3) relates η and X^0 photoproduction. The relevant relations are as follows:

$$\frac{\beta_\rho(\eta)}{\beta_\rho(\pi^0)} = \frac{\beta_B(\eta)}{\beta_B(\pi^0)} = \sqrt{3}\,A_P\,,$$

$$\frac{\beta_\omega(\eta)}{\beta_\omega(\pi^0)} = \frac{A_P}{3\sqrt{3}} + \frac{2A_VB_P}{3\sqrt{3}\,B_V}\,,$$

$$\frac{\beta_H(\eta)}{\beta_H(\pi^0)} = \frac{A_P}{3\sqrt{3}} + \frac{2A_AB_P}{3\sqrt{3}\,B_A}\,,$$

$$\frac{\beta_\rho(X^0)}{\beta_\rho(\pi^0)} = \frac{\beta_B(X^0)}{\beta_B(\pi^0)} = \sqrt{3}\,B_P\,,$$

$$\frac{\beta_\omega(X^0)}{\beta_\omega(\pi^0)} = \frac{B_P}{3\sqrt{3}} - \frac{2A_PA_V}{3\sqrt{3}\,B_V}\,,$$

$$\frac{\beta_H(X^0)}{\beta_H(\pi^0)} = \frac{B_P}{3\sqrt{3}} - \frac{2A_PA_A}{3\sqrt{3}\,B_A}\ .$$

The pseudoscalar mixing angle has been taken as $-10.3°$ from the (mass)^2 formula. For θ_A we have used 35.3° since it is expected that the B nonet will be moderately strongly mixed and there are as yet no measurements of the isoscalar masses which would determine the deviation of θ_A from this value.

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Method for Calculating Electromagnetic Mass Differences of Hadrons: Application to the Pion and Kaon*

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A method is presented for the calculation of hadronic electromagnetic mass differences and is applied to the calculation of $m_{\pi^\pm}-m_{\pi^0}$ and $m_{K^\pm}-m_{K^0}$. Our results, although far from conclusive, include an explanation of the observed sign and general magnitude of $m_{K^\pm}-m_{K^0}$. In contrast with the familiar Cottingham approach, the method apparently fails for $m_{\pi^\pm}-m_{\pi^0}$.

I. INTRODUCTION

The observed values of the $\Delta I=1$ electromagnetic mass differences have not yet yielded to a convincing calculation,[1] in contrast to the known $\Delta I=2$ differences, $m(\pi^+)-m(\pi^0)$ and $m(\Sigma^+)+m(\Sigma^-)-2m(\Sigma^0)$, which appear relatively eager to reveal their origin.[2] An appealing reason for this dichotomy, as first proposed by Harari,[3] is based upon the relative enhancement of the strong-interaction dynamics in channels with $I=1$. In particular, Harari observed that the existence of the $I=1$ A_2 meson implies the necessity of subtraction terms in the Cottingham[4] formulas for the $\Delta I=1$ mass differences. Although there exist arguments[5] that these subtraction terms can be ob-