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# Intersecting Real Regge Trajectories in $\pi^- p \to \pi^0 n$ and the $\rho'$ Puzzle

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A new and very simple interpretation of the reaction $\pi^- p \to \pi^0 n$ based on intersecting, real, $\rho$ and $\rho'$ Regge trajectories is proposed. The most dominant physical features of the differential cross section and polarization data directly determine the main characteristics of the $\rho'$, which turn out to be completely different from those based on a belief in parallel trajectories. The $\rho'$ trajectory thus determined is compatible with the recently discovered spin-one resonance of mass 1968 MeV.

## I. INTRODUCTION

The absence of any physical $\rho'$ particle has long been an embarrassment in Regge analysis. On the one hand, the necessity for something like a $\rho'$ trajectory in $\pi p$ and $np$ charge-exchange scattering has been well established for some time.[1,2] On the other hand, no trace of a *physical* vector meson $\rho'$ has been found in the mass region $1000 < m_{\rho'} < 1300$ MeV, where the $\rho'$ would show itself, assuming that the slope of its trajectory is the same as that of the $\rho$. The idea of the "universality" of the slopes of the Regge trajectories, which is justified only on the grounds of simplicity, is very much emphasized in the Veneziano model,[3] which makes the specific prediction $m_{\rho'} = m_f = (1264 \pm 10)$ MeV. A careful experimental search has established that there is no evidence for $\rho'$ in the above-mentioned mass region.[4]

A very simple resolution of this dilemma is suggested by the new data on polarization in $\pi p$ charge-exchange scattering at 5 and 8 GeV/$c$.[5] Since the polarization is extremely large for small negative $t$, strong interference between $\rho$ and $\rho'$ is called for. However, the polarization appears to vanish near $t \simeq -1$ (GeV/$c$)$^2$, indicating the need for the $\rho$ and $\rho'$ contributions to have the same phase, i.e., the need to have $\alpha_{\rho'} = \alpha_\rho$ for a $t$ value in the above-mentioned region. Now the $\rho$ trajectory is rather well determined from numerous sources, and it is also fairly well established that at $t=0$, $\alpha_{\rho'}(0)$ should be of the order of 0.[6] The conjunction of these known results with the need for crossing trajectories in the region $t \simeq -1$ is enough to establish that $\alpha_{\rho'}$ is an extremely flat trajectory,[7] and in consequence that a spin-one $\rho'$ particle should exist only in the very high mass region

$$m_{\rho'} \approx 2 \text{ GeV}.$$

It is remarkable that there has recently been a report by Benvenuti *et al.*[8] of a sizable "bump" in $\bar{p}p \to K_L K_S$ which they identify as a $\rho'$ of mass 1968 MeV.

It is thus evident that a simple model based on intersecting $\rho$ and $\rho'$ trajectories will provide an elegant explanation of the polarization in $\pi p$ charge-exchange scattering and at the same time resolve the dilemma of the $\rho'$ by predicting for it a very large mass.

Armed with the basic idea outlined above and reinforced by the experimental indication of a new particle compatible with our proposed picture of the $\rho'$, we have constructed an extremely simple model for $\pi p$ charge-exchange scattering involving a minimal number of parameters and in which the $t$ variation of all functions is kept to an absolute minimum. A remarkable feature is that we are able to establish a close link between the structure of the various physical features of the scattering and the values of individual parameters of the model. Thus it will be shown that qualitative arguments alone suffice to pin down the form of the model, and the detailed computer fits to the data serve only to provide a quantitative confirmation of these arguments.

It would perhaps be well to point out that the model suggested here, with intersecting *real* $\rho$ and $\rho'$ trajectories, has almost nothing in common with some of the recent models based on the use of *complex* trajectories.[9] Indeed a certain prejudice has built up that trajectories which intersect have to become complex. This is largely a result of an unintended nuance in the original paper on the subject,[10] from which one could easily draw the false conclusion that the case of trajectories crossing and becoming complex is in some sense "more natural" than the case in which they remain real. It is perfectly obvious, though, that there is nothing at all untoward in the picture of real intersecting trajectories.

In this connection, it is interesting to note that in practice it is not even possible to construct a *simple* model with *complex* trajectories which is compatible with certain general features of the data. A plot of the "effective" trajectory, $\alpha_{\rm eff}$, versus $t$ supports the picture of crossing trajectories, since it indicates a break, or change of slope, in the region $t \lesssim -0.7$ (see Figs. 1 and 2). However, if one tries to use complex intersecting trajectories then firstly it requires very strongly $t$-dependent imaginary parts of $\alpha$ in order to get $\alpha_\rho$ and $\alpha_{\rho'}$ to pass satisfactorily through the $\rho$, $g$, and $\rho'$ particles, respectively, and secondly one does not get zero polarization at the point of intersection, so that one loses the physical connection between the polarization zero and the point of intersection of the trajectories.

In Sec. II the model is defined and in Sec. III it is shown how the detailed properties of the model can be deduced from the structure of the data. Section IV discusses the quantitative numerical fit to the data, and in Sec. V a comparison with other models of $\pi^- p \to \pi^0 n$ is made. In Sec. VI we comment rather speculatively about other reactions in which the mechanism of intersection of Regge trajectories could play a role. Conclusions follow in Sec. VII.

## II. DEFINITION OF THE MODEL

We deal throughout this paper with the amplitudes $A'^{(-)}$ and $B^{(-)}$ in terms of which one has

$$A'(\pi^- p \to \pi^0 n) = -\sqrt{2}\, A'^{(-)}\,,$$
$$B(\pi^- p \to \pi^0 n) = -\sqrt{2}\, B^{(-)}\,, \qquad (1)$$

and we shall, for simplicity, write $A'^{(-)}$ and $B^{(-)}$ as $A'$ and $B$ everywhere in what follows.

The differential cross section and polarization for $\pi^- p \to \pi^0 n$ are then given by

$$\frac{d\sigma}{dt} = \frac{m^2}{8\pi s q^2}\left[\left(1-\frac{t}{4m^2}\right)|A'|^2 - \frac{t}{4m^2}\left(\frac{4m^2 p_L{}^2 + st}{4m^2 - t}\right)|B|^2\right], \qquad (2)$$

$$P\frac{d\sigma}{dt} = -\frac{\sin\theta}{8\pi\sqrt{s}}\,\mathrm{Im}(A'B^*)\,, \qquad (3)$$

where $q$ and $p_L$ are the c.m. and lab momenta of the pion respectively.

The difference of the $\pi^- p$ and $\pi^+ p$ total cross sections is

$$\Delta\sigma \equiv \sigma(\pi^- p) - \sigma(\pi^+ p) = \frac{2}{p_L}\,\mathrm{Im}A' \quad (t=0)\,. \qquad (4)$$

We put

$$A' = A'_\rho + A'_{\rho'}\,,$$
$$B = B_\rho + B_{\rho'} \qquad (5)$$

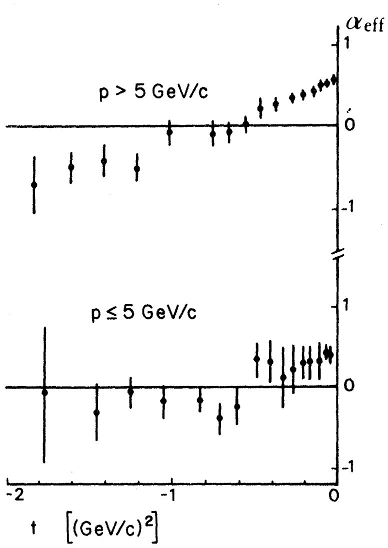


FIG. 1. Plot of $\alpha_{\rm eff}$ versus $t$.

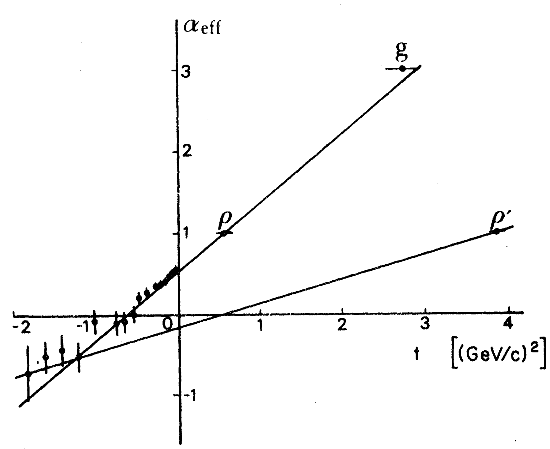


FIG. 2. Chew-Frautschi plot of $\alpha_\rho$ and $\alpha_{\rho'}$ showing their relation to $\alpha_{\rm eff}$.

in which

$$A'_\rho = a_\rho(t)(1+\alpha_\rho)[i+\tan(\tfrac{1}{2}\pi\,\alpha_\rho)](s/s_\rho)^{\alpha_\rho}\,,$$
$$B_\rho = b_\rho(t)\alpha_\rho(1+\alpha_\rho)[i+\tan(\tfrac{1}{2}\pi\,\alpha_\rho)](s/s_\rho)^{\alpha_\rho-1}\,, \qquad (6)$$

and analogous expressions for the $\rho'$ contribution. The functions $a(t)$ and $b(t)$ are reduced residue functions and it will be a basic requirement of the model to make them as simple and constant as possible. The expressions used in (6) are quite standard and do not merit detailed discussion here. $s_\rho$ and $s_{\rho'}$ are scale factors. Substituting (6) and (5) into (3) gives a very highly instructive expression for the polarization

$$P\frac{d\sigma}{dt} = \frac{\sin\theta}{8\pi s^{3/2}}\left(\frac{s}{s_\rho}\right)^{\alpha_\rho}\left(\frac{s}{s_{\rho'}}\right)^{\alpha_{\rho'}}(1+\alpha_\rho)(1+\alpha_{\rho'})(\tan\tfrac{1}{2}\pi\,\alpha_\rho - \tan\tfrac{1}{2}\pi\,\alpha_{\rho'})(s_\rho a_\rho \alpha_{\rho'} b_{\rho'} - s_{\rho'} a_{\rho'} \alpha_\rho b_\rho) \qquad (7)$$

in which it is assumed that $\alpha_\rho(t)$ and $\alpha_{\rho'}(t)$ are real, linear trajectories.

## III. PHYSICAL DETERMINATION OF THE PARAMETERS

We shall demonstrate in this section how the gross qualitative features of the data can be used to determine the essential properties of the parameters occurring in Eq. (6).

(i) The assumption that $\alpha_\rho(t)$ is a linear trajectory

$$\alpha_\rho(t) = \alpha_\rho(0) + \alpha_\rho' t \qquad (8)$$

and that it passes through the values 1 and 3 at $t = m_\rho^2 = 0.585\ \text{GeV}^2$ and $t = m_g^2 = 2.76\ \text{GeV}^2$ respectively,[11] serves to fix $\alpha_\rho(t)$ almost completely; i.e., one has

$$\alpha_\rho(0) \simeq 0.5,$$
$$\alpha_\rho' \simeq 0.9\,. \qquad (9)$$

(ii) The assumption that $\alpha_{\rho'}(t)$ is also linear, i.e.,

$$\alpha_{\rho'}(t) = \alpha_{\rho'}(0) + \alpha_{\rho'}' t\,, \qquad (10)$$

together with the requirement $\alpha_{\rho'} = 1$ at the mass of the newly discovered $\rho'$ meson,[12] i.e., at $t = m_{\rho'}^2 = 3.88\ \text{GeV}^2$ relates $\alpha_{\rho'}(0)$ to $\alpha_{\rho'}'$. By looking at the plot of $\alpha_{\text{eff}}$ versus $t$ (Figs. 1 and 2) or alternatively by looking at the region in which the polarization vanishes (Fig. 5) and associating this vanishing with the intersection of the $\rho$ and $\rho'$ trajectories, one can roughly pin down the point of intersection (let us call it $t_I$) to $t_I \approx -1$ and thereby obtain separate information on $\alpha_{\rho'}(0)$ and $\alpha_{\rho'}'$. One has

$$\alpha_{\rho'}(0) \lesssim 0,$$
$$\alpha_{\rho'}' \approx 0.25\,. \qquad (11)$$

It should be stressed that the break in $\alpha_{\text{eff}}$ and the vanishing of the polarization are two quite independent pieces of experimental information so that their compatibility with the single physical mechanism of intersecting trajectories is noteworthy.

In fact, one can go even further. The intersection of two real trajectories will rather generally lead to some sort of structure in $d\sigma/dt$ as well. We shall see in (vi) that the second maximum of $d\sigma/dt$ can also be linked to the intersection point.

(iii) From data on the electromagnetic form factors of the nucleon one has long had some information on the signs and magnitudes of the $\rho N\bar{N}$ electric and magnetic coupling constants. Translated in terms of our parameters one has, at $t = m_\rho^2$,

$$a_\rho(m_\rho^2) > 0\,, \quad b_\rho(m_\rho^2) > 0, \quad \text{with } b_\rho \gg a_\rho\,. \qquad (12)$$

In order to make our model as simple as possible and to introduce the minimum number of parameters we should like to be able to take $a_\rho(t)$ and $b_\rho(t)$ as constants (call them $a_\rho$ and $b_\rho$, respectively), in which case we would have (12) holding for all $t$, i.e.,

$$a_\rho > 0, \quad b_\rho > 0, \quad b_\rho \gg a_\rho\,. \qquad (13)$$

We shall see in (vii) below that this is impossible and that $a_\rho(t)$ must change sign in the physical region.

(iv) Since the total cross section difference defined in (4) is positive and since it varies approximately like

$$\Delta\sigma = d/\sqrt{s} \qquad (14)$$

with $d > 0$ (see Fig. 3) we must have, from (4) and (6), that

$$a_\rho(0) > 0 \qquad (15)$$

which is, of course, compatible with (13).

(v) From the shape of $d\sigma/dt$ at very small $t$ (see Fig. 4) it is well known that the spin-flip amplitude dominates the nonflip amplitude, i.e.,

$$|b_\rho(t)| \gg |a_\rho(t)| \qquad (16)$$

in a region near the forward direction. One would then naturally expect the term in Eq. (7) involving $b_\rho$ to dominate the polarization for very small $t$. Then, using (7) and the fact that the polarization

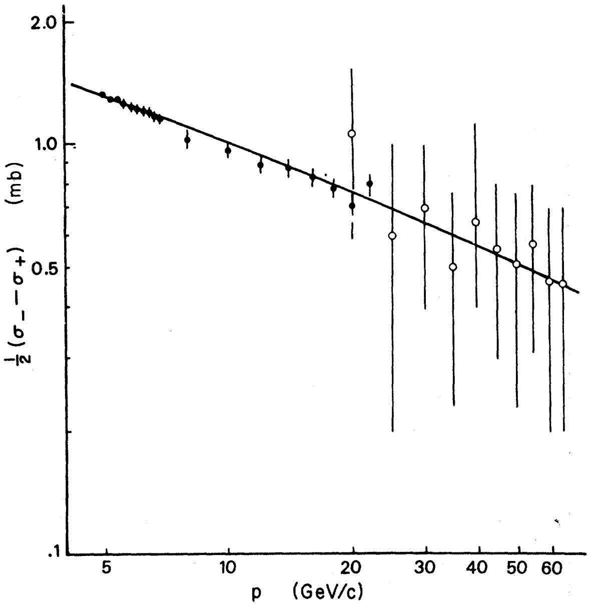


FIG. 3. Fit to the total cross-section difference $\frac{1}{2}(\sigma\{\pi^- p\} - \sigma\{\pi^+ p\})$ versus laboratory momentum.

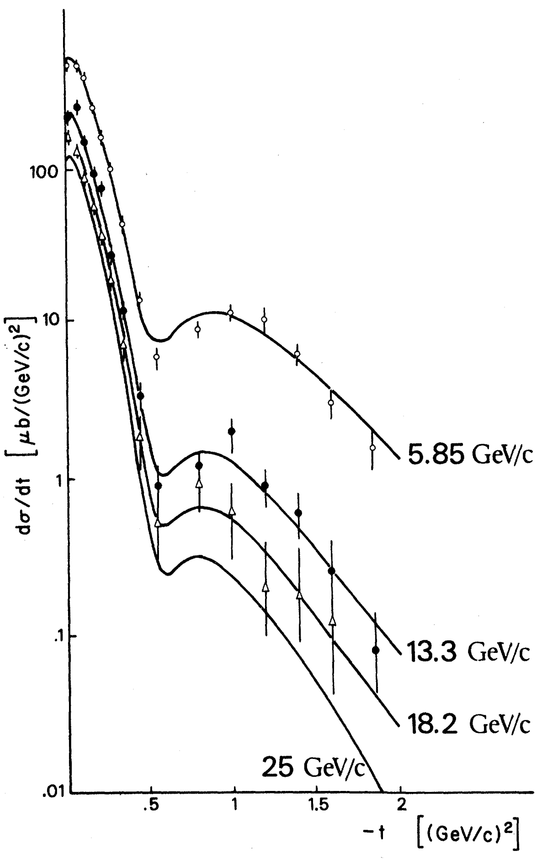


FIG. 4. Fit to the differential cross section for $\pi^- p \to \pi^0 n$. Also shown is the prediction for a lab momentum of 25 GeV/$c$.

is positive for very small $t$, we can assume that

$$a_{\rho'}(t) b_\rho(t) < 0 \tag{17}$$

for very small $t$. However, $b_\rho(t)$ is multiplied by $\alpha_\rho(t)$ in (7) so that as we move out to the point at which $\alpha_\rho(t) = 0$, i.e., $t \approx -0.6$, it must be the term involving $a_\rho \alpha_{\rho'} b_{\rho'}$ which then controls the polarization. But as shown in Fig. 5, $P$ is large and positive in the region $t \sim -0.6$ and thus, since $\alpha_{\rho'} < 0$, we must have

$$a_\rho(t) b_{\rho'}(t) < 0 \tag{18}$$

over some region around $t = -0.6$.

(vi) Consider now the second maximum of $d\sigma/dt$ which occurs at $t = -0.9 \pm 0.1$. Since $A'$ (and similarly $B$) can be considered in an Argand diagram as a vector given by the sum of the vectors $A'_\rho$ and $A'_{\rho'}$ it is clear that as the directions of $A'_\rho$ and $A'_{\rho'}$ vary, a vector $A'$ of maximum length will result whenever $A'_\rho$ and $A'_{\rho'}$ are parallel and pointing in the same direction. In other words we expect that $|A'|$ will reach a maximum when the phases of $A'_\rho$ and $A'_{\rho'}$ are the same. To produce the second maximum we thus require

$$\alpha_{\rho'}(t) \approx \alpha_\rho(t), \tag{19}$$

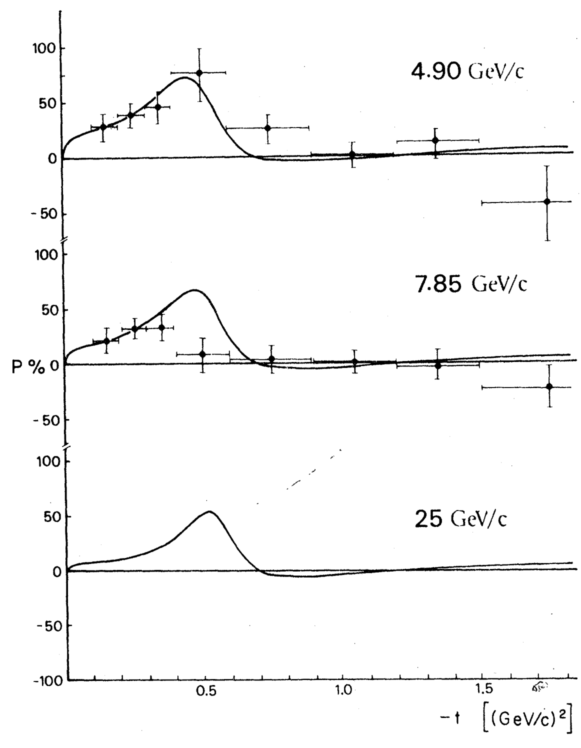


FIG. 5. Fit to the new polarization data for $\pi^- p \to \pi^0 n$. Also shown is the prediction for lab momentum of 25 GeV/$c$.

$$a_\rho(t)a_{\rho'}(t) > 0\,, \tag{20}$$

and

$$b_\rho(t)b_{\rho'}(t) > 0 \tag{21}$$

in a region around $t = -0.9$.

(vii) We can now ask whether it is possible to make the ultimate simplification of taking all of $a_\rho$, $b_\rho$, $a_{\rho'}$, and $b_{\rho'}$ independent of $t$. Comparing (17) and (21) under the assumption that $a_{\rho'}$, $b_{\rho'}$, and $b_\rho$ are constants leads to

$$a_{\rho'}b_{\rho'} < 0\,, \tag{22}$$

which is not incompatible with any other conditions on $a_{\rho'}$ or $b_{\rho'}$. On the other hand, the regions involved in (18) and (21) essentially overlap, so that $b_{\rho'}(t)$ can be eliminated, giving

$$a_\rho(t)b_\rho(t) < 0 \quad \text{for} \quad -0.9 \lesssim t \lesssim -0.6$$

which would contradict (12) if we tried to take both $a_\rho(t)$ and $b_\rho(t)$ as constants. Thus, bearing in mind (15), either $a_\rho(t)$ changes sign somewhere in the region $t<0$ or $b_\rho(t)$ changes sign for $t < m_\rho^2$. Since $b_\rho(t)$ dominates $d\sigma/dt$, a zero in $t<0$ would certainly be incompatible with the data. So *if* $b_\rho$ has the zero, it lies in $0 \leq t < m_\rho^2$. But, we have

$$|b_\rho(t)| \gg |a_\rho(t)|$$

both at $t = m_\rho^2$ *and* at $t \lesssim 0$ so a zero in $b_\rho(t)$ would imply that $b_\rho(t)$ varies extremely strongly with $t$. We thus choose the smoother of the two possibilities and take

$$b_\rho(t) \equiv b_\rho > 0 \tag{23}$$

and

$$a_\rho(t) = a_\rho(0)(1+ct) \quad (c>0). \tag{24}$$

From (23) and (17) we can now take

$$a_{\rho'}(t) \equiv a_{\rho'} < 0 \tag{25}$$

and from (23) and (21)

$$b_{\rho'}(t) \equiv b_{\rho'} > 0. \tag{26}$$

Moreover (24), (26), and (18) now imply a value for $c$ such that zero of $a_\rho(t)$ lies in the range $-0.6 \lesssim t < 0$. In practice the zero in $a_\rho(t)$ is much more closely determined than implied above, since a zero at very small $t$ or very close to $-0.6$ gives too large or too small a value for $d\sigma/dt$ at the dip, respectively.

(viii) Lastly, we note that the exponential cutoff in $t$ is provided in Eq. (6) by the factors

$$(s/s_\rho)^{\alpha_\rho(t)} \quad \text{and} \quad (s/s_{\rho'})^{\alpha_{\rho'}(t)}$$

or more precisely by

$$(s/s_\rho)^{\alpha_\rho' t} \quad \text{and} \quad (s/s_{\rho'})^{\alpha_{\rho'}' t}\,.$$

Since experimentally the falloff of $d\sigma/dt$ is extremely sharp and is known to be compatible with a $\rho$ contribution in which $s_\rho \approx 1$ GeV$^2$, and since $\alpha_{\rho'}' < \alpha_\rho'$ it is clear that we must have $s_{\rho'} < s_\rho$ in order that the $\rho'$ contribution cut itself off sufficiently fast in $t$.

In summary, we see that granted the assumption of maximal simplicity, and smoothness for the model parameters, there exists a very close and direct connection between the values of the parameters and the gross features of the experimental data. Our final model is then defined by Eqs. (23)–(26).

## IV. QUANTITATIVE FIT TO THE DATA

With the parameters given by the simple forms discussed in Sec. III, we have carried out a numerical fit to the data using the MINUIT minimization program on the University of London CDC 6600.

We have utilized the following data:

(a) The difference between $\sigma(\pi^- p)$ and $\sigma(\pi^+ p)$ for

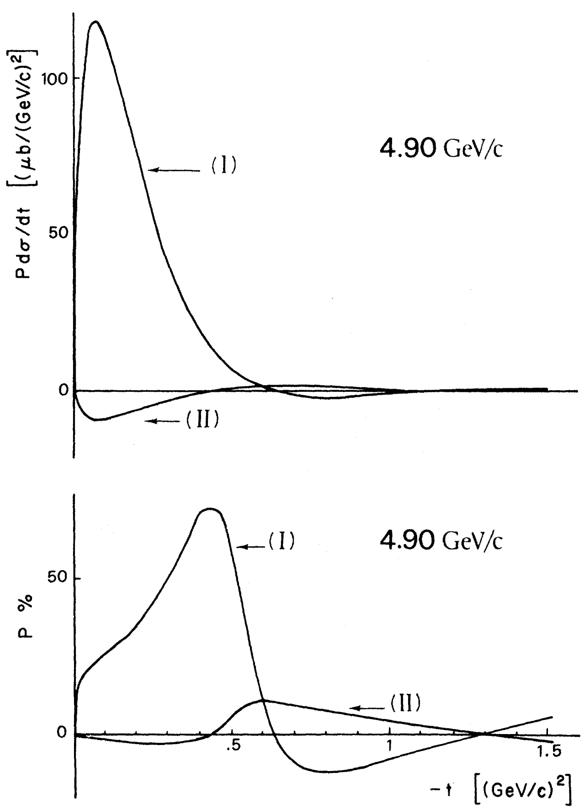


FIG. 6. Relative contributions to $P$ and $d\sigma/dt$ of the two terms I and II of Eq. (7), as defined in Sec. IV.

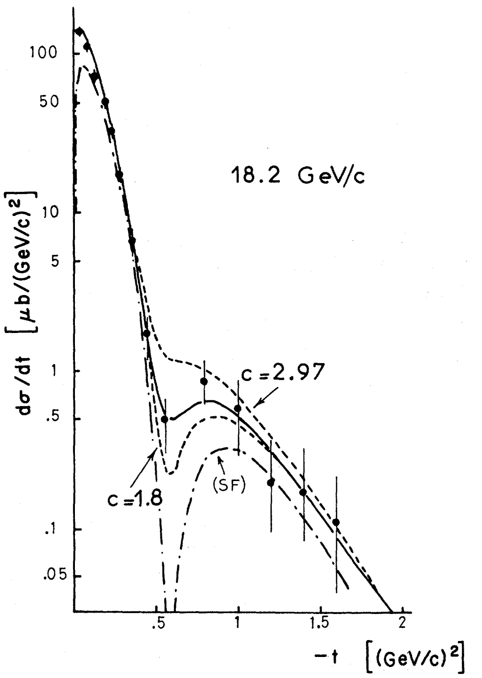


FIG. 7. Sensitivity of $d\sigma/dt$ to variations in $c$ or, equivalently, to variation in the position of the zero in $A'_\rho$. Also shown is the contribution of the spin-flip amplitude to $d\sigma/dt$.

$5 \leq p_L \leq 65$ GeV/$c$ (see Refs. 13 and 14).

(b) Measurements of $d\sigma/dt$ for

$-2 \leq t \leq 0$ (GeV/$c$)$^2$

and

$5.85 \leq p_L \leq 18.2$ GeV/$c$ (see Ref. 15).

(c) The new polarization measurements [5] for

$-2 \leq t \leq 0$ (GeV/$c$)$^2$

and

$p_L = 4.90$ and $7.85$ GeV/$c$ .

The best fit to these data is shown respectively in Figs. 3, 4, and 5. It can be seen that an excellent fit is obtained. The $\chi^2$ value is 1.3 per data point. The final value of the parameters are listed below.

$$\rho \text{ trajectory } \begin{cases} \alpha_\rho(0) = 0.51\,, \\ \alpha_\rho{}' = 0.8 \text{ (GeV}/c)^{-2}. \end{cases}$$

Scale factor: $s_\rho = 0.72$ GeV$^2$,

$$\rho' \text{ trajectory } \begin{cases} \alpha_{\rho'}(0) = -0.15\,, \\ \alpha_{\rho'}{}' = 0.3 \text{ (GeV}/c)^{-2}. \end{cases}$$

Scale factor: $s_{\rho'} = 0.47$ GeV$^2$,

$$\rho \text{ residues } \begin{cases} a_\rho(t) = 70(1 + 2.27t) \ (\mu\text{b})^{1/2}, \\ b_\rho = 3129 \ (\mu\text{b})^{1/2} \ (\text{GeV}/c)^{-1}. \end{cases}$$

This corresponds to $c = 158.9$ (GeV/$c$)$^{-2}$ in Eq. (24).

$$\rho' \text{ residues } \begin{cases} a_{\rho'} = -139 \ (\mu\text{b})^{1/2}, \\ b_{\rho'} = 1624 \ (\mu\text{b})^{1/2} \ (\text{GeV}/c)^{-1}. \end{cases}$$

It is seen that the best-fit parameters are in complete agreement with the qualitative estimates made for them in Sec. III on the basis of the dominant features of the experimental data.

The relative contribution of the terms involving $a_{\rho'}\alpha_\rho b_\rho$ and $a_\rho\alpha_{\rho'}b_{\rho'}$, to the polarization [see Eq. (7)] is of some interest. These contributions (labeled I and II respectively) to $Pd\sigma/dt$ and $P$ are shown in Fig. 6.

We have also examined the sensitivity of our results to variations in $c$ [see Eq. (24)] and $\alpha_{\rho'}{}'$.

The effect of these variations is shown in Figs. 7, 8, and 9. As mentioned in (vii) of Sec. III the value of $c$ [or equivalently the position of the zero in $a_\rho(t)$] is rather strongly restricted. In Fig. 7 we show also to what extent the spin-flip amplitude dominates the differential cross section.

It should be noted that the energy variation of the polarization, as given by a literal interpretation of the data point at $t = -0.5$ (see Fig. 5), seems quite unphysical, and would be almost impossible to reproduce in any Regge-like model. If future experiments give a smaller polarization at $p_L = 5$ GeV/$c$ and $t = -0.5$, say $P \sim (30-50)\%$, then the energy variation of $P$ will be completely compatible

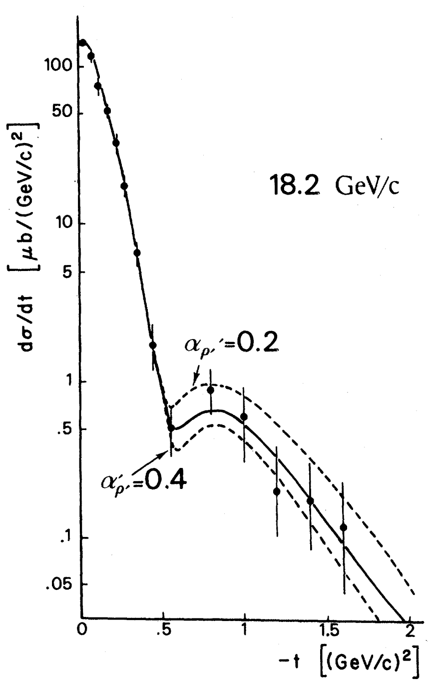


FIG. 8. Sensitivity of $d\sigma/dt$ to variations in $\alpha_{\rho'}{}'$.

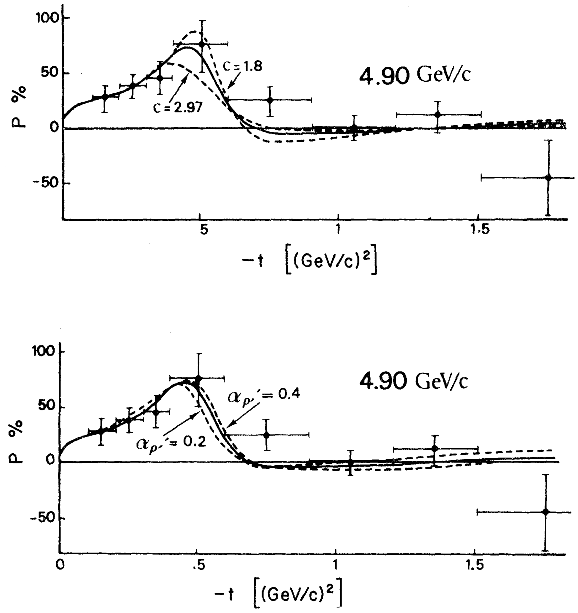


FIG. 9. Sensitivity of $P$ to variations in $c$ and $\alpha_{\rho'}'$.

with that given by the $\rho+\rho'$ model and one will have greater flexibility in fixing the parameters; in particular one will have the zero in $A_\rho'(t)$ at a smaller value of $|t|$.

## V. COMPARISON WITH OTHER MODELS OF $\pi^- p\to\pi^0 n$

It is of some interest to compare our interpretation of charge-exchange scattering with those of earlier work.

### A. Barger and Phillips Model

These authors [6] made an analysis of $\pi N$ scattering in terms of five Regge poles ($P$, $P'$, $P''$, $\rho$, and $\rho'$). They supplemented the high-energy data with low-energy data by using continuous-moment sum rules.

One of the interesting points concerning the isospin-1 exchanges is the fact that the $\rho$ and $\rho'$ parameters tend to be correlated. Therefore, in order to separate the $\rho$ and $\rho'$ contributions to the amplitudes, Barger and Phillips were forced to *assume* that the $\rho$ and $\rho'$ trajectories are spaced well apart and have *similar* slopes; i.e., their *nonintersecting* $\rho$ and $\rho'$ are not a result of the continuous-moment sum rules, but simply an hypothesis of their model. In our model the very different slopes of the $\rho$ and $\rho'$ Regge trajectories are a consequence of the connection between the *zero in the polarization* near $t\simeq -1$ $(\text{GeV}/c)^2$ and the *intersection* of the trajectories in this $t$ region; the intercepts of $\rho$ and $\rho'$ being *similar* to those obtained by Barger and Phillips.

Another difference lies in the physical interpretation of the $\rho'$. There being no $\rho'$ resonance at $m_{\rho'}\approx 1000$ MeV Barger and Phillips assumed that what is fitted as a "$\rho'$ Regge pole" is really an "effective $\rho'$ pole," i.e., a combination of poles and cuts.

It is not possible, in our case, to *prove* that the $\rho'$ is a pole. However, the existence of a potential $\rho'$ resonance at 1968 MeV, as discussed earlier, suggests that perhaps the pole picture is the correct one. A further comment on this question is made in Sec. V D.

Finally we want to underline that a clear-cut experimental test in order to distinguish between the Barger and Phillips model and our model can be made by careful measurements of the polarization in $\pi^- p\to\pi^0 n$ for more energies and large $t$. Namely, in both models the energy variation of the polarization is formally the same,

$$P\propto s^{\alpha_{\rho'}(t)-\alpha_\rho(t)}=s^{\alpha_{\rho'}(0)-\alpha_\rho(0)-(\alpha_\rho'-\alpha_{\rho'}')t}\ .$$

However, in the Barger and Phillips model the slopes of the $\rho$ and $\rho'$ are the same, and they predict a *rapid* and $t$-independent decrease of the polarization with energy, while in our model $\alpha_\rho' - \alpha_{\rho'}'\sim 0.5$ and therefore we predict a *slow* decrease with energy at larger $|t|$. New experiments on the polarization in $\pi^- p\to\pi^0 n$ would therefore be extremely interesting.

### B. Kogitz and Logan Model

This model [16] includes a $\rho$ Regge pole plus a "background" term, which is represented by a *fixed-pole* singularity, $\tilde\alpha_{\rho'}=-0.47$ which is assumed to contribute only to the $A'$ amplitude. The Kogitz and Logan model presents two features in common with our model: (a) For small $t$ the scattering amplitude is dominated by the $\rho$ Regge term, while for large $t$ the "background" ($\rho'$) term dominates; (b) the zero in the polarization at $t\approx -1$ $(\text{GeV}/c)^2$ is obtained when the fixed pole crosses the $\rho$ Regge pole. However, in order to obtain the second maximum of $d\sigma/dt$ at $t=-0.9\pm0.1$, Kogitz and Logan are forced to introduce a complicated *ad hoc* $t$ dependence of the "background" term,

$$e^{-[a+b(t-t_0)^2]^{1/2}}\ ,$$

where $a$, $b$, and $t_0$ are free parameters. As explained in Sec. III, we obtain the second maximum in $d\sigma/dt$ simply because around the intersection point ($\alpha_\rho=\alpha_{\rho'}$) the phases of the $\rho$ and $\rho'$ contributions are the same.

### C. Cut Models

In a recent review paper [17] Field shows that, using the standard prescriptions for calculating $\rho$-

Pomeranchukon Regge cuts, it is impossible to fit the new polarization data[5] with Regge cuts and the $\rho$ alone. Both the weak-cut model and the strong-cut model violently disagree with the polarization data, giving a zero in the polarization for small values of $t$ ($-t \simeq 0.2-0.4$), clearly not seen in the data.

### D. Vasavada Model

In this model[18] the $\pi^- p \to \pi^0 n$ polarization arises from interference between the $\rho$ Regge-pole term and the $\rho - P'$ Regge cut. The parameters of this cut,

$$\alpha_c(0) = 2\alpha_\rho(0) - 1, \quad \alpha_c{}' = \tfrac{1}{2}\alpha_\rho{}' ,$$

with $\alpha_\rho(0) \sim 0.5$ and $\alpha_\rho{}' \sim 0.8$ $(\text{GeV}/c)^{-2}$, are not far from the parameters of our $\rho'$. Therefore similar predictions are given in both models [the form of $\alpha_{\text{eff}}$ in the region $0 < -t < 2$ $(\text{GeV}/c)^2$, the variation of the polarization with energy, etc.], but the basic ideas are different. Also, it has been argued by Worden[19] that the $\rho - P'$ and $\omega - A_2$ cuts should approximately cancel in $\pi^- p \to \pi^0 n$, and there is a similar cancellation also in $\gamma p \to \pi^0 p$. Thus there is some doubt as to whether the $\rho'$ (or a possible $\omega'$) can be considered as an effective cut contribution.

### E. Halzen and Michael Amplitude Analysis of $\pi N$ Scattering at 6 GeV/$c$

Using data on $\pi^\pm p \to \pi^\pm p$ and $\pi^- p \to \pi^0 n$ the authors[20] attempt to deduce the $I = 0$ and $I = 1$ *s-channel amplitudes* in a model-independent fashion. However, there is *in principle* one arbitrary $t$-dependent phase, which they chose as the phase of the $I = 0$ amplitude given by the five-pole model of Barger and Phillips.[6] Thus the real and imaginary parts of the Halzen-Michael $I = 1$ amplitudes depend inherently on their assumed $I = 0$ phase. Nevertheless it is of interest to compare the real and imaginary parts of our $I = 1$ amplitudes with the $\perp$ and $\parallel$ $I = 1$ amplitudes, respectively, of Halzen and Michael. Their shapes and general structure are very similar. However, the zero that appears in $(F^1_{++})_\parallel$ at $t \simeq -0.2$ $(\text{GeV}/c)^2$, and which is directly connected to the crossover zero in the $\pi^\pm p \to \pi^\pm p$ differential cross sections, occurs in our $\text{Im}\, F^1_{++}$ at $t \simeq -0.4$ $(\text{GeV}/c)^2$. There is no disagreement here, since a ratio $\text{Re}F^0_{++}/\text{Im}F^0_{++} \sim -\frac{1}{3}$ (a not unrealistic phase[21]) in this region of $t$ is sufficient to make the positions of the zeros compatible with each other.

## VI. SOME CONSEQUENCES OF THE MODEL

The simplest consequences obtained by extrapolating our results to higher energies are shown in Figs. 4 and 5. It will be very interesting to compare the predictions with experimental results at these energies. In particular we predict no dramatic changes with energy in either $d\sigma/dt$ or $P$, but the shape of $P$ is rather unusual, being somewhat like a "pulse" in the region $-0.7 \leq t \leq 0$ and very small for $t \leq -0.7$.

Since the principal features of our results for $\pi^- p \to \pi^0 n$ follow from the intersection of the $\rho$ and $\rho'$ trajectories it is clear that one should expect some consequences of this property to appear in any reaction which is dominated by $\rho$ exchange, i.e., one might expect to find some structure in the differential cross section, and the vanishing of some spin density matrix elements at roughly the point corresponding to the intersection.

However, it is not trivial to find reactions where it is consistent to assume that $J^P = 1^-$ exchange alone dominates. For example, in $\pi^+ p \to \omega\Delta^{++}$ and $\pi^+ n \to \omega p$, although the $\rho$ is expected to dominate, the density matrix elements show quite clearly that some admixture of $J^P = 1^+$ is needed. This is usually provided by either the $B$ meson or an absorptive correction.[22]

The only class of reactions in which we can be absolutely sure that $I^G = 1^+$ and $J^P = 1^-$ alone is exchanged are those of the type

$$\pi^\pm X \longleftrightarrow \pi^0 Y .$$

As an example we show in Fig. 10 $d\sigma/dt$ for the reaction[23]

$$\pi^+ p \to \pi^0 \Delta^{++} \quad \text{at 3–4 GeV}/c .$$

It is seen that the second maximum occurs in the same region as for $\pi^- p \to \pi^0 n$ .

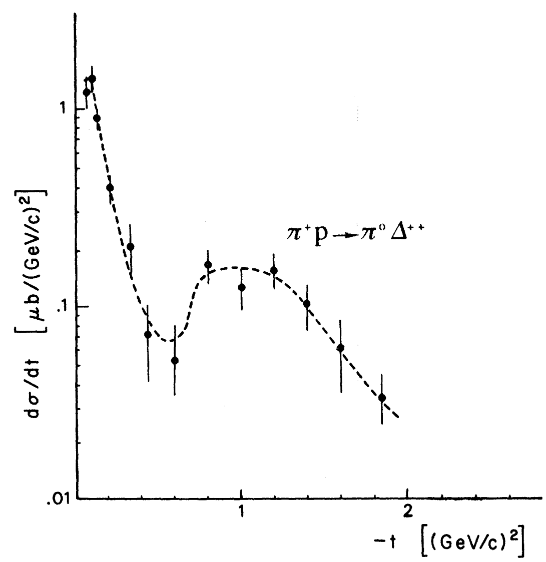


FIG. 10. $d\sigma/dt$ for the reaction $\pi^- p \to \pi^0\Delta^{++}$ at 3–4 GeV/$c$. The curve is simply a fit by eye.

It is also probable that the zero found in the nonflip amplitude $A'_\rho$ at $t\sim -0.45$ will, via factorization, have some effect in other processes. However, it does not seem possible to discuss these without a detailed treatment of each reaction.

Our simple picture involving intersecting real $\rho$ and $\rho'$ trajectories raises also the tantalizing question as to what is happening in the closely related channel with quantum numbers of the $\omega$.

Just as $\pi^- p\to\pi^0 n$ isolates $\rho$ exchange, so, as was shown by Contogouris *et al.*,[24] the combination

$$X_\sigma=\frac{d\sigma}{dt}(\pi^-p\to\rho^-p)+\frac{d\sigma}{dt}(\pi^+p\to\rho^+p)-\frac{d\sigma}{dt}(\pi^-p\to\rho^0p)$$

isolates exactly an exchange with $I=0$, $G=-1$, i.e., with the quantum numbers of the $\omega$.

A recent measurement of the above combination by Crennell *et al.*[25] at 6 GeV/$c$ seems to indicate a very clean, Regge-like exchange amplitude, much as $d\sigma/dt(\pi^-p\to\pi^0n)$ did for $\rho$ exchange.

Bearing in mind our experience with $\pi^-p\to\pi^0n$, it becomes imperative to measure the *polarization* in the above combination of reactions, i.e.,

$$X_P=P\frac{d\sigma}{dt}_{\pi^-p\to\rho^-p}+P\frac{d\sigma}{dt}_{\pi^+p\to\rho^+p}-P\frac{d\sigma}{dt}_{\pi^-p\to\rho^0n},$$

which also isolates the quantum numbers of the $\omega$.

In this way one will be able to answer directly the following intriguing questions:

(i) Are exchange mechanisms, after all, *simple*, i.e., somehow free of Regge-cut complications, as is indeed suggested by the above-mentioned data on $\rho$ and $\omega$ exchange?

(ii) Is the $\omega$ channel really *pure* $\omega$ exchange, or is there some influence of an $\omega'$? (From exchange degeneracy one might expect an $\omega'$.) The polarization data would give an immediate answer.

It is of some interest to note that although we have imposed no duality constraints, and although our $\rho$ and $\rho'$ trajectories are nonparallel; nevertheless, exchange degeneracy in the $\pi\pi$ system still holds approximately. This is because the $\rho'$ trajectory, as obtained from the high-energy charge-exchange data, is compatible (see Fig. 2) with a spin-zero meson with mass approximately equal to $m_\rho$, i.e., with an $\epsilon$. Thus one can have approximate $\rho-f$ degeneracy and one can consider $\rho'-\epsilon$ as its degenerate, nonparallel, first daughter. The possibility of nonparallel daughters in $\pi\pi$ scattering was first suggested, in the framework of Veneziano model, by Copley and Eilbeck,[26] but it has not been investigated in depth.

Finally, let us note that the new $\rho'$ particle (and, possibly, an $\omega'$ particle) could also have some interesting consequences for the pion, kaon, and nucleon form factors. As remarked by Bernardini[27] the relatively high cross section observed at Adone in the $e^+e^-\to p\bar{p}$ reaction at $2\times 1.05$ GeV (i.e., in the measurement of the proton form factors at $s=4.3$ GeV$^2$) could be related to the bump discovered by Benvenuti *et al.*[8] at 1968 MeV, and interpretable as a $\rho'$. Also, the pion form factor seems to be better fitted with a $\rho'$ of mass around 2 GeV (in addition to the $\rho$) rather than with a $\rho'$ of lower mass.[28] The recently discovered very broad peak in the cross section of $e^+e^-\to\pi^+\pi^-\pi^+\pi^-$ (Ref. 29) between 1500 and 1800 MeV could equally be the manifestation of a new vector meson of high mass.[30] Obviously, however, much more accurate experimental data are necessary in order to conclude with any certainty the location, the width, and the coupling constants of the $\rho'$ particle.

## VII. CONCLUSIONS

The underlying physical picture used by us is very different from the absorptive-type models and the models based on intersecting complex trajectories. Indeed it is suggested that in the latter case it would be very difficult to arrange to fit both $\alpha_{\rm eff}$ and $\rho$, $g$, and $\rho'$ particles with any reasonable parametrization of the trajectories. Also, with complex trajectories, the polarization would not in general vanish at the point of intersection. Insofar as absorption is concerned it is perhaps significant that absorptive corrections are usually important in those reactions in which they can contribute with opposite normality $[P(-1)^J]$ to that of the dominant exchanged pole. Such a situation cannot arise in $\pi^-p\to\pi^0n$ since the $\pi\pi$ system couples only to positive normality.

It will have been noticed that we have made no attempt to explain quantitatively the infamous crossover zero, i.e., the point at which the differential cross sections for $\pi^+p\to\pi^+p$ and $\pi^-p\to\pi^-p$ cross each other, usually quoted as occurring at $t=-0.15$, and which must be controlled by the interference between $I=0$ and $I=1$ amplitudes. It is common practice[6] to insert this zero by forcing $A'_\rho$ to vanish at or very close to the crossover point. We have verified (as can be seen from the results discussed in Sec. IV) that it is quite impossible to shift the zero which the charge-exchange data alone forces into $A'_\rho$ at $t=-0.45$, nearer to the crossover point. (See however the discussion at the end of Sec. IV.)

On the other hand, the addition of the $\rho'$ shifts this zero to $t=-0.3$ in the imaginary part of the *total* amplitude $A'$. The precise position of the crossover zero can only be determined from a detailed knowledge of the $I=0$ amplitude. For example, if $\mathrm{Re}A'^{(+)}/\mathrm{Im}A'^{(+)} \approx -\frac{1}{3}$ around $t=-0.2$, then we can obtain a crossover zero near $t=-0.15$. A more quantitative study will be given elsewhere.

The model presented here relies on the intersection of linear real $\rho$ and $\rho'$ Regge trajectories, in order to explain both the structure of the polarization in $\pi^- p \to \pi^0 n$ at high energies and the break in the plot of "$\alpha$ effective" versus $t$. It was found possible to fit all the data using an extremely simple parametrization of the residue functions and it was shown that this structure is essentially minimal in the sense that a very close connection could be established between the dominant features of the data and each of the parameters of the model. In particular, the present paper differs from all previous work in that a *direct* physical link is established between an important characteristic of the data, namely, the zero in the polarization near $t \approx -1$ $(\mathrm{GeV}/c)^2$ and the properties of the $\rho'$. It is this feature which leads to *intersecting* trajectories, which avoids the embarrassment of the usual low-mass $\rho'$, and which leads to the prediction that there should exist a $\rho'$ with mass $m_{\rho'} \approx 2$ GeV.

A fundamental question which remains is: Is the $\rho'$ a genuine Regge pole or just a parametrization of our ignorance of the $I=1$ exchange mechanism? The finding by Benvenuti *et al.*[8] of a possible $\rho'$ at mass 1968 MeV makes the interpretation given in our model all the more interesting. But, of course, more experimental effort is needed to confirm the existence of the $\rho'$ with mass $m_{\rho'} \approx 2$ GeV.

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# Quark-Model Predictions for Reactions with Hyperon Beams*

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Quark-model predictions are discussed for Primakoff excitation of hyperon resonances, total hyperon-nucleon cross sections, and diffractive excitation. The $U$-spin selection rule forbidding electromagnetic excitation of negatively charged decuplet resonances is shown to hold even in the presence of large SU(3)-symmetry breaking. A new model for diffractive excitation is presented which suggests the existence of new hyperon resonances, not yet discovered, which would be observed in diffractive excitation but only weakly coupled to two-body formation and decay channels. The SU(3) partners of the Roper resonance $N(1470)$ might be such states and be found with hyperon beams.

The availability of hyperon beams raises the possibility of observing new strong-interaction phenomena not previously available to experiment. The presence of a strange baryon in the initial state allows the study of strange-baryon transitions without strangeness exchange. The present discussion considers three types of strangeness-conserving hyperon transitions that appear to be of interest: (1) electromagnetic transitions, (2) hadron reactions with exchanges of nonstrange Reggeons ($\rho$, $\omega$, etc.), and (3) diffractive excitation. Electromagnetic transitions can be studied by the Primakoff effect, for which a strong excitation of the $\frac{3}{2}^+$ decuplet is expected. However, a $U$-spin selection rule[1] forbids this excitation for negatively charged hyperons but allows it for neutral and positive baryons. The extent to which this selection rule is violated by SU(3)-symmetry breaking is of particular interest, since the most readily available hyperon beams have negative charge. The couplings of nonstrange bosons and Reggeons to strange baryons is of interest because of still untested quark-model and symmetry predictions for these couplings. Diffractive excitation might produce new hyperon resonances which are not excited by strangeness exchange and which appear only weakly in phase-shift analyses – e.g., the SU(3) partners of the Roper resonance $N(1470)$ and other diffractively excited nonstrange baryon resonances.

Theoretical understanding of these questions is not very well founded, but the quark model seems to give a good description of hadron systematics and spectroscopy. These points will therefore be examined with the aid of the quark model to see if any new insight can be obtained.

## I. ELECTROMAGNETIC EXCITATION OF BARYON RESONANCES

Decuplet baryons can be excited by the electromagnetic reactions