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Estimate of the σ Term in $K^{\pm}$-p Scattering

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The σ term in K-p scattering is calculated using on-mass-shell dispersion relations, available data on the $K^{\pm}p$ total cross sections, and real parts of the forward scattering amplitude. The subthreshold contribution is minimized by introducing a factor which vanishes at the $Y^*_{0,1}$ peaks; this also yields information on the $Y^*_{0,1}pK$ coupling constants. Our result $\sigma^{Kp}\sim 160$ MeV is rather insensitive to the choice of the ΛpK and ΣpK coupling constants. Within the $(3, 3^*)+(3^*, 3)$ model our value for σ^{Kp} favors small values of $\sigma^{\pi p}$.

I. INTRODUCTION

It is widely believed that the strong-interaction Hamiltonian can be meaningfully separated into a part that is invariant under SU(3)×SU(3) and a part that breaks this symmetry. The most popular model, due to Gell-Mann, Oakes, and Renner[1] (GMOR), suggests a symmetry breaking of the type $(3, 3^*)+(3^*, 3)$ which involves two nonets of scalar and pseudoscalar operators, $u_i(x)$ and $v_i(x)$, respectively.

In this model the energy density is of the form

$$H=H_0-u_0(x)-cu_8(x),$$

where H_0 commutes with the generators of SU(3) ×SU(3). The parameter c has been estimated by GMOR to be about -1.25. The GMOR model can be tested in meson-nucleon scattering where the matrix element $\langle N|u_0+cu_8|N\rangle$ is related to the so-called σ term, the equal-time commutator of an axial charge and the divergence of an axial-vector current. Since $\langle N|u_8|N\rangle \simeq 170$ MeV, as obtained from the octet mass splittings, the determination of the σ term in a single reaction will fix $\langle N|u_0|N\rangle$. This means that the σ term in K-N scattering will be uniquely determined within the framework of the model.

Unfortunately, in spite of the large amount of data on π-N scattering there seems to be no consensus on the value of the π-N σ term.[2] Values range from 25 MeV (Ref. 3) to 110 MeV (Ref. 4). The predictions for the K-N σ term will vary accordingly from about 200 to 1300 MeV (for $c=-1.25$).

In this note we shall evaluate the K-N σ term directly from on-mass-shell dispersion relations. We hope this will allow one to discriminate between the theoretical predictions even though the data on K-N scattering are less abundant. The σ term is proportional to the K-p scattering amplitude at the point where both kaons have zero 4-momentum. There have been previous attempts[3,5] to calculate the K-N σ term from off-mass-shell dispersion relations using the elegant method of Fubini and Furlan[6] which relates the current-algebra soft-meson point to the scattering amplitude at threshold. It was however pointed out by Brown, Pardee, and Peccei[7] that threshold is not a suitable point to relate the σ term to, since the contribution of the essentially unknown continuum in the dispersion integral is of the same order, $(m/M)^2$ (where m is the mass of the K meson and M that of the nucleon), as the σ term itself. A better method would be to determine the amplitude at

$$\nu\equiv\frac{(p+p')\cdot(q+q')}{4M}=0,$$

$$\nu_B\equiv-\frac{q\cdot q'}{2M}=0,\quad q^2=q'^2=m^2,$$

where p (p') and q (q') denote the momenta of the incoming (outgoing) baryon and meson, respectively, by on-mass-shell dispersion relations, and then to use a linear expansion in q^2 and q'^2 to go off the mass shell.

The purpose of our calculation is to incorporate as much information as possible about the forward K-N scattering amplitude. Following a calculation by Adler,[8] we carry out the extrapolation to the unphysical point $\nu=\nu_B=0$ in two steps: First in the forward direction from $\nu=m$, $\nu_B=-m^2/2M$ (threshold) to the point $\nu=0$, $\nu_B=-m^2/2M$ and then to the point $\nu=\nu_B=0$. The first part of the extrapolation can be done quite reliably using the accurately known total cross-section data.[9] The second, over a much smaller distance, is done in a narrow-resonance approximation.

One complicating factor in K-N dispersion relations is the contribution of the unphysical region below the elastic threshold. This contribution is expected to be dominated by the $Y_0^*(1405)$ and $Y_1^*(1385)$. A way to handle such a difficulty is to multiply the amplitude by factors which vanish at the peak of the resonances,[10] thus minimizing their contribution. By comparison with the original dispersion relations this method will also yield information on the Y_0^*pK and Y_1^*pK coupling constants.

Another difficulty comes from the uncertainty in the ΛpK and ΣpK coupling constants. As we shall see, however, our results are quite insensitive to their exact values.

II. THE σ TERM

We continue the amplitude for the process

$$K^{\pm}(q)+P(p)\to K^{\pm}(q')+P(p')$$

off the mass shell by means of the definition[11]

$$T^{a,b}(\nu,\nu_B,q^2,q'^2)=i\frac{2M}{f_K{}^2m^4}(m^2-q^2)(m^2-q'^2)\int dy\,e^{iq'y}\langle P(p')\,|\,TD^b(y)D^a(0)\,|\,P(p)\rangle\,, \tag{1}$$

where $D^{a,b}\equiv\partial^\mu A_\mu^{a,b}$ with $(a,b)=(\pm,\mp)$ and f_K is the kaon decay constant defined by

$$\langle 0|D^{a,b}(0)|K^{a,b}\rangle=f_Km^2\,,\quad f_K\approx 125\text{ MeV}\,.$$

The generalized Ward-Takahashi identity relates the amplitude [Eq. (1)] at $q=q'=0$ to the σ term:

$$T^{a,b}(0,0,0,0)\frac{f_K{}^2}{2M}=-\sigma_{Kp}^{a,b}\,, \tag{2}$$

where

$$\sigma_{Kp}^{a,b}=-i\int dy\,e^{iq'y}\delta(y_0)\times\langle P(p')\,|\,[A_0^b(y),D^a(0)]\,|\,P(p)\rangle\,. \tag{3}$$

$\sigma^{a,b}$ is symmetrical, $\sigma^{a,b}=\sigma^{b,a}=\sigma$. As has been pointed out in Ref. 7, a consistent calculation of the σ term should make use of the even amplitude

$$T\equiv T^{ab}+T^{ba}\,.$$

T cannot be measured directly at the current-algebra point $q=q'=0$. However, it can be related to the Cheng-Dashen[4] point $\nu=\nu_B=0$, $q^2=q'^2=m^2$, which can be reached by on-mass-shell dispersion relations, via the reflection property

$$T(0,0,0,0)=-T(0,0,m^2,m^2)+O(\,(m/M)^4)\,. \tag{4}$$

To obtain an estimate of the σ term we shall have to neglect the part $O(\,(m/M)^4)$. Since σ itself is expected to be $O(\,(m/M)^2)$ this approximation introduces an ~30% error. We think this constitutes one of the major sources of uncertainty in our calculation. The assumption of the existence of a linear expansion in q^2 and q'^2 on which Eq. (4) is based has been recently questioned[12] in view of possible appearance of logarithmic terms. These terms are however expected to be small[13] and are therefore not likely to change our result drastically.

III. DISPERSION RELATION IN ν

We start this section using forward dispersion relations to relate $T(\nu=0,\nu_B=-m^2/2M,q'^2=m^2)$ to the total cross-section data. As mentioned in the introduction we shall minimize the uncertainty arising from the contribution of the unphysical region by considering the function[14]

$$\overline{T}(\nu)=T(\nu)\frac{\nu^2-\nu_0{}^2}{\nu^2-\overline{\nu}^2} \tag{5}$$

rather than T itself. The value of the parameter ν_0 is chosen such that $\overline{T}$ vanishes at the position of the resonance. As the masses of the $Y_0^*(1405)$ and $Y_1^*(1385)$ are very close, one such multiplying factor suffices. The denominator is introduced in order to keep the asymptotic behavior unchanged. The dispersion relation is evaluated for 39 different values of $\overline{\nu}$ all lying on the physical cut where $\text{Re}\,T(\overline{\nu})$ is known.[15]

Using the fact that $\overline{T}$ is even under the substitution $\nu\to-\nu$ and subtracting at $\nu=0$, one can write the dispersion relation

$$\overline{T}(\nu=0)=\text{Re}\,\overline{T}(\nu)-\frac{\overline{\nu}^2-\nu_0{}^2}{\nu^2-\overline{\nu}^2}\frac{\nu^2}{\overline{\nu}^2}\text{Re}\,T(\overline{\nu})-\frac{2}{\pi}\nu^2\,\text{P}\int_0^\infty d\nu'\,\frac{\text{Im}\,\overline{T}(\nu')}{(\nu'^2-\nu^2)\nu'}\,. \tag{6}$$

We next separate explicitly the contribution of the Λ and Σ Born terms, use the optical theorem to express the continuum in terms of total cross sections, and obtain at $\nu=m$

$$\overline{T}(0)=\text{Re}\,\overline{T}(m)+\frac{\overline{\nu}^2-\nu_0{}^2}{\overline{\nu}^2-m^2}\frac{m^2}{\overline{\nu}^2}\text{Re}\,T(\overline{\nu})-2m^2\frac{\nu_\Lambda{}^2-\nu_0{}^2}{\nu_\Lambda{}^2-\overline{\nu}^2}\frac{M_\Lambda-M-\nu_\Lambda}{\nu_\Lambda(\nu_\Lambda{}^2-m^2)}G_{\Lambda pK}{}^2-2m^2\frac{\nu_\Sigma{}^2-\nu_0{}^2}{\nu_\Sigma{}^2-\overline{\nu}^2}\frac{M_\Sigma-M-\nu_\Sigma}{\nu_\Sigma(\nu_\Sigma{}^2-m^2)}G_{\Sigma pK}{}^2$$
$$-\frac{4Mm^2}{\pi}\,\text{P}\int_m^\infty d\nu'\,\frac{\nu'^2-\nu_0{}^2}{\nu'^2-\overline{\nu}^2}\frac{1}{\nu'(\nu'^2-1)^{1/2}}[\sigma_{K^+p}(\nu')+\sigma_{K^-p}(\nu')]\,, \tag{7}$$

where $\nu_{\Lambda,\Sigma}=(M_{\Lambda,\Sigma}{}^2-M^2-m^2)/2M$.

The contribution of the unphysical region from the $\Lambda\pi$ to the elastic threshold ($\nu=m$) is being neglected. We believe this approximation to be quite safe since the multiplying factor vanishes at the peak of the virtual bound states and changes sign there, thus leading to cancellations.

To evaluate the principal-value integral reliably, we subdivide the fitting curves to the total cross sections in the region between threshold and $\nu'=55$ GeV into small intervals over which the cross sections can be approximated by a straight line and the integration performed analytically. For the asymptotic region we assume $\sigma^{\text{total}}(\nu')\simeq\sigma^{\text{total}}$ (55 GeV) as indicated by recent data.[16] At any rate, for reasonably well-behaved cross sections, the contribution of this region is small as the dispersion relation converges rapidly. In this way we calculate $T(\nu=0)$ for 39 different values of the "subtraction point" $\bar{\nu}$ ranging from 790 MeV to 1320 MeV for which $\text{Re}\,T(\bar{\nu})$ is known.[15] The result is shown in Fig. 1 for the particular choice of $G_{\Lambda pK}{}^2/4\pi=7.0$ and $G_{\Sigma pK}{}^2/4\pi=2.1$, a choice which is consistent with the input for $\text{Re}\,T(\bar{\nu})$.[10,13] It is seen that $T(0)$ is remarkably stable with respect to variation of $\bar{\nu}$. Taking the average of the 39 values for $T(0)$ we obtain

$$T(0)=357\pm17\,.$$

The error is statistical and does not include the error due to neglect of the subthreshold contribution to the dispersion integral. However, the latter should be negligible since the part of the integral which we have taken into account is itself small (~-10 for T_{K^-p}).

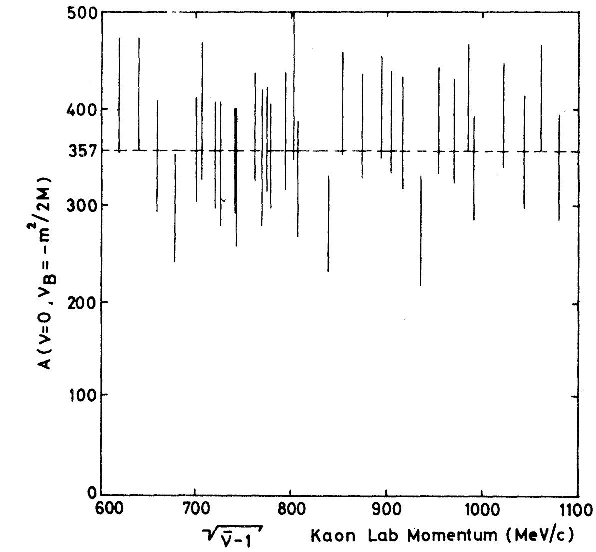

FIG. 1. The amplitude $A(\nu=0,\ \nu_B=-m^2/2M)$ for different values of the "subtraction point" $\bar{\nu}$ and $G_{\Lambda pK}{}^2/4\pi=7.0$, $G_{\Sigma pK}{}^2/4\pi=2.1$. The error bars trace back to errors in the real part of the forward scattering amplitude at $\nu=\bar{\nu}$.

IV. EXTRAPOLATION IN ν_B

Our next task is to reach the point $\nu_B=0$ from $\nu_B=-m^2/2M$ keeping $\nu=0$ fixed. This extrapolation is more ambiguous in principle since we cannot make use of as much physical data as in the previous section. Fortunately the distance over which we extrapolate is much shorter (~25%). We think this justifies the sole use of the hyperon Born terms and narrow-resonance Y_0^* and Y_1^* terms in effecting the extrapolation in ν_B. We get

$$\begin{aligned}A(0)=A(-m^2/2M)&-\frac{4Mm^2}{(M_\Lambda+M)(M_\Lambda{}^2-M^2-m^2)}G_{\Lambda pK}{}^2\\&-\frac{4Mm^2}{(M_\Sigma+M)(M_\Sigma{}^2-M^2-m^2)}G_{\Sigma pK}{}^2-\frac{4Mm^2}{(M_{Y_0^*}-M)(M_{Y_0^*}{}^2-M^2-m^2)}G_{Y_0^*pK}{}^2\\&+\frac{8M}{3}\frac{1}{M_{Y_1^*}{}^2}\frac{m^2}{M_{Y_1^*}{}^2-M^2-m^2}(M+M^*)\left(1-\frac{m^2}{M^{*2}}\right)G_{Y_1^*pK}{}^2-\frac{2}{3}\frac{M^2}{M_{Y_1^*}{}^2-M^2}\frac{m^4}{M_{Y_1^*}{}^4}G_{Y_1^*pK}{}^2,\end{aligned}\qquad(8)$$

where $A(\nu_B)\equiv T(\nu=0,\ \nu_B)$ and

$$\langle P(p)|j_\pi|Y_1^*(k)\rangle\equiv\frac{iG_{Y_1^*pK}}{M_{Y_1^*}}u_\mu(p)u(k)(k-p)^\mu\,.$$

This result is obtained by writing for $A(\nu,\nu_B)$ an unsubtracted dispersion relation in ν for arbitrary ν_B and saturating it with the Λ, Σ, Y_0^*, Y_1^* poles. The approximation made is equivalent to retaining only the corresponding Feynman graphs in perturbation theory. Though the dispersion integral is not expected to converge, Eq. (8) should be a reasonable approximation for the variation of A between two relatively close points.[17] The contribution of the Y_1^* was calculated using only the nonambiguous pole part of the spin-$\frac{3}{2}$ propagator. The variation of the model-dependent nonpole part of the spin-$\frac{3}{2}$ propagator between the points $\nu=0$, $\nu_B=-m^2/2M$, and $\nu=0$, $\nu_B=0$ was found to amount to less than 5% of the variation of the pole term in the three specific models discussed for instance by

Achuthan, Hite, and Höhler.[18] Substituting numerical values for masses, Eq. (8) becomes

$$A(0)=A(-m^2/2M)-46\frac{G_{\Lambda pK}{}^2}{4\pi}-18\frac{G_{\Sigma pK}{}^2}{4\pi}-29\frac{G_{Y_0^*pK}{}^2}{4\pi}-10\frac{G_{Y_1^*pK}{}^2}{4\pi}. \qquad (9)$$

V. DETERMINATION OF COUPLING CONSTANTS

To obtain a numerical value for $A(0,0)$ one needs the coupling constants $G_{\Lambda pK}$, $G_{\Sigma pK}$, $G_{Y_0^*pK}$, and $G_{Y_1^*pK}$, none of which is accurately known. The Y_1^* is a p-wave resonance, and it is argued[19] that $G_{Y_1^*pK}$ can be taken from SU(3) and the knowledge of the $Y_1^* \to \Lambda\pi$ width since the centripetal barrier keeps the particles within the short-range SU(3)-invariant interaction potential. Then

$$\frac{G_{Y_1^*pK}{}^2}{4\pi}=5.6\,.$$

On the other hand, $G_{Y_0^*pK}{}^2$ being an s-wave resonance can differ markedly from the SU(3) value.[20] Our method of dealing with the subthreshold continuum allows the determination of this coupling constant in a simple way:

Equating $A(0,-m^2/2M)$ of our analysis to a value obtained by keeping the Y_0^* and Y_1^* poles, one gets a relation between the coupling constants. $G_{\Lambda pK}$ and $G_{\Sigma pK}$ can be taken from independent dispersion-theoretical calculations available in the literature[21] though the values obtained there differ widely (e.g., $G_{\Lambda pK}{}^2/4\pi$ may be somewhere between 4 and 15). Fortunately $A(0,0)$ is quite insensitive to the exact values of $G_{\Lambda pK}$ and $G_{\Sigma pK}$. Taking $G_{\Sigma pK}{}^2/4\pi=0.3G_{\Lambda pK}{}^2/4\pi$, which follows from SU(3) and which is consistent with most dispersion-theoretical calculations, we determine $A(0,0)$ for values of $G_{\Lambda pK}{}^2/4\pi$ ranging from 1 to 15. The results are shown in Fig. 2. We also obtain $G_{Y_0^*pK}{}^2/4\pi=0.43$ quite independent of the assumed values of $G_{\Lambda pK}{}^2/4\pi$. This result agrees with other empirical calculations.[20,3]

VI. RESULTS AND CONCLUSIONS

Using Eqs. (2) and (4) and the values of $A(0,0)$ taken from Fig. 2 we are now in a position to calculate the K-p σ term

$$\sigma=-4.0A(0,0)\ \text{MeV}\,.$$

σ varies from 200 MeV to 100 MeV corresponding to a variation of $G_{\Lambda pK}{}^2/4\pi$ from 4 to 15. If we adopt $G_{\Lambda pK}{}^2/4\pi=7.0$ obtained by Restignoli and Violini[10] who used a method similar to ours, we find

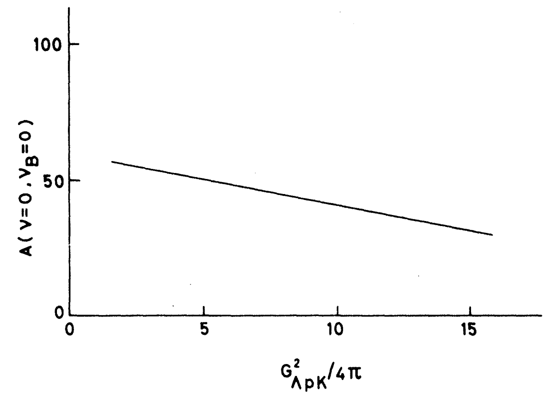

FIG. 2. Variation of $A(\nu=0,\ \nu_B=0)$ with $G_{\Lambda pK}{}^2/4\pi$ taking $G_{\Sigma pK}{}^2=0.3\,G_{\Lambda pK}{}^2$.

$$\sigma\approx 160\ \text{MeV} \qquad (10)$$

to be the value most consistent with our calculations. Another frequently adopted choice, $G_{\Lambda pK}{}^2/4\pi=11.4$ obtained from an SU(3) fit to the experimental data,[22] would give

$$\sigma\simeq 100\ \text{MeV}\,.$$

The error in result (1) is rather difficult to estimate. As mentioned in Sec. II, the neglect of terms $O((m/M)^4)$ may introduce an error of ~30%. The second major source of uncertainty lies in the extrapolation in ν_B. However, more sophisticated methods of extrapolation are not likely to yield more accurate results since they require as an input more accurately known experimental data than are available.

We test our method by applying it to π-N scattering. Using the value of the even π-N amplitude at $\nu=0$, $\nu_B=-m_\pi{}^2/2M$, $q^2=q'^2=m_\pi{}^2$ given by Samaranayake and Woolcock[23] we obtain

$$\sigma^{\pi p}\simeq 35\ \text{MeV}\,.$$

It is interesting to note that our results for σ^{Kp} and $\sigma^{\pi p}$ are consistent with the GMOR model; they also agree with the ones of Kim and von Hippel, who used the method of Fubini and Furlan.

Other more recent calculations of σ^{Kp} by Thomson[24] and Köpp, Walsh, and Zerwas,[25] using different methods, obtain $\sigma^{Kp}=-370$ MeV and $\sigma^{Kp}=+380$ MeV, respectively.

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