the amplitudes, the predictions for the R measurement are made (see Fig. 5). The points correspond to predictions made by Halzen and Michael.[1]

Figures 6 and 7 show the contributions to the s channel for the ρ alone (Fig. 6) and for the cut terms (Fig. 7).

We have examined the helicity amplitudes of isospin one for pion-nucleon scattering in terms of three models, each having four free parameters. By looking at the helicity amplitudes (Fig. 4), we find that there is serious difficulty for the weak-cut model. Although the strong-cut model appears to have the approximate form for the amplitudes, it has the wrong polarization structure and energy dependence for differential cross sections. The conspiracy appears to have no serious difficulty for any of the experimental quantities.

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Lower Bound on the Magnitude of the Rate of $K^+ \to \pi^+ + e^+ + e^-$ †

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It is shown that a lower bound on the decay rate $K^+ \to \pi^+ e^+ e^-$ can be obtained by calculation of the absorptive part of the amplitude to which only the connected three-pion intermediate state contributes significantly. Some remarks on pole-model calculations are also included.

I. INTRODUCTION

A great deal of effort has been expended on the search for neutral currents in weak interactions; even if such currents are not present in the basic weak Hamiltonian, forbidden processes such as $\nu + p \to \nu + p$ should appear at some point because of higher-order weak-interaction effects. This has led to an impressive number of experimental searches for either neutral currents or higher-order weak-interaction effects, with as of yet no evidence for the existence of either, other than double β decay and the $K_L - K_S$ mass difference (both due presumably to higher-order weak interactions).

Some processes, forbidden to order G_F (the weak-interaction decay constant equal to $10^{-5}/M_N^2$, where M_N is the nucleon mass), are allowed by a combination of weak and electromagnetic interactions. The prime example is $K_L \to \mu^+\mu^-$ proceeding by way of an intermediate two-photon state. Despite an intensive search,[1] this rare decay mode has not been seen; moreover one obtains a lower bound on the decay rate by making use of unitarity,[2] the known decay rate for $K_L \to \gamma\gamma$, and the easily calculated matrix element for $\gamma\gamma \to \mu^+\mu^-$. This lower bound has now been shown to be violated.

This apparent puzzle led us to ask where else such bounds might be established and experimentally tested. The natural place seemed to be in the decay modes $K^+ \to \pi^+ e^+ e^-$ and $K^0 \to \pi^0 e^+ e^-$. Extensive literature[3-10] on these decays exists already, the various estimates all suggesting that 10^{-6}–10^{-7} of all K^+ decays should occur through such a mode. Since this is about the present experimental limit, it seemed worthwhile to find out if one could establish with some degree of reliability a unitarity bound for the decay rate.

The answer is yes, and the calculation is given in Sec. II; the result is unexpectedly small, of the order of 4×10^{-10}, for a set of reasons explained in detail in Sec. II, so that it seems quite hopeless to see whether a paradox similar to that for $K_L \to \mu^+\mu^-$ also occurs in the decays of $K^+ \to \pi^+ e^+ e^-$.

In Sec. III we comment on the work of previous authors in estimating the importance of pole diagrams in $K^+ \to \pi^+ + e^+ + e^-$ and show what conclusions may be drawn with the aid of Ward identities.

An appendix on the relation between the absorptive part of an amplitude and the unitarity bound is also included.

II. UNITARITY BOUND FOR $K^+ \to \pi^+ e^+ e^-$

We use formulas (A6) and (A10) of the Appendix to obtain a lower bound for the process $K^+ \to \pi^+ e^+ e^-$. Neglecting second-order weak processes and higher-order electromagnetic effects, all the diagrams considered will be of the general form of Fig. 1, where the photon of momentum k^μ is virtual but all other particles are on the mass shell. The possible intermediate states which will be considered are[11]

Mode	Branching ratio
$K^+ \to \pi^+\pi^0$	2.1×10^{-1}
$\to \pi^+\pi^+\pi^-$	5.6×10^{-2}
$\to \pi^0\pi^0\pi^+$	1.7×10^{-2}
$\to \pi^+\pi^0\gamma$	$<1.9\times10^{-4}$.

Since the unitarity sum includes only physical states (on the mass shell), the intermediate state $\pi^+\gamma$ does not appear, because $K^+ \to \pi^+\gamma$ is a forbidden zero-to-zero transition. The experimental upper limits for the process $K^+ \to \pi^+ e^+ e^-$, and the similar $K^+ \to \pi^+ \mu^+ \mu^-$ are[11]

$$K^+ \to \pi^+ e^+ e^- \quad <4\times10^{-7}$$
$$\to \pi^+ \mu^+ \mu^- \quad <2.4\times10^{-6}.$$

The notation of Fig. 1 will be used throughout. $l^\mu = \bar{u}(p_1)\gamma^\mu v(p_2)$ is the lepton current, and $e>0$ is the electronic charge. Also $k^\mu = (\omega, \vec{k})$ and $p_i^\mu = (E_i, \vec{p}_i)$.

We now proceed to a discussion of the possible intermediate states.

(a) $\pi^+\pi^0$. In our lowest-order electromagnetic treatment this intermediate state gives zero contribution. The T matrix elements may be written

$$T(K^+ \to \pi^+\pi^0) = \text{constant}, \tag{2.1}$$
$$T(\pi^+\pi^0 \to \pi^+ e^+ e^-) = f(k^2, q_1\cdot k, q_2\cdot k)\epsilon_{\mu\nu\rho\sigma} q_1^\mu q_2^\nu k^\rho l^\sigma,$$

where q_1^μ, q_2^μ are the 4-momenta of the intermediate π^+, π^0, respectively, and f is a scalar function. Regardless of the form of f it is clear that after the intermediate phase-space integrations the contribution will vanish, since one matrix element is symmetric in the pion relative momentum and the other antisymmetric (alternatively one can say that the indices of $\epsilon_{\mu\nu\rho\sigma}$ cannot be saturated).

(b) $K^+ \to 3\pi$. Here the simplest process occurs only in the mode $K^+ \to \pi^+\pi^+\pi^-$, and is shown in Fig. 2. However, this process also gives zero contribution in the case where the $K^+ \to 3\pi$ matrix element is taken to be constant over the Dalitz plot. This is most easily seen in the center-of-mass frame in the channel $K^+\pi^- \to \pi^+\pi^- \to e^+e^-$ where the expression $(q_2 - q_3)^\mu l_\mu = 2q_2^\mu l_\mu$ becomes $-2\vec{q}_2 \cdot \vec{l}$, which vanishes when integrating over the momenta $\vec{q}_2, \vec{q}_3$. This result is independent of the choice of pion electromagnetic form factor and depends only on the assumed constancy of the $K^+ \to 3\pi$ matrix element.

A nonzero result may be obtained by allowing the pions to interact strongly with each other. We may, for example, use the tree-graph approximation with the effective Lagrangian of the nonlinear σ model, for which the 4-pion interaction may be

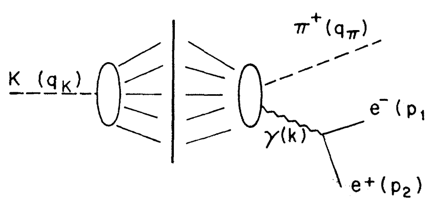

FIG. 1. General unitarity diagram for $K^+ \to \pi^+ e^+ e^-$.

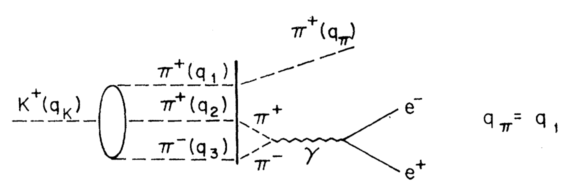

FIG. 2. Unitarity diagram for $K^+ \to \pi^+\pi^+\pi^- \to \pi^+ e^+ e^-$ with disconnected π^+.

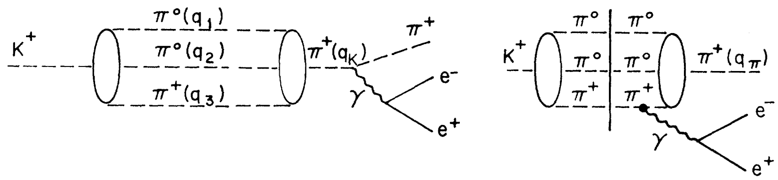

FIG. 3. Connected unitarity diagram for $K^+ \to \pi^0\pi^0\pi^+ \to \pi^+e^+e^-$.

written[12]

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \vec{\phi})^2 - \tfrac{1}{2}m_\pi{}^2\vec{\phi}^2 + \frac{1}{2f_\pi{}^2}(\vec{\phi}\cdot\partial_\mu\vec{\phi})^2 - \frac{m_\pi{}^2}{8f_\pi{}^2}(\vec{\phi}^2)^2, \qquad (2.2)$$

where $f_\pi = 94$ MeV is the charged-pion decay constant, and $\vec{\phi}$ is the pion field. This interaction leads to no additional electromagnetic coupling under the minimal substitution $\partial_\mu\phi_\pm \to \partial_\mu\phi_\pm \pm ieA_\mu$ (where A_μ is the electromagnetic field) so that the only tree diagrams in the $\pi^0\pi^0\pi^+$ mode are those shown in Fig. 3. In the mode $K^+ \to \pi^+\pi^+\pi^-$ there will be four diagrams, since all the intermediate pions are charged. We may easily obtain an estimate for these processes by assuming minimal electromagnetic coupling of pions and neglecting the 3-momenta $\vec{q}_1$, $\vec{q}_2$, $\vec{q}_3$ of the pions. In this approximation

$$T(K^+ \to \pi^0\pi^0\pi^+) = A\,, \quad \text{a constant} \qquad (2.3a)$$

$$T(K^+ \to \pi^+\pi^+\pi^-) = 2A\,, \qquad (2.3b)$$

$$T(\pi^0\pi^0\pi^+ \to \pi^+e^+e^-) = -\frac{6e^2m_\pi{}^2}{f_\pi{}^2(m_K + m_\pi)}\,\frac{l^0}{k^2 - 2m_\pi\omega}\,, \qquad (2.3c)$$

$$T(\pi^+\pi^+\pi^- \to \pi^+e^+e^-) = -\frac{12e^2m_\pi{}^2}{f_\pi{}^2(m_K + m_\pi)}\,\frac{l^0}{k^2 - 2m_\pi\omega}\,, \qquad (2.3d)$$

where ω and l^μ are as defined previously, and where (2.3c) and (2.3d) are the gauge-invariant sum of contributing diagrams, with the photon propagator having been canceled by a k^2 factor in the numerator. For (2.3) see Zemach's[13] discussion of 3π decays. The intermediate phase-space integration is trivial, so that the absorptive part $A_{\beta\alpha}$ is

$$A_{\beta\alpha} = \frac{15Ae^2m_\pi{}^2}{(m_K + m_\pi)f_\pi{}^2}\rho_{3\pi}\frac{l^0}{k^2 - 2m_\pi\omega}\,, \qquad (2.4)$$

where $\rho_{3\pi}$ is the 3π phase space.

(c) $\pi^+\pi^0\gamma$. Here we have the diagram shown in Fig. 4. This diagram does give a nonzero absorptive part, but it is negligible compared to the 3π contribution. The decay $K^+ \to \pi^+\pi^0\gamma$ is at least two orders of magnitude slower than $K^+ \to 3\pi$, and the $\pi^0\gamma\gamma$ vertex is far weaker than the pion electromagnetic vertex in the 3π case (the 4π vertex there is of order unity).

We conclude that the 3π intermediate states give the dominant part of the absorptive part for $K^+ \to \pi^+e^+e^-$. From Eq. (2.4)

$$\frac{\Gamma(K^+ \to \pi^+e^+e^-)}{\Gamma(K^+ \to \pi^0\pi^0\pi^+)} \geq \left(\frac{15m_\pi{}^2\alpha}{f_\pi{}^2}\right)^2 \frac{\rho_{3\pi}}{\pi(m_\pi + m_K)^2} \times \int \frac{dE_1dE_2(4E_1E_2 - k^2)}{(k^2 - 2m_\pi\omega)^2} \simeq 1.08\times 10^{-7}I\,, \qquad (2.5)$$

where I is the integral appearing in the second line, and $\alpha = e^2/4\pi \approx 1/137$. Hence

$$\frac{\Gamma(K^+ \to \pi^+e^+e^-)}{\Gamma(K^+ \to \text{all})} \geq 1.8\times 10^{-9}I\,.$$

Neglecting the electron mass, and setting $m_K = 3m_\pi$ to avoid kinematical inconsistency with the neglect of the 3-momenta in the intermediate state, we ob-

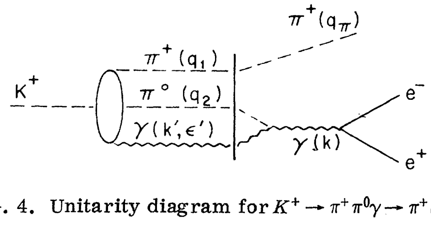

FIG. 4. Unitarity diagram for $K^+ \to \pi^+\pi^0\gamma \to \pi^+e^+e^-$.

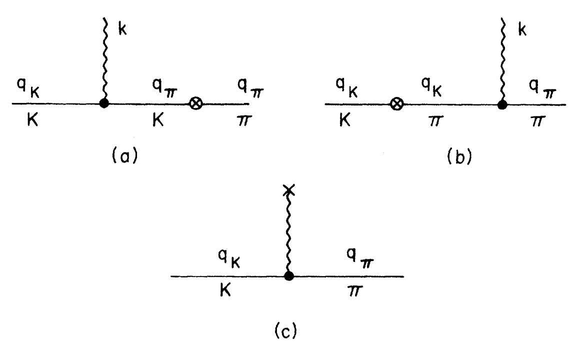

FIG. 5. Pole and contact diagrams for $K^+ \to \pi^+ + \gamma$ (virtual).

tain

$$I=\frac{1}{24m_\pi^2}\int_{4m_\pi/3}^{2m_\pi} d\omega\frac{(4m_\pi-\omega)^{3/2}}{(2m_\pi-\omega)^{1/2}}\simeq 0.2\,.$$

Our lower bound for the branching ratio is thus about 4×10^{-10}.

III. WARD IDENTITIES

There are several Feynman diagrams, which, though not contributing to the absorptive part of the amplitude for $K^+\to\pi^+e^+e^-$ discussed in II, are nonvanishing and may in fact give the dominant terms in calculating the rate. They are of the nature of pole and contact terms, as depicted in Figs. 5(a)–5(c), where the photon is virtual with mass k^2 and is eventually to be connected to the electron positron pair. The weak interactions allow a K to π transition which we characterize by a function of the momentum q, $C(q^2)$. The third diagram [Fig. 5(c)] corresponds to a possible contact term in which the weak and electromagnetic interactions operate effectively at the same point.

Aside from renormalization constants, Figs. 5(a) and 5(b) (with $q_K{}^2=m_K{}^2$, $q_\pi{}^2=m_\pi{}^2$) are given by

$$(2\pi)^4\delta(q_K-q_\pi-k)M_\mu=iC(q_\pi{}^2)\int\int d^4x\,d^4y\,e^{-iq_\pi\cdot x-ik\cdot y}\langle K^+|T(J_\mu(y)\phi_{K^-}(x))|0\rangle$$
$$+iC(q_K{}^2)\int\int d^4x\,d^4y\,e^{iq_K\cdot x-ik\cdot y}\langle 0|T(J_\mu(y)\phi_{\pi^+}(x))|\pi^+\rangle\,, \qquad (3.1)$$

with $J_\mu(y)$ being the electromagnetic current. This includes the full propagator for the off-mass-shell particle. We may also write

$$M_\mu(q_K,q_\pi,k)=(q_K+q_\pi)_\mu F_1(k^2,q_K{}^2,q_\pi{}^2)+(q_K-q_\pi)_\mu F_2(k^2,q_K{}^2,q_\pi{}^2)\,. \qquad (3.2)$$

Since $q_K-q_\pi=k$, F_2 will not contribute when dotted into the external electron-positron current, but for the moment we keep it.

Let us first discuss the case when $C(q_K{}^2)=C(q_\pi{}^2)=C$, a constant. Then we have the Ward identity

$$k^\mu M_\mu=0=(q_K{}^2-q_\pi{}^2)F_1(k^2,q_K{}^2,q_\pi{}^2)+k^2F_2(k^2,q_K{}^2,q_\pi{}^2)\,, \qquad (3.3)$$

or

$$(m_K{}^2-m_\pi{}^2)F_1(k^2,m_K{}^2,m_\pi{}^2)=-k^2F_2(k^2,m_K{}^2,m_\pi{}^2)\,, \qquad (3.4)$$

which implies

$$F_1(0,m_K{}^2,m_\pi{}^2)=0\,, \qquad (3.5)$$

which must of course hold since $K^+\not\to\pi^++\gamma$ for a real photon.

After using Eq. (3.4) dotting into the lepton current l^μ, and introducing the photon propagator $1/k^2$, we find the matrix element T, for $K^+\to\pi^+e^+e^-$, proportional to

$$T=\frac{l^\mu M_\mu}{k^2}=l^\mu(q_K+q_\pi)_\mu\frac{F_2(k^2,m_K{}^2,m_\pi{}^2)}{(m_\pi{}^2-m_K{}^2)}\,, \qquad (3.6)$$

so, e.g., for small k^2, one may expand $F_2(k^2,m_K{}^2,m_\pi{}^2)$ in a Taylor series about $k^2=0$.

If $C(p^2)$ is not independent of p^2, the Ward identity yields

$$(m_K{}^2-m_\pi{}^2)F_1(k^2,m_K{}^2,m_\pi{}^2)+k^2F_2(k^2,m_K{}^2,m_\pi{}^2)=k^\mu M_\mu=C(m_K{}^2)-C(m_\pi{}^2)\,, \qquad (3.7)$$

so, by gauge invariance we require a contact term R_μ to be generated by a graph of the type shown in Fig. 5(c), with the form

$$R_\mu=(q_K+q_\pi)_\mu\frac{C(m_K{}^2)-C(m_\pi{}^2)}{m_\pi{}^2-m_K{}^2}\,. \qquad (3.8)$$

These pole approximations are basically the types of models authors have used to calculate the decay rate for $K^+\to\pi^++e^++e^-$; e.g., Baker and Glashow[5] try to estimate the difference $C(m_K{}^2)-C(m_\pi{}^2)$. An exception is Bég's work[6] in which the pole contributions are set equal to zero; he, however, assumes the existence of a neutral intermediate vector boson coupled to weak hadronic currents and then a particular form for the $K\to3\pi$ matrix element which allows for dependence on the final pion momenta so that the partially connected 3π state contributes to the absorptive part.

We can also let the pion four-momentum $q_\pi\to0$ and see what can be learned by the use of partial conservation of axial-vector current (PCAC) and axial-vector-current Ward identities. The answer is not much; we cannot let $q_K\to0$ as well since then $k\to0$ and we know that the amplitude vanishes as $k\to0$. Letting only $q_\pi\to0$, and keeping track of σ terms, we can arrive at the relation [when $C(m_K{}^2)=C(0)$]

$$F_1(m_K{}^2, m_K{}^2, 0) + F_2(m_K{}^2, m_K{}^2, 0) = 0 , \qquad (3.9)$$

which, however, is just relation (3.4), with $q_\pi = 0$.

More detailed calculations can be done by specifying the model of weak interactions to be used. For instance in an intermediate-vector-boson (IVB) theory one has a well-defined set of graphs to calculate, which must include seagull diagrams and diagrams in which the photon interacts with the charged IVB. In such a theory one can also calculate the K-π transition element $C(p^2)$ at $p^2 = 0$ by Weinberg[14] spectral sum rules; the uncertainties in such estimates, due to divergence and extrapolation problems, do not seem to us to warrant such a detailed calculation.

IV. CONCLUSIONS

A series of calculations of the partial decay ratio $K^+ \to \pi^+ e^+ e^-$ has shown it to be of the order of magnitude of 10^{-6}–10^{-7}. Since this decay mode has not yet been seen, and it is conceivable to improve the present experimental limit of 4×10^{-7} by at least an order of magnitude, we attempted to find whether one could put a testable bound on the rate. The result of the calculation is that one does get a bound since only one state, the connected three pion state, contributes appreciably to the absorptive part of the amplitude. Its contribution, however, is very small, of the order of 4×10^{-10}, chiefly because of the smallness of the intermediate phase space. Hence it does not seem possible to test this lower bound.

Some discussion of pole model approximations to this decay rate are also given. It is shown that Ward identities and PCAC are of little practical value in obtaining estimates of pole diagram contributions to this decay rate.

APPENDIX: NOTATION AND THE UNITARITY BOUND

1. Notation

We use the Bjorken-Drell[15] metric and γ-matrix notation. Particle states are normalized according to

$$\langle p'\lambda' | p\lambda\rangle = (2\pi)^3 2p^0 \delta^{(3)}(\vec{p} - \vec{p}\,')\delta_{\lambda\lambda'} . \qquad (A1)$$

Dirac spinors are normalized to $2m$, where m is the mass of the particle, but are otherwise as in Ref. 15. The T matrix is expressed in terms of the S matrix by

$$S = 1 + R , \qquad \langle \beta | R | \alpha\rangle = -i(2\pi)^4 \delta^{(4)}(p_\beta - p_\alpha) T_{\beta\alpha} . \qquad (A2)$$

In this notation the T matrix is related to the invariant amplitude M by

$$T_{\beta\alpha} = iM_{\beta\alpha} , \qquad (A3)$$

unmodified by any kinematical factors, or powers of 2π. The unitarity relation $SS^\dagger = 1$ becomes

$$i(T_{\beta\alpha} - T^*_{\alpha\beta}) = \sum_n T_{n\beta} T^*_{n\alpha} (2\pi)^4 \delta^{(4)}(p - p_n) , \qquad (A4)$$

where $p = p_\alpha = p_\beta$, and the sum is over all free-particle states n.

2. The Unitarity Bound

The absorptive part $A_{\beta\alpha}$ and dispersive part $D_{\beta\alpha}$ of a transition amplitude $T_{\beta\alpha}$ are defined by

$$T_{\beta\alpha} = D_{\beta\alpha} + iA_{\beta\alpha} , \quad D_{\beta\alpha} = \tfrac{1}{2}(T_{\beta\alpha} + T^*_{\alpha\beta}) , \quad A_{\beta\alpha} = \frac{1}{2i}(T_{\beta\alpha} - T^*_{\alpha\beta}) . \qquad (A5)$$

$A_{\beta\alpha}$ may be obtained from the unitarity relation (A4).

$$A_{\beta\alpha} = -\tfrac{1}{2}\sum_n T_{n\beta} T^*_{n\alpha} (2\pi)^4 \delta^{(4)}(p - p_n) . \qquad (A6)$$

In $K_L \to \mu^+\mu^-$ the value $|A_{\beta\alpha}|^2$, calculated from the dominant intermediate state 2γ, has been widely used[2] as a lower bound for $|T_{\beta\alpha}|^2$, but it is not always made clear why this is valid. In order to make this conclusion it is clearly necessary to relate $T_{\alpha\beta}$ to $T_{\beta\alpha}$ in some way. Let us specify the spins of particles by their helicities, which are invariant under both time reversal and rotations.

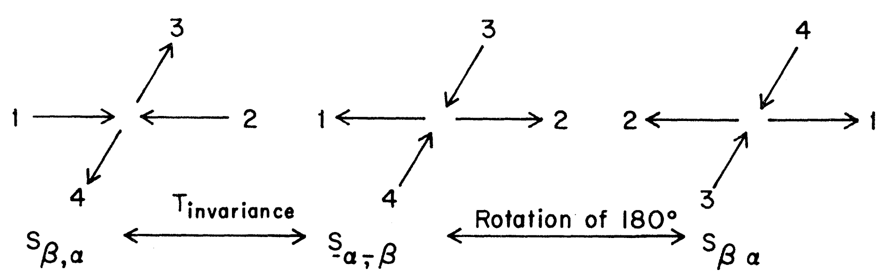

FIG. 6. Effect of T invariance and 180° rotation on scattering process.

Then assuming T invariance, for example, in a $2\to 2$ process, we have ($\alpha=1+2, \beta=3+4$) the situation depicted in Fig. 6. $-\alpha$, $-\beta$ denote the motion-reversed counterparts of α and β. Thus we conclude

$$T_{\alpha\beta}=e^{2i\lambda}T_{\beta\alpha}, \tag{A7}$$

where λ is a phase, which depends on the states α and β. That is, it depends on the intrinsic time-reversal parities of the particles (which for non-self-conjugate particles are not physically measurable) and the way in which the phases of the various orientations of the particle states are specified. The above is the principle of detailed balance, which actually holds for the individual spin states, provided helicity states are used. It uses T invariance, and the fact that there exists a frame in which the 3-momenta of the particles are coplanar. This latter will always be the case if the process involves no more than four external particles. If we write $T_{\beta\alpha}$ in the form

$$T_{\beta\alpha}=re^{i\Phi}, \quad r, \Phi \text{ real}, \tag{A8}$$

then the expressions for $D_{\beta\alpha}$ and $A_{\beta\alpha}$ become

$$\begin{aligned} D_{\beta\alpha}&=re^{-i\lambda}\cos(\Phi+\lambda),\\ A_{\beta\alpha}&=re^{-i\lambda}\sin(\Phi+\lambda). \end{aligned} \tag{A9}$$

So

$$|T_{\beta\alpha}|^2=|D_{\beta\alpha}|^2+|A_{\beta\alpha}|^2\geq|A_{\beta\alpha}|^2. \tag{A10}$$

We see that if $\lambda=0$ ($T_{\beta\alpha}=T_{\alpha\beta}$) then $D_{\beta\alpha}$ and $A_{\beta\alpha}$ are the real and imaginary parts of $T_{\beta\alpha}$, whereas Eq. (A10) is valid regardless of the value of λ. However, there are two cases where Eq. (A10) may fail: (1) If the process is CP-violating, and so by the CPT theorem T-violating. Recent discussions[16] of the $K_L^0\to\mu^+\mu^-$ puzzle have appealed to CP-violating effects to reduce the theoretical lower limit to the process, and the breakdown of Eq. (A10) may be a further complication. (2) In the decay of a particle into four or more other particles the relation may also break down since in general it will not be possible to obtain the states $|\alpha\rangle$ and $|\beta\rangle$ from $|-\alpha\rangle$ and $|-\beta\rangle$ by means of a rotation, as was done in the derivation.

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‡ Thouron Fellow.

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