mass;[8] and also the failure to observe A_1 peaks in nondiffractive reactions.[10,11]

We have neglected the finite width of the ρ, and the effect of Bose statistics for the final like-charge pions. We have also neglected nondiffractive processes, such as the real part and energy dependence of elastic scattering. These effects would not change our basic conclusion, although including the energy dependence would raise the cross section and decrease the slope somewhat. We have also neglected loop diagrams in which the $\rho\pi$ reform a π one or more times. These diagrams would enforce two-particle unitarity, making the total probability to find a π or $\rho\pi$ at the target equal to 1. They are currently under investigation. Other possibilities are that the function $F(z)$ is not monotonic, or that the basic assumption of dominance of the intermediate state by quasi-real $\pi+\rho\pi$ is incorrect.

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Conspiracy and Regge Cuts in Pion-Nucleon Amplitudes*

Ronald W. Hanson
Department of Physics, Arizona State University, Tempe, Arizona 85281
(Received 3 May 1972)

The pion-nucleon amplitudes are analyzed in terms of the conspiracy and strong- and weak-cut Regge-pole models. The weak-cut model has serious difficulty, in that it predicts the wrong sign of the helicity-flip amplitude. The strong-cut model appears to have the correct form for the amplitudes, but has the wrong energy dependence for differential cross sections and polarizations. The conspiracy model is found not to suffer from any serious difficulty.

INTRODUCTION

Recently a model-independent analysis was used to determine the pion-nucleon amplitudes at 6-GeV/c momentum.[1] It is intended here to analyze these data in terms of three models: (1) the conspiracy model, which consists of the ρ plus the conspiring ρ' trajectories,[2,3] (2) the strong-cut model, which has the ρ plus a large absorptive cut,[4] and, finally, (3) the weak-cut model which has the ρ plus a small absorptive cut.[5]

The strong-cut model uses a smooth residue of the form $1/(t-M_\rho^2)$, with no nonsense wrong-signature zeros, in order to integrate the absorptive cut analytically. The conspiracy and weak-cut models both use Veneziano-type residues of the form $1/\Gamma(\alpha)$, which give nonsense wrong-signature zeros.[6] Thus in the weak-cut model the absorptive cuts have to be integrated numerically.

FORMALISM

The t-channel isospin decomposition of the amplitudes[7] is

$$\pi^{\pm}p \to \pi^{\pm}p = F^0 \pm F^1 ,$$
$$\pi^- p \to \pi^0 n = -\sqrt{2}F^1 . \tag{1}$$

This normalization is chosen so that when the amplitudes are crossed over to the s channel by the SU(3) crossing matrix one has relations which satisfy

$$[A(\pi^+p \to \pi^+p) - A(\pi^-p \to \pi^-p)] = \sqrt{2}\,A(\pi^-p \to \pi^0 n) . \tag{2}$$

From the optical theorem and experimental values for $\sigma_T(\pi^+p)$ and $\sigma_T(\pi^-p)$, it follows that $\mathrm{Im}F^1(t=0)$ must be negative.

The analysis will only consider s-channel helicity amplitudes of isospin 1, which corresponds to charge-exchange scattering. The amplitudes are chosen to correspond to the following formulas[8]:

$$\frac{d\sigma}{dt} = |F_{++}|^2 + |F_{+-}|^2 , \tag{3}$$

$$P\,\frac{d\sigma}{dt} = 2\,\mathrm{Im}(F_{++}F_{+-}^*) , \tag{4}$$

and

$$R\,\frac{d\sigma}{dt} = -\cos\theta_R(|F_{++}|^2 - |F_{+-}|^2) + \sin\theta_R\,\mathrm{Re}(2F_{++}F_{+-}^*) , \tag{5}$$

where θ_R is the laboratory proton recoil angle and is given by

$$\cos\theta_R = \left(\frac{-t}{4M^2 - t}\right)^{1/2}\left(\frac{E+M}{P}\right) .$$

The incident laboratory pion has energy E and momentum P, and M is the proton mass.

Both cut models are similar in origin and calculation. They both add an absorptive term to the s-channel helicity amplitudes as follows:

$$F_{\mu'\mu} = F^{\rho}_{\mu'\mu} + \lambda_{\mu'\mu}F^{\mathrm{cut}}_{\mu'\mu} , \tag{6}$$

where $F^{\rho}_{\mu'\mu}$ is the plain Regge-pole amplitude. The factor λ is used to adjust the strength of the cut contribution, and is believed to have a value of 1 to 2. Too large a λ would cause overabsorption in the smaller partial waves.

The cut term is calculated from the integral[9]

$$F^{\mathrm{cut}}_{\mu'\mu} = \frac{-i}{32\pi^2}\int d\Omega\, F^{\rho}_{\mu'\mu}F^{\mathrm{el}}_{\mu'\mu} . \tag{7}$$

The difference in strong- and weak-cut models comes from the form of the ρ amplitude used. This double integral can be reduced by using the following approximation for the elastic scattering amplitude:

$$F^{\mathrm{el}}_{\mu'\mu} = -\delta_{\mu'\mu}(i+\rho)s\sigma_T e^{Gt} , \tag{8}$$

where ρ is the ratio of the real to the imaginary part of the forward πN peak, and σ_T is the πN total cross section. The experimental values[10] used are $\rho=0$, $\sigma_T = 25$ mb, and $G = 3.75$ (GeV/c)$^{-2}$. More complicated forms for $F^{\mathrm{el}}_{\mu'\mu}$ have been suggested,[11] where the phase is a function of energy. However, eleven parameters were required as opposed to the four used here.

The cut integral is reduced to

$$F^{\mathrm{cut}}_{\mu'\mu} = -(\sigma_T/4\pi)(1-i\rho)e^{Gt} \times \int_{-\infty}^{0} \tfrac{1}{2}\,dt'\,e^{Gt'}I_n(2G(tt')^{1/2})F^{\rho}_{\mu'\mu}(t') , \tag{9}$$

where $n = |-\mu' + \mu|$ and $I_n(Z) = (-i)^n J_n(iZ)$. The cut is thus approximately constant in phase, and 180° out of phase with the ρ amplitude.

STRONG-CUT MODEL

The strong-cut model is identical to the original model of the Michigan group.[4] The structure of the scattering amplitude is derived from the strong interference between the pole term and the cut term. The amplitudes are chosen to have no nonsense wrong-signature zeros.

The s-channel helicity amplitudes are

$$F^1_{++} = \frac{-M_\rho \gamma_{++} i e^{-i\pi\alpha_\rho(t)/2}}{8q\sqrt{2\pi s}\,(t - M_\rho^2)}\left(\frac{E}{E_0}\right)^{\alpha_\rho(t)} , \tag{10}$$

$$F^1_{+-} = \frac{-M_\rho \gamma_{+-} i\sqrt{-t}\, e^{-i\pi\alpha_\rho(t)/2}}{8q\sqrt{2\pi s}\,(t - M_\rho^2)}\left(\frac{E}{E_0}\right)^{\alpha_\rho(t)} , \tag{11}$$

TABLE I. Least χ^2 for conspiracy and Regge pole-cut models.

Assumptions	χ^2 (total) 149 points	χ^2 ($d\sigma/dt$) 84 points	χ^2 (polarization) 41 points	χ^2 (amplitude) 24 points
Conspiracy	413	121	100	192
Strong cut	468	327	107	40
Weak cut	1375	142	177	1056

TABLE II. Parameter values obtained in least-χ^2 fit to data.

Assumptions	β_ρ^n	β_ρ^f	λ_{++}	λ_{+-}	$\beta_{\rho'}^n$	$\beta_{\rho'}^f$	γ_{++}	γ_{+-}
Conspiracy	−16.0	−204.0	···	···	41.8	−59.9	···	···
Strong cut	···	···	1.37	1.52	···	···	−21.2	77.0
Weak cut	−22.9	214.6	1.25	0.19	···	···	···	···

where M_ρ is the mass of the ρ, and q is the center-of-mass momentum. The ρ trajectory is chosen to be

$$\alpha_\rho(t) = \alpha_0 + \alpha_1 t, \tag{12}$$

where $\alpha_0 = 0.5$ and $\alpha_1 = 0.9$ $(\text{GeV}/c)^{-2}$. This corresponds to the exchange-degenerate trajectory through the ρ and A_2 mesons.[12]

The amplitudes are substituted into Eq. (9) and integrated analytically. The cut terms are

$$F_{++}^{\text{cut}} = \frac{-\lambda_{++}\sigma_T(1-i\rho)A_{++}}{64\pi q\sqrt{2\pi s G}} \times\left(1+\frac{d^2}{2M_\rho^4 dB^2}\right)\frac{G}{G+B}\exp\left(\frac{GBt}{G+B}\right) \tag{13}$$

and

$$F_{+-}^{\text{cut}} = \frac{-\lambda_{+-}\sigma_T\sqrt{-t}\,(1-i\rho)A_{+-}}{64\pi q\sqrt{2\pi s G}} \times\left(1+\frac{d^2}{2M_\rho^4 dB^2}\right)\left(\frac{G}{G+B}\right)^2\exp\left(\frac{GBt}{G+B}\right), \tag{14}$$

where

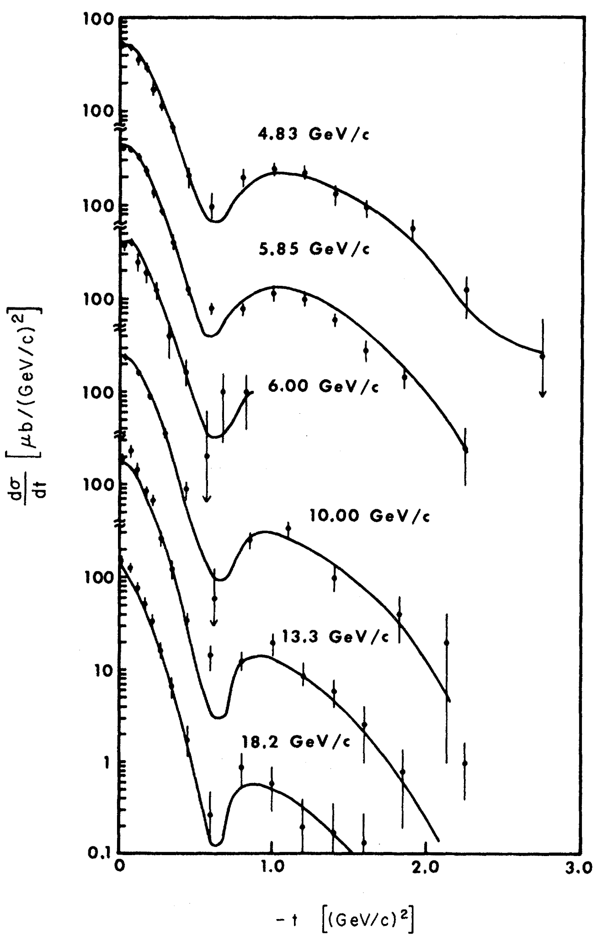

FIG. 1. Conspiracy fit to $d\sigma/dt$ for $\pi^- p \to \pi^0 n$, at laboratory momenta 4.83, 5.85, 6.00, 10.00, 13.3, and 18.2 GeV/c.

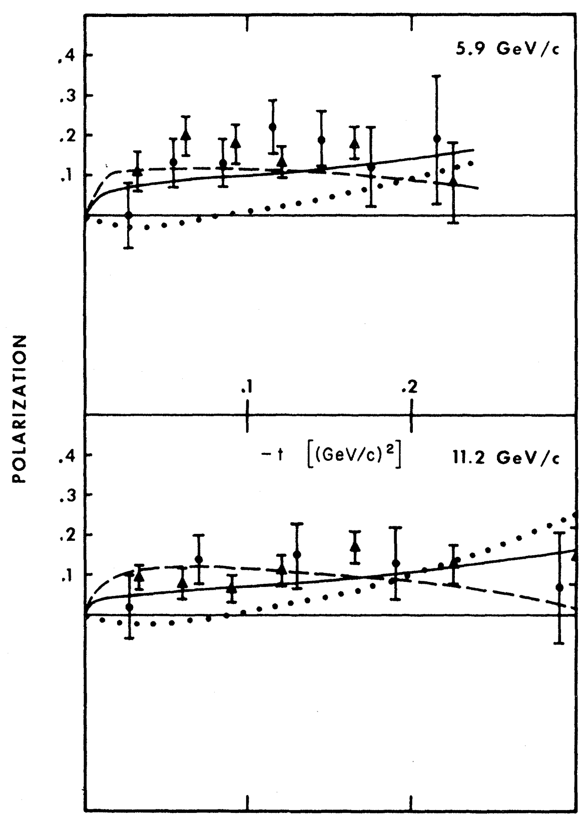

FIG. 2. Polarization of $\pi^- p \to \pi^0 n$ at 5.9- and 11.2-GeV/c momenta. Solid curve is conspiracy model, dashed curve is strong-cut model, and dotted line is weak-cut model.

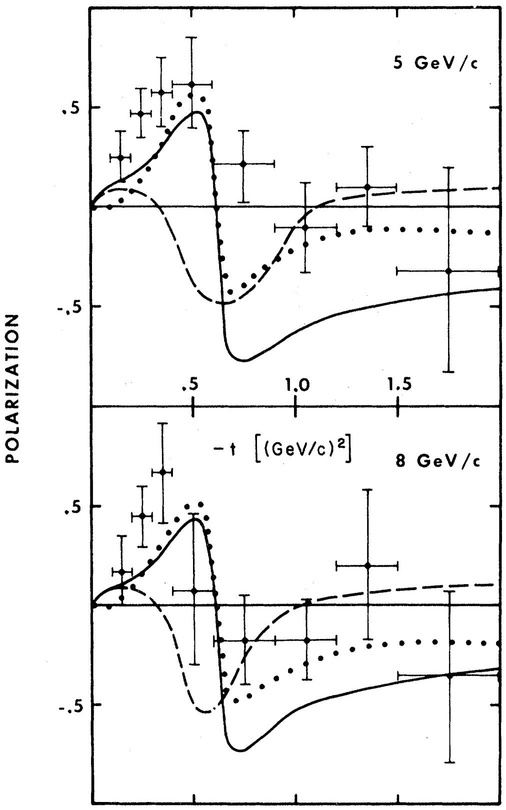

FIG. 3. Polarization of $\pi^- p \to \pi^0 n$ at 5- and 8-GeV/c momenta. Solid curve is conspiracy model, dashed curve is strong-cut model, and dotted curve is weak-cut model.

$$A_{+\pm} = \gamma_{+\pm} i e^{-i\pi\alpha_0/2}(E/E_0)^{\alpha_0},$$

$$B = (1/M_\rho^2) + \alpha_1[\ln(E/E_0) - \tfrac{1}{2}i\pi]\,.$$

The strong-cut model is left with four free parameters: two residue parameters γ_{++}, γ_{+-} and two cut-strength parameters $\lambda_{++}, \lambda_{+-}$. The value E_0 was fixed at 0.165 GeV^2.

WEAK-CUT MODEL

Both the weak-cut and conspiracy models are parametrized in the t-channel amplitudes, which are then crossed over to the s channel. The crossing matrix is

$$F_{++} = \frac{2m(1+t/4q^2)^{1/2}}{8q\sqrt{2\pi s}}\left[A' + \left(\omega - \frac{\omega + t/4m}{1 - t/4m^2}\right)B\right], \tag{15}$$

$$F_{+-} = \frac{\sqrt{-t}}{8\sqrt{2\pi s}\, q^2}\left\{EA' + \left[m(\sqrt{s} - E) - E\,\frac{\omega + t/4m}{1 - t/4m^2}\right]B\right\}, \tag{16}$$

where

$$E = \frac{s + m^2 - \mu^2}{2\sqrt{s}}, \qquad \omega = \frac{s - m^2 - \mu^2}{2m}\,.$$

The t-channel amplitudes are parametrized by

$$A' = \frac{\beta_\rho^\eta\, \xi_\rho (as)^{\alpha_\rho}}{\Gamma(\alpha_\rho)} \tag{17}$$

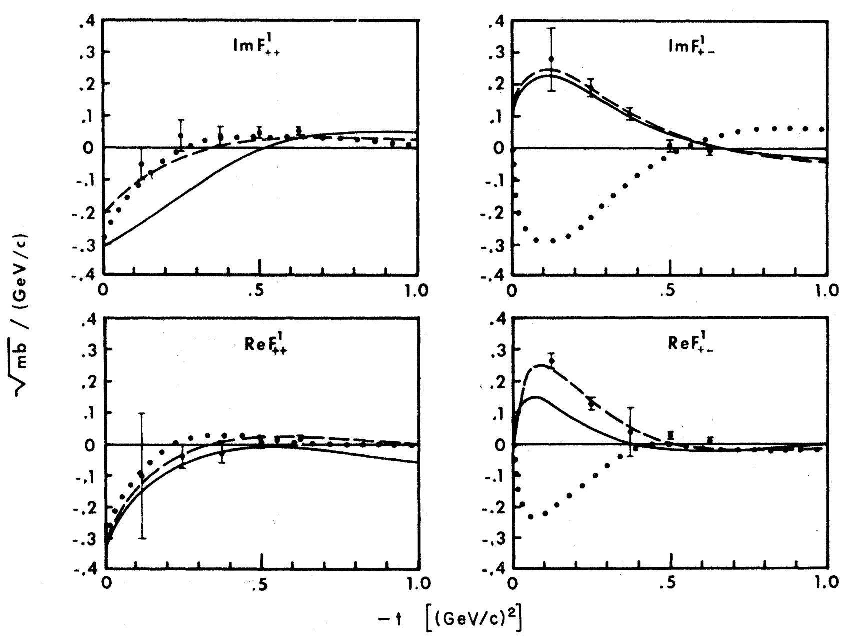

FIG. 4. Helicity amplitudes at 6-GeV/c momentum. Data points are from Halzen and Michael, Ref. 1. Solid curve is conspiracy model, dashed curve is strong-cut model, and dotted curve is weak-cut model.

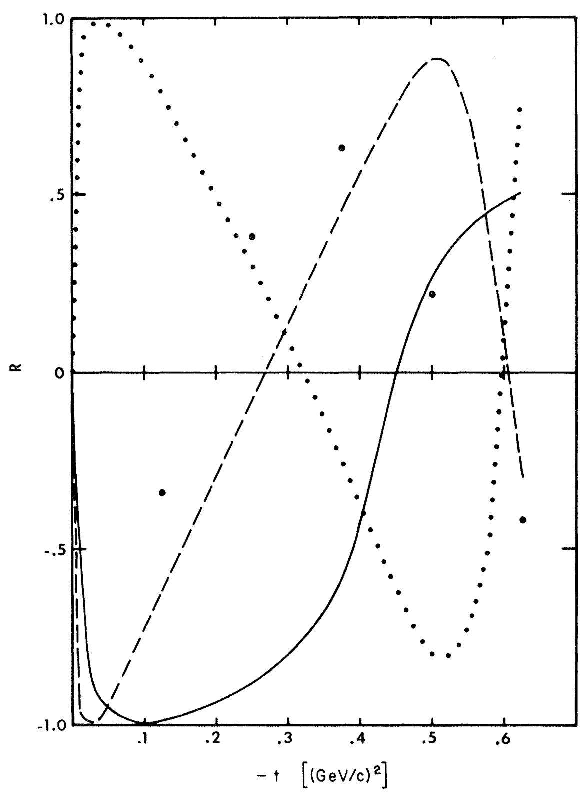

FIG. 5. R-parameter predictions based on helicity amplitudes. Points are from Halzen and Michael, Ref. 1. Solid curve is conspiracy model, dashed curve is strong-cut model, and dotted line is weak-cut model.

and

$$B=\frac{\beta_\rho^f \xi_\rho (as)^{\alpha_\rho -1}}{\Gamma(\alpha_\rho)}\ , \tag{18}$$

where $\xi_\rho \mp (1-e^{-i\pi\alpha_\rho})/\sin(\pi\alpha_\rho)$ is the signature factor and the trajectory is given by

$$\alpha_\rho = 0.5 + at, \quad a = 0.9\ (\text{GeV}/c)^{-2}\ . \tag{19}$$

The amplitudes are chosen to have nonsense wrong-signature zeros.

The t-channel amplitudes [Eqs. (17) and (18)] are crossed over to the s channel by Eqs. (15) and (16). This then constitutes the plain ρ contribution to Eq. (6). The cut term is thus integrated numerically and added to $F^\rho_{\mu'\mu}$ to form the weak-cut model. The weak-cut model is left with two residue parameters β^n_ρ, β^f_ρ and two cut-strength parameters λ_{++}, λ_{+-}.

CONSPIRACY MODEL

The conspiracy model assumes that in addition to the ρ trajectory there exists a conspiring ρ' which has the same quantum numbers but is exchange-degenerate with the π-B trajectory. This trajectory is given by

$$\alpha_{\rho'} = -0.02 + at, \quad a = 0.9\ (\text{GeV}/c)^{-2}\ . \tag{20}$$

Thus the t-channel amplitudes become

$$A' = A'_\rho + A'_{\rho'}\ , \tag{21}$$

$$B = B_\rho + B_{\rho'}\ , \tag{22}$$

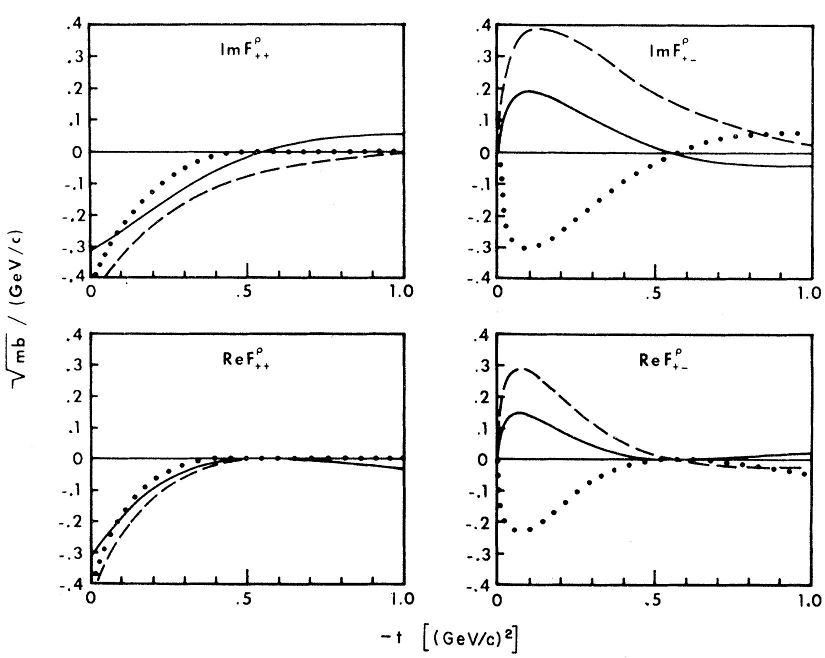

FIG. 6. The ρ contribution to the s-channel amplitude. Solid curve is conspiracy model, dashed curve is strong-cut model, and dotted line is weak-cut model.

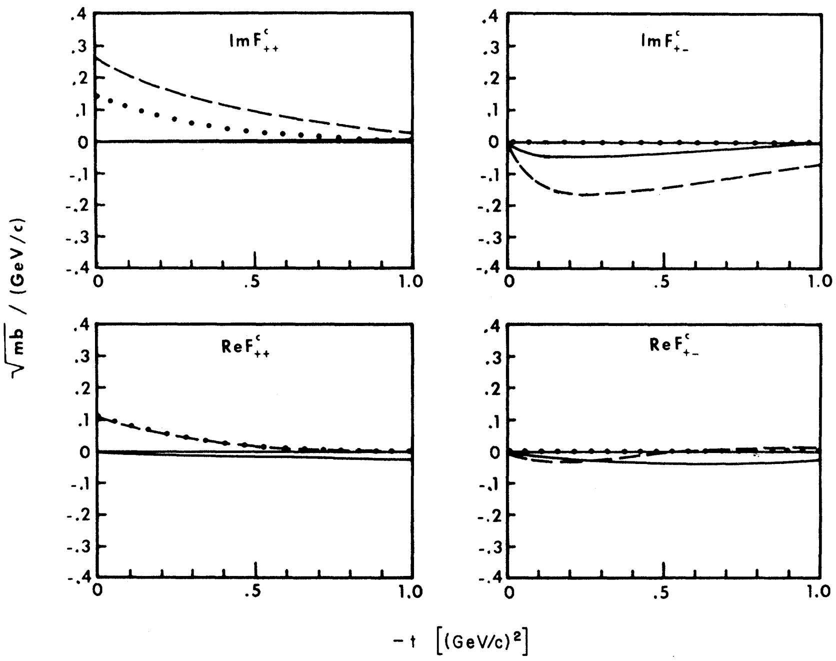

FIG. 7. The cut and conspiracy contribution to the s-channel amplitudes. Solid curve is conspiracy model, dashed curve is strong-cut model, and dotted line is weak-cut model.

where the ρ' amplitudes are given by

$$A'_{\rho'} = \frac{t\beta^n_{\rho'}\xi_{\rho'}}{\Gamma(\alpha_{\rho'})}(as)^{\alpha_{\rho'}}, \tag{23}$$

$$B_{\rho'} = \frac{\beta^f_{\rho'}\xi_{\rho'}(as)^{\alpha_\rho -1}}{\alpha_{\rho'}\Gamma(\alpha_{\rho'})}. \tag{24}$$

The factor t in Eq. (23) is a requirement of the conspiracy model.[13] Thus the conspiracy model is left with four free residue parameters β^n_ρ, β^f_ρ, $\beta^n_{\rho'}$, and $\beta^f_{\rho'}$. Unlike the cut models, the conspiring term changes phase with t, such that there is a constant phase difference of 47° between the ρ and ρ' terms.

RESULTS

The three models were used to fit simultaneously π^-p charge-exchange differential cross sections, polarizations, and helicity amplitudes. It was found that if the amplitudes were fitted separately, the resulting solutions gave unacceptable results for the differential cross sections and polarizations. There were 84 differential cross-section data points[14] used, varying in laboratory momentum from 4.83 to 18.2 GeV/c and having t values out to -3 (GeV/c)^2. There were 41 polarization points,[15] ranging in laboratory momentum from 5 to 11.2 GeV/c and having t values out to -2 (GeV/c)^2. The 24 helicity amplitude points come from Ref. 1.

The resulting best χ^2 values are given in Table I, with the corresponding parameter values in Table II. An additional solution was found for the weak-cut model. This solution gave the correct sign for the s-channel helicity-flip amplitude; however, the λ factor was approximately zero, which corresponds to a plain Regge-pole and gives a zero for polarization. This result is in agreement with the orginal amplitude analysis of Halzen and Michael.[1]

Figure 1 shows the best fit of differential cross sections for the conspiracy model. The weak-cut model gave very similiar results, but the strong-cut model did poorly (see Table I), especially around the dip at $t=-0.6$ (GeV/c)^2 and at larger t values.

The polarization results for all three models are given in Figs. 2 and 3. It should be noted that the strong-cut model has a zero around $t=-0.3$, which the other two models do not show.

Figure 4 shows the fits to amplitudes at 6 GeV/c. The data points are from Halzen and Michael.[1] Although the data are available only out to $t=-0.6$ (GeV/c)^2, they clearly indicate serious difficulty for the weak-cut model. As mentioned earlier, there is another solution for the weak-cut model, but this corresponds to a simple ρ model which gives zero for the polarization. On the basis of the amplitudes, the predictions for the R measurement are made (see Fig. 5). The points correspond to predictions made by Halzen and Michael.[1]

Figures 6 and 7 show the contributions to the s channel for the ρ alone (Fig. 6) and for the cut terms (Fig. 7).

We have examined the helicity amplitudes of isospin one for pion-nucleon scattering in terms of three models, each having four free parameters. By looking at the helicity amplitudes (Fig. 4), we find that there is serious difficulty for the weak-cut model. Although the strong-cut model appears to have the approximate form for the amplitudes, it has the wrong polarization structure and energy dependence for differential cross sections. The conspiracy appears to have no serious difficulty for any of the experimental quantities.

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Lower Bound on the Magnitude of the Rate of $K^+ \to \pi^+ + e^+ + e^-$ †

Gino Segrè and David Wilkinson‡

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

(Received 3 August 1972)

It is shown that a lower bound on the decay rate $K^+ \to \pi^+ e^+ e^-$ can be obtained by calculation of the absorptive part of the amplitude to which only the connected three-pion intermediate state contributes significantly. Some remarks on pole-model calculations are also included.

I. INTRODUCTION

A great deal of effort has been expended on the search for neutral currents in weak interactions; even if such currents are not present in the basic weak Hamiltonian, forbidden processes such as $\nu + p \to \nu + p$ should appear at some point because of higher-order weak-interaction effects. This has led to an impressive number of experimental searches for either neutral currents or higher-order weak-interaction effects, with as of yet no evidence for the existence of either, other than double β decay and the $K_L - K_S$ mass difference (both due presumably to higher-order weak interactions).

Some processes, forbidden to order G_F (the weak-interaction decay constant equal to $10^{-5}/M_N^2$, where M_N is the nucleon mass), are allowed by a combination of weak and electromagnetic interactions. The prime example is $K_L \to \mu^+\mu^-$ proceeding by way of an intermediate two-photon state. Despite an intensive search,[1] this rare decay mode has not been seen; moreover one obtains a lower bound on the decay rate by making use of unitarity,[2] the known decay rate for $K_L \to \gamma\gamma$, and the easily calculated matrix element for $\gamma\gamma \to \mu^+\mu^-$. This