Eikonal Interpretation of the Deck Effect*

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The Deck model for diffraction dissociation is revised to satisfy an eikonal representation. The theory is applied unsuccessfully to the A_1 enhancement in $\pi^- p \to \rho^0\pi^- p$. The conventional double-Regge model of this reaction is shown to be inadequate also. Further directions are suggested.

Diffractive dissociation is supposed to dominate reactions for which the exchanged quantum numbers are those of the vacuum, and produce approximately constant cross sections at high energy. Consider the reaction $\pi^- p \to \rho^0\pi^- p$ at low $\rho\pi$ invariant mass. From an optical-model point of view,[1] the incident π can be thought of as a superposition of virtual "constituents" – which might be quarks, partons, or possibly physical particles like $\rho\pi$, $N\bar{N}$, etc. The relative amplitudes in this superposition are altered by the absorptive interaction with the proton, resulting in a final state which contains π^-, $\rho^0\pi^-$, and other systems with the same internal quantum numbers as the original π^-. These systems need not have the same spin and parity as the pion, because the spatial characteristics of the superposition have been changed. A second view of diffractive dissociation[2,3] is represented by the "Deck effect" diagram of Fig. 1(a). This diagram has a pole near the physical region in $(k_0 - k_1)^2$. With any reasonable form factor, it produces a low-mass $\rho\pi$ enhancement, whose magnitude is constrained by the known $\rho\pi\pi$ coupling, and whose asymptotic energy dependence is the same as elastic scattering.

This paper attempts to synthesize the above points of view by including only physical π and $\rho\pi$ states in the superposition of constituents. The attempt is based on ideas contained in a previous paper,[4] to which the reader is referred for details. We use old-fashioned perturbation theory, in an "infinite"-momentum frame, such as the laboratory one at very high energy. In this frame, the dissociation into constituents takes place a long distance, $\propto s$, in front of the target. The interactions between constituents are slowed down by time dilation, and are therefore negligible inside the target, i.e., the "impulse approximation" is valid. Virtual particles are on their mass shells in old-fashioned perturbation theory, and energy is not conserved at vertices. We assume that off-shell effects depend on the distance from the energy shell at infinite momentum, rather than on the Lorentz-scalar distance from the mass shell. This assumption is necessary to maintain an optical model, and makes our result different from all previous calculations of the Deck effect.

Our amplitude is given by the diagrams of Fig. 1. The double-scattering terms [1(c), 1(d)] are calculated in the eikonal limit.[5] There is no diagram where both 0 and the 12 system scatter because of the impulse approximation. The amplitude is derived in Ref. 4, with the omission of Fig. 1(e). Another derivation follows from expressing the elastic amplitudes as sums of exchanges of a fictitious vector meson, including all crossed exchanges in the eikonal limit[5]; then summing all such exchanges to particles 0, 1, and 2 before and after dissociation. With spin and off-shell effects to be inserted later, the result is

$$M=\frac{1}{2\pi^2}\int d\vec{b}_1 d\vec{b}_2 \exp(i\vec{b}_1\cdot\vec{q}_1+i\vec{b}_2\cdot\vec{q}_2)\{\exp[i\chi_1(|\vec{b}_1|)+i\chi_2(|\vec{b}_2|)]-\exp[i\chi_0(|\vec{b}_0|)]\}\int d\vec{q}\exp[i(\vec{b}_1-\vec{b}_2)\cdot\vec{q}]/z\,, \quad (1)$$

where particles 1 and 2 have transverse momenta $\vec{q}_1$ and $\vec{q}_2$, and longitudinal momenta which are fractions x and $(1-x)$ of the "infinite" momentum, with $0<x<1$. The χ_j's are eikonal phase shifts, which are related to the elastic scattering amplitudes by

$$f_j(t)=-4\pi i\int b\,db\,J_0(b\sqrt{-t})\{\exp[i\chi_j(b)]-1\}\,, \quad (2)$$

with normalization $\sigma_j^{\text{total}}=\text{Im} f_j(0)$. $\vec{b}_1$ and $\vec{b}_2$ are the impact parameters of 1 and 2, and $\vec{b}_0=x\vec{b}_1+(1-x)\vec{b}_2$ is the impact parameter of 0 by angular momentum

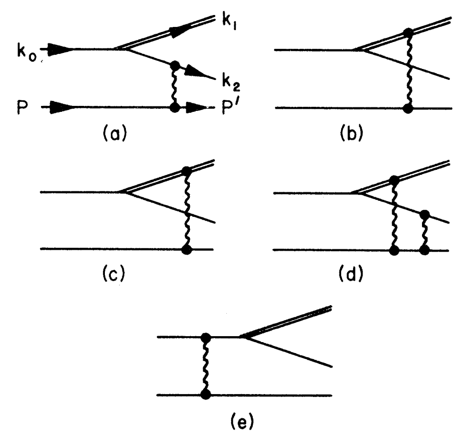

FIG. 1. Feynman diagrams for diffractive dissociation. The four-momenta for $\pi^- p \to \rho^0 \pi^- p$ are k_0, P, k_1, k_2, P'. A wavy line denotes elastic scattering.

conservation. $\vec{q}$ is the transverse momentum of 1 before interaction with the target in Figs. 1(a)–1(d). The denominator

$$z = m_{\text{int}}^{\;2} - m_0^{\;2} = \frac{\vec{q}^{\,2} + m_1^{\;2}}{x} + \frac{\vec{q}^{\,2} + m_2^{\;2}}{1-x} - m_0^{\;2} \tag{3}$$

is proportional to the distance off the energy shell, i.e., to the nonconservation of energy at the dissociation vertex. m_{int} is the invariant mass of the 12 system after dissociation, but before scattering in 1(a)–1(d). The amplitude is proportional to the difference between the absorption factor

$$\exp[i\chi_1(|\vec{b}_1|) + i\chi_2(|\vec{b}_2|)]$$

of the $\rho\pi$ system and the absorption factor

$$\exp[i\chi_0(|\vec{b}_0|)]$$

of the undissociated π. It is therefore *peripheral*, i.e., biggest at the surface of the target, unlike elastic scattering. The individual contributions from single scattering [1(a), 1(b), 1(e)] and double scattering [1(c) + 1(d)] can be separated using

$$\begin{aligned} e^{i\chi_1 + i\chi_2} - e^{i\chi_0} &= (e^{i\chi_1} - 1) + (e^{i\chi_2} - 1) \\ &\quad - (e^{i\chi_0} - 1) \\ &\quad + (e^{i\chi_1} - 1)(e^{i\chi_2} - 1)\,. \end{aligned} \tag{4}$$

All of the diagrams contribute to a pole at $z = 0$ in the integrand of (1). The integration region covers $(m_1 + m_2)^2 - m_0^{\;2} < z < \infty$. For the single-scattering terms, the $\vec{b}_1$ and $\vec{b}_2$ integrals produce a δ function, such that in the forward direction $z = m^{*2} - m_0^{\;2}$, where m^* is the final $\rho\pi$ invariant mass. To take account of off-shell effects, we *introduce a function F(z) into (1)*, which reduces the amplitude for dissociation into virtual states of high mass. We require $F(0) = 1.0$ to preserve the residue at the pole, and assume F falls to zero on the scale of 1 GeV^2. The three pole terms then have approximately the same magnitude [1(e) having opposite sign], since they are about the same distance from the energy shell. This is what one would expect according to an optical model. It is very different from the conventional Feynman-diagram[2] or double-Regge[3] point of view, where form factors depend on the distance to the pole in four-momentum transfer squared, and the "pion-exchange" pole [1(a)] dominates, since it alone is $\approx m_\pi^{\;2}$ from the physical region.

Spin must be introduced according to old-fashioned perturbation theory also, in order to preserve the eikonal picture. In Fig. 1(e), for $\pi^- p \to \rho^0 \pi^- p$, we put

$$g_{\rho\pi\pi}\,\epsilon_\lambda^*(k_1)\cdot(k_0' + k_2)\,,$$

where

$$\vec{k}_0' = \vec{k}_1 + \vec{k}_2 \quad \text{and} \quad (k_0')_0 = (\vec{k}_0'^{\,2} + m_0^{\;2})^{1/2}\,.$$

As a result, this diagram contributes both 0^- and 1^+ $\rho\pi$ systems. Interpreting the other diagrams similarly, and assuming s-channel helicity conservation for the ρ elastic scattering,[6] leads in each case to the same function of $\vec{q}$ and $\vec{x}$. The result is a wave-function factor

$$g_{\rho\pi\pi} \times \begin{cases} \dfrac{\sqrt{2}}{x}\,(\mp q_x + i q_y) & (\lambda = \pm 1) \\[2ex] \dfrac{2-x}{2m_\rho x(1-x)}\,[\vec{q}^{\,2} + x^2 m_\pi^{\;2} - (1-x) m_\rho^{\;2}] & (\lambda = 0) \end{cases} \tag{5}$$

to be included in (1), where λ is the ρ helicity in the infinite-momentum frame. In Figs. 1(a)–1(d), the π dissociates into 0^- and 1^+ $\rho\pi$ systems only, but interaction with the target produces additional spin parities. Our normalization is defined by

$$\frac{d\sigma}{dt\,dm^*\,d\Omega} = \pi p x(1-x)\frac{d\sigma}{d\vec{q}_1\,d\vec{q}_2\,dx} = \frac{p}{256\pi^4}\sum_\lambda |M_\lambda|^2\,, \tag{6}$$

where

$$m^{*2} = (k_1 + k_2)^2 = \frac{\vec{q}_1^{\;2} + m_1^{\;2}}{x} + \frac{\vec{q}_2^{\;2} + m_2^{\;2}}{1-x} - (\vec{q}_1 + \vec{q}_2)^2$$

and

$$t = (P - P')^2 = -(\vec{q}_1 + \vec{q}_2)^2\,.$$

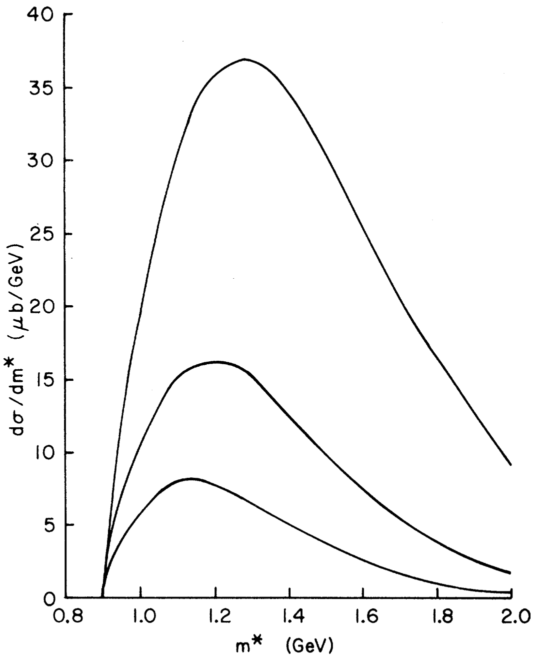

FIG. 2. Calculated mass distributions for $\pi^- p \to \rho^0 \pi^- p$, with various values of the parameter c (see text).

The momentum is

$$p = [m^{*2} - (m_1 + m_2)^2]^{1/2} [m^{*2} - (m_1 - m_2)^2]^{1/2} / 2m^*$$

and Ω is the solid angle of 1 in the 12 rest frame.

The $\rho\pi$ mass distribution predicted by this model is shown in Fig. 2. The parameters used are $f(t) = i\sigma e^{at/2}$ with $a = 10\ \text{GeV}^{-2}$ and $\sigma_\rho = \sigma_\pi = 25$ mb, $g_{\rho\pi\pi}{}^2/4\pi = 2.4$, and $F(z) = e^{-cz}$ with various values of c. The general shape of this curve can be made to agree with experiments from 7 to 20 GeV/c,[7] except of course that the A_2 resonance peak is omitted, by choosing $c = 1.2\ \text{GeV}^{-2}$. However, the magnitude of $d\sigma/dm^*$ is then too small by about a factor of 50. This comes about partly because the "form factor" $F(z)$ decreases significantly between the pole, at $z = 0$, and the beginning of the physical region, at $z = 0.8\ \text{GeV}^2$, if it is made to fall sharply enough to reproduce the observed shape of $d\sigma/dm^*$. A second difficulty with the model is that the dependence on momentum transfer is too strong. At small $|t|$, we have $d\sigma/dt dm^* \propto e^{At}$, where A decreases with increasing m^*, but is almost 20 GeV^{-2} at $m^* = 1.1$ GeV, whereas experimentally,[7,8] $A \approx 12\ \text{GeV}^{-2}$. This comes about because the range of interaction in impact parameter space is a combination of the range associated with elastic scattering and the separation $\vec{b}_1 - \vec{b}_2$ of the virtual π and ρ; and because the amplitude is peripheral. For larger $|t|$, there is a break at $|t| \approx 0.3$ followed by a smaller slope as a result of double scattering.

It is widely believed that the pion-exchange diagram [Fig. 1(a)] by itself can explain the data, if the pion is Reggeized.[3] This "double-Regge" hypothesis is highly questionable for the following reasons. If the pole term is calculated with no form factor, using either the usual $2g_{\rho\pi\pi} k_0 \cdot \epsilon_\lambda^*(k_1)$ spin coupling, or our infinite momentum coupling, $d\sigma/dm^*$ is found to have about the right magnitude at $m^* = 1.1$ GeV, but to rise monotonically with m^*. One would expect that a form factor (Regge or otherwise) which is strong enough to produce the desired peak at $m^* = 1.1$ GeV would then make the magnitude too small. When the matrix element squared is calculated in Ref. 3, the sum over ρ helicities

$$\sum_\lambda |k_0 \cdot \epsilon_\lambda^*(k_1)|^2 = \frac{(k_0 \cdot k_1)^2}{m_\rho{}^2} - m_\pi{}^2$$

is replaced by its value at the pole, $m_\rho{}^2/4 - m_\pi{}^2$. This makes $d\sigma/dm^* \to 0$ as $m^* \to \infty$. However, it is not equivalent to the more defensible procedure of replacing the amplitude factors $k_0 \cdot \epsilon_\lambda^*(k_1)$ by their values at the pole. In fact, it makes the cross section for helicity ±1 in the laboratory frame negative. Secondly, the $\rho\pi$ masses of interest are so small that one is uncertain how, as well as whether, to Reggeize the pion. A Regge factor $(\text{"}s\text{"}/s_0)^{\alpha(t_{\pi p})}$ is used in Ref. 3, where

$$\text{"}s\text{"} = m^{*2} - t_{pp} - m_\pi{}^2 - (m_\rho{}^2 - m_\pi{}^2 - t_{\pi p}) \frac{t_{\pi p} + t_{pp} - m_\pi{}^2}{2t_{\pi p}}$$

and $s_0 = 1.0\ \text{GeV}^2$. "s"$/s_0$ is not large compared to 1, but rather lies between 0.4 and 0.8 at $m^* = 1.1$ GeV. Since $\alpha(t_{\pi p}) < 0$ for the pion trajectory, $(\text{"}s\text{"}/s_0)^{\alpha(t_{\pi p})}$ is actually larger than 1. An appeal to duality[9] in this situation is unpersuasive, since duality is expected to work for imaginary parts, while pion exchange is mainly real; and also since the duality hypothesis would have to be applied to a four-point function which has the Pomeranchukon as one of its legs.

We conclude that a satisfactory understanding of diffractive three-pion production has not been achieved using the Deck effect,[2] interpreted either according to the double-Regge model,[3] or to the eikonal model presented here. In order to make the eikonal model work, it would be necessary to somehow favor dissociation into low-mass $\rho\pi$ systems at the expense of high-mass ones. This could result from a resonance in the $\rho\pi$ system, or perhaps equally well from a nonresonant attractive interaction. The existence of an A_1 meson[10] therefore remains unproved, and in fact, it appears unlikely, in view of the failure to observe rapid variation in the phase of the 1^+ $\rho\pi$ system relative to other partial waves, as a function of mass;[8] and also the failure to observe A_1 peaks in nondiffractive reactions.[10,11]

We have neglected the finite width of the ρ, and the effect of Bose statistics for the final like-charge pions. We have also neglected nondiffractive processes, such as the real part and energy dependence of elastic scattering. These effects would not change our basic conclusion, although including the energy dependence would raise the cross section and decrease the slope somewhat. We have also neglected loop diagrams in which the $\rho\pi$ reform a π one or more times. These diagrams would enforce two-particle unitarity, making the total probability to find a π or $\rho\pi$ at the target equal to 1. They are currently under investigation. Other possibilities are that the function $F(z)$ is not monotonic, or that the basic assumption of dominance of the intermediate state by quasi-real $\pi + \rho\pi$ is incorrect.

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Conspiracy and Regge Cuts in Pion-Nucleon Amplitudes*

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The pion-nucleon amplitudes are analyzed in terms of the conspiracy and strong- and weak-cut Regge-pole models. The weak-cut model has serious difficulty, in that it predicts the wrong sign of the helicity-flip amplitude. The strong-cut model appears to have the correct form for the amplitudes, but has the wrong energy dependence for differential cross sections and polarizations. The conspiracy model is found not to suffer from any serious difficulty.

INTRODUCTION

Recently a model-independent analysis was used to determine the pion-nucleon amplitudes at 6-GeV/c momentum.[1] It is intended here to analyze these data in terms of three models: (1) the conspiracy model, which consists of the ρ plus the conspiring ρ' trajectories,[2,3] (2) the strong-cut model, which has the ρ plus a large absorptive cut,[4] and, finally, (3) the weak-cut model which has the ρ plus a small absorptive cut.[5]

The strong-cut model uses a smooth residue of the form $1/(t-M_\rho^2)$, with no nonsense wrong-signature zeros, in order to integrate the absorptive cut analytically. The conspiracy and weak-cut models both use Veneziano-type residues of the form $1/\Gamma(\alpha)$, which give nonsense wrong-signature zeros.[6] Thus in the weak-cut model the absorp-