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# Low-Energy Limit in Hard-Pion Amplitudes and Magnetic Moment of Charged $\rho$ Mesons

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A method previously used for deriving Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) -type relations is applied here to the radiative decay $\rho^+ \to \pi^+\pi^0\gamma$. Comparing the amplitude calculated with the hard-pion technique to its exactly calculable (photon) low-energy limit, one obtains as consistency conditions the first Weinberg sum rule, the modified KSRF relation, and the magnetic moments of $A_1$ and $\rho^+$. The value for the last one is further investigated in a model devoid of the single-particle approximation, the result consisting of upper and lower bounds for it, namely, $16\pi^2\alpha^2 g_\rho^2/(m_\rho^4 \int \sigma_{e^+e^-\to n}\, ds) < \mu_\rho < 2$.

## I. INTRODUCTION

It has been shown recently[1] that by considering the low-energy limit for the process $\rho^0 \to \pi^+\pi^-\gamma$ in conjunction with the amplitude calculated by the hard-pion technique, a modified Kawarabayshi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation is obtained as a consistency condition. The method suggested in Ref. 1, which was applied there to radiative decays of various neutral vector mesons in order to derive KSRF-type relations, consists in short of the following. Terms up to order zero in the photon momentum $k_\mu$ of the radiative amplitude $V \to P + P' + \gamma$ (where $V$ denotes a vector meson and $P, P'$ denote pseudoscalar mesons) are exactly calculable in terms of the strong vertex $F_{VP'P}$, by virtue of the low-energy theorem.[2-4] On the other hand, an explicit calculation of the $V \to P + P' + \gamma$ process is performed by the use of current algebra, partial conservation of axial-vector current (PCAC), and the hard-pion technique as developed by Schnitzer, Weinberg, and Gerstein.[5,6] Equating the terms to order $k^{-1}$ or to order $k^0$ of the two calculations, KSRF-type relations with all particles on the mass shell were obtained,[1] thus overcoming certain ambiguities present in previous derivations.

In the present work we apply in Sec. II this method to the appropriate radiative decay of charged $\rho$ mesons, namely, $\rho^+ \to \pi^+\pi^0\gamma$. Equating the terms to order $k^{-1}$ and $k^0$ of the amplitude calculated by current algebra and the hard-pion technique to those obtained from the low-energy theorem expansion, we derive as consistency conditions (i) the first Weinberg sum rule,[7] (ii) the modified on-mass-shell KSRF relation, (iii) the magnetic moment of $\rho^+$, (iv) the anomalous magnetic moment of $A_1$.

The results of Sec. II are obtained by using single-particle dominance for the various propagator functions involved. In Sec. III we do not make this approximation and we study further the $\rho^+$ magnetic moment with the aid of Nutbrown's model[8] for the three-point function of three vector currents. Our results are summarized and discussed in Sec. IV.

## II. COMPARISON OF THE HARD-PION AMPLITUDE AND THE LOW-ENERGY EXPANSION FOR $\rho^+ \to \pi^+\pi^0\gamma$

According to the low-energy theorem,[2-4] terms up to order zero in the photon momentum of the radiative amplitude $\rho^+ \to \pi^+\pi^0\gamma$ are exactly calculable in terms of the strong vertex $f_{\rho\pi\pi}(m_\rho^2, m_\pi^2, m_\pi^2)$. We also recall that the radiative decay amplitude can be separated into an inner-bremsstrahlung part and a direct part, which are separately gauge-invariant. The latter has only terms of order $k$ and higher[1,3,4]; thus the low-energy theorem involves essentially the first two terms of the expansion of the inner-bremsstrahlung part. We use the following definitions for the interaction among $\pi$'s, $\rho$'s, and the electromagnetic current $J^{(\text{em})}$, with self-evident notation (throughout this article we employ units in which $\hbar = c = 1$):

$$(2\pi)^3 \langle \pi^+(p) | J_\mu^{(\text{em})}(0) | \pi^+(q) \rangle = eK((p-q)^2)(p+q)_\mu, \quad K(0)=1 \tag{1}$$

$$i(2\pi)^3 \langle \pi^+(p)\pi^0(q) | \rho^+(Q), \lambda \rangle = f_{\rho\pi\pi}(p-q)\cdot\epsilon(\lambda), \tag{2}$$

$$(2\pi)^3 \langle \rho^+(p), \lambda_1 | J_\mu^{(\text{em})} | \rho^+(q), \lambda_2 \rangle = e\{p\cdot\epsilon(\lambda_2) q\cdot\epsilon^*(\lambda_1) T_1(t)(p+q)_\mu - \epsilon^*(\lambda_1)\cdot\epsilon(\lambda_2) T_2(t)(p+q)_\mu + [\epsilon_\mu(\lambda_2)\epsilon^*(\lambda_1)\cdot q + \epsilon_\mu^*(\lambda_1)\epsilon(\lambda_2)\cdot p]\, T_3(t)\}, \tag{3}$$

where $\epsilon(\lambda)$ are the $\rho^+$ polarization vectors and $t = (p-q)^2$. The form factors $T_1(t)$, $T_2(t)$, $T_3(t)$ are related to the static multipole moments of the charged vector meson by[9]

$$T_1(0)=\frac{1}{2m_\rho}(Q_\rho+1-\mu_\rho)\ , \tag{4a}$$

$$T_2(0)=1\ , \tag{4b}$$

$$T_3(0)=\mu_\rho\ . \tag{4c}$$

$\mu_\rho$ is the magnetic dipole moment of the charged $\rho$ meson (in units of $\rho$ magnetons) and $Q_\rho$ its electric quadrupole moment (in units of $m_\rho^{-2}$).

Now using (1)–(3) and the low-energy theorem, we find the following expression for the amplitude $\rho^+_{\lambda_2}(Q)\to\pi^+(p)+\pi^0(q)+\gamma_{\lambda_1}(k)$, where $Q,p,q,k$ are the four-momenta of the particles and $\lambda_1,\lambda_2$ are the helicities:

$$\begin{aligned}T_{\lambda_1\lambda_2}(\rho^+\to\pi^+\pi^0\gamma)&=ef_{\rho\pi\pi}\left\{(p-q)\cdot\epsilon(\lambda_2,Q)\left[\frac{p\cdot\epsilon^*(\lambda_1,k)}{p\cdot k}-\frac{Q\cdot\epsilon^*(\lambda_1,k)}{Q\cdot k}\right]-\epsilon^*(\lambda_1,k)\cdot\epsilon(\lambda_2,Q)+\frac{[k\cdot\epsilon(\lambda_2,Q)][p\cdot\epsilon^*(\lambda_1,k)]}{p\cdot k}\right.\\&\left.+\frac{\mu_\rho/e}{2Q\cdot k}\{k\cdot(p-q)[\epsilon^*(\lambda_1,k)\cdot\epsilon(\lambda_2,Q)]-[(p-q)\cdot\epsilon^*(\lambda_1,k)]\cdot[k\cdot\epsilon(\lambda_2,Q)]\}\right\}+O(k)+\cdots\ .\end{aligned} \tag{5}$$

We note that in (5) the terms of order $k^{-1}$ and $k^0$ are separately gauge-invariant and $f_{\rho\pi\pi}$ is the strong coupling constant with all particles on the mass shell, i.e., $f_{\rho\pi\pi}\equiv f_{\rho\pi\pi}(m_\rho^2,m_\pi^2,m_\pi^2)$.

In order to apply the technique[1] outlined in Sec. I, we now need the expression for the $\rho^+\to\pi^+\pi^0\gamma$ calculated by using current algebra and the hard-pion technique. Such a calculation was already performed by Chaudhuri and Dutt.[10] Those authors obtained the radiative $\rho^+\to\pi^+\pi^0\gamma$ amplitude with the aid of the hard-pion current-algebra technique developed by Gerstein and Schnitzer.[6] In arriving at the expression for this amplitude one uses the SU(2)⊗SU(2) chiral algebra, as well as the field-current identity and partial conservation of the axial-vector current in the form, respectively,

$$J^\mu_a(x)=g_\rho\rho^\mu_a(x)\ , \tag{6}$$

$$\partial_\mu A^\mu_a(x)=F_\pi m_\pi^2\pi_a(x)\ . \tag{7}$$

$\rho^\mu_a(x),\pi_a(x)$ are the extrapolating fields for the $\rho$ and $\pi$ mesons, and $J^\mu_a(x),A^\mu_a(x)$ are the vector and axial-vector currents. For the electromagnetic current one takes

$$J^\mu_{\text{em}}(x)=e\left[J^\mu_3(x)+\frac{1}{\sqrt3}J^\mu_8(x)\right]\ . \tag{8}$$

Using formula (38) of Ref. 6 for the four-point function as well as the same notation as these authors, we arrive at the expression for $\rho^+\to\pi^+\pi^0\gamma$, of which we present here explicitly the terms to order $1/k$ and $k^0$:

$$\begin{aligned}T_{\lambda_1\lambda_2}(\rho^+\to\pi^+\pi^0\gamma)=\frac{eg_\rho^3}{F_\pi^2m_\rho^2}&\left\{-\frac{C_AC_V^{-2}}{m_A^2}[(2+\delta)(\epsilon^*\cdot q)(\epsilon\cdot p)-(1+\delta)(\epsilon^*\cdot p)(\epsilon\cdot q)-(\epsilon^*\cdot\epsilon)(p\cdot q)]+\frac{C_V^{-1}}{2}(\epsilon^*\cdot\epsilon)\right.\\&-\frac{C_V^{-1}}{4k\cdot Q}y[2(\epsilon\cdot k)(\epsilon^*\cdot(p-q))+2(\epsilon^*\cdot(p+q))(\epsilon\cdot(p-q))-2(\epsilon^*\cdot\epsilon)(k\cdot(p-q))]\\&+\frac{C_AC_V^{-2}}{m_A^2}\left[(\epsilon^*\cdot q)(\epsilon\cdot p)-(\epsilon^*\cdot\epsilon)\left(m_\rho^2-m_\pi^2+\delta\frac{m_\rho^2}{2}\right)+\delta((\epsilon^*\cdot p)(\epsilon\cdot q)-(\epsilon^*\cdot q)(\epsilon\cdot p))\right]\\&\left.-\frac{F_\pi^{-2}}{2}(1-2xC_V^{-1})y\frac{(\epsilon\cdot q)(\epsilon^*\cdot p)}{p\cdot k}\right\}+[\sigma\text{ terms}]+O(k)+\cdots\ .\end{aligned} \tag{9}$$

In (9) we used the following abbreviations:

$$\epsilon^*=\epsilon^*(\lambda_1,k),\quad \epsilon=\epsilon(\lambda_2,Q),\quad y=1-(1+\delta)\frac{C_Am_\rho^2}{C_Vm_A^2}\ ,\qquad x=C_A-\tfrac12C_V\ . \tag{10}$$

$C_V,C_A$ are defined in terms of vector (axial-vector) spectral functions

$$C_{V,A}=\int\frac{da}{a}\rho_{V,A}(a)\ , \tag{11}$$

and δ, the anomalous magnetic moment of $A_1$, is defined in Eq. (47) of Ref. 6. One should remark that there are no contributions from $J_8$ in (9), as only the $I=1$ part of the current is allowed in the electromagnetic transitions involved.

If we now require the identity of the coefficients of the $O(k^{-1})$ terms in (5) and (9), we obtain from the $(1/kQ)$ term the modified KSRF relation

$$f_{\rho\pi\pi}(m_\rho^2, m_\pi^2, m_\pi^2) = \frac{g_\rho^3 C_V^{-1}}{2F_\pi^2 m_\rho^2} y \,, \tag{12}$$

and from the $(1/k\cdot p)$ term the first Weinberg sum rule

$$F_\pi^2 + C_A^2 = C_V^2 \,. \tag{13}$$

[In comparing (12) with the identical result of Ref. 1, one has to identify (in the single-particle approximation) $C_V = g_\rho^2/m_\rho^2$, $g_\rho = m_\rho^2/f_\rho$, $C_A = g_A^2/m_A^2$, $g_A = m_A^2/f_A$.]

Comparing now the $O(k^0)$ terms in (5) and (9), and using (12), we arrive at two additional consistency relations:

$$\mu_\rho = 2 \tag{14}$$

and

$$\delta = -\tfrac{1}{2} \,. \tag{15}$$

In obtaining these relations we assumed that the following commutation relation holds for the σ term:

$$[A_a^0(x), \partial_\mu A_b^\mu(y)]\, \delta(x^0 - y^0) = \delta_{ab}\, \sigma(x) \delta(x-y) \,. \tag{16}$$

Thus, the σ is taken to be an isospin scalar and belongs with the pion to the $(\frac{1}{2}, \frac{1}{2})$ representation of SU(2) ⊗SU(2).

At this point, we want to emphasize that relations (12) and (14) are contained in the formalism of hard pions with current algebra, and our procedure is an alternative way of their derivation, free of various ambiguities in previous derivations. Furthermore, relation (13) is a consequence of Ward identities, and as such is again already built into the four-point function we used. Relation (15) could be changed in principle if the σ term had an $I=2$ component.

## III. MAGNETIC MOMENT OF CHARGED ρ MESONS

The value $\mu_\rho = 2$ for the total magnetic moment obtained in the previous section is obviously dependent on the single-particle approximation for the propagator structure functions used in Ref. 6. In this section we shall use Ward identities and the symmetry properties of the three-point function for three vectors, in order to obtain maximum information on $\mu_\rho$, which follows from the SU(2) current algebra.

The three-point function is defined as

$$A_{\mu\nu\lambda}^{abc}(p_1, p_2, p_3) = \int e^{-ip_1x} e^{-ip_2y} e^{-ip_3z} \langle 0|\, T\{J_a^\mu(x) J_b^\nu(y) J_c^\lambda(z)\}|\, 0\rangle\, dx dy dz \,, \tag{17}$$

$$p_1 + p_2 + p_3 = 0 \,.$$

We reduce the function $A_{\mu\nu\lambda}^{abc}$ in the symmetry and Lorentz indices, and we define, with Ref. 6,

$$A_{\mu\nu\lambda}^{abc}(p_1, p_2, p_3) = \Delta_{aa'}^{\mu\mu'}(p_1) \Delta_{bb'}^{\nu\nu'}(p_2) \Delta_{cc'}^{\lambda\lambda'} \Gamma_{\mu'\nu'\lambda'}^{a'b'c'}(p_1, p_2, p_3) \,. \tag{18}$$

Decomposing now the three-point vertex function $\Gamma_{\mu\nu\lambda}^{abc}(p_1, p_2, p_3)$ in terms of invariant amplitudes $H_i = H_i(p_1^2, p_2^2, p_3^2)$, we have

$$\Gamma_{\mu\nu\lambda}^{abc}(p_1, p_2, p_3) = \epsilon^{abc}[g_{\mu\nu}(p_1)_\lambda H_i + g_{\mu\nu}(p_2)_\lambda H_2 + g_{\mu\lambda}(p_1)_\nu H_3 + (g_{\mu\lambda})(p_2)_\nu H_4 + g_{\nu\lambda}(p_1)_\mu H_5$$
$$+ g_{\nu\lambda}(p_2)_\mu H_6 + \text{terms of the form } (p_i)_\mu (p_j)_\nu (p_k)_\lambda F_{ijk}] \,. \tag{19}$$

Because of the symmetry properties of the vertex, one can express the various $H_i$ in terms of $H_1$. For example, the following relations hold:

$$H_3(p_1^2, p_2^2, p_3^2) = -H_1(p_3^2, p_1^2, p_2^2) - H_1(p_1^2, p_3^2, p_2^2) \,, \tag{20a}$$

$$H_5(p_1^2, p_2^2, p_3^2) = H_1(p_3^2, p_2^2, p_1^2) = -H_2(p_2^2, p_3^2, p_1^2) \,. \tag{20b}$$

Using SU(2) current-algebra relations and the conservation of the vector current, we obtain the following

Ward identities:

$$(p_1^2+p_1\cdot p_2)H_1+(p_2^2+p_1\cdot p_2)H_2=[\Delta_T^{-1}(p_2)-\Delta_T^{-1}(p_1)]C_V^{-1}\ , \tag{21a}$$

$$[F_{111}(p_1^2+p_1\cdot p_2)+F_{112}(p_2^2+p_1\cdot p_2)]+(H_3+H_5)=\frac{1}{p_1^2}\Delta_T^{-1}(p_1)C_V^{-2}[\Delta_T(p_1)-C_V]\ , \tag{21b}$$

with

$$\Delta^{\mu\nu}(p)=-\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)\Delta_T(p)-\frac{p^\mu p^\nu}{p^2}C_V=-\int\frac{\rho_V(a)}{a-p^2}\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{a}\right)da\ . \tag{22}$$

Now comparing the decomposition (18),(19) with (3) we can identify the expression for the magnetic moment of the $\rho^+$ (taking $p_1^2, p_2^2\to m_\rho^2$, $p_3^2\to 0$) as

$$T_3(0)=\mu_\rho=H_3(m_\rho^2,m_\rho^2,0)g_\rho^2C_V\ . \tag{23}$$

From Eqs. (21a) and (20b) we obtain in the limit $p_1^2, p_2^2\to m_\rho^2$, $p_3^2\to 0$

$$H_1(m_\rho^2,m_\rho^2,0)=-g_\rho^{-2}C_V^{-1}\ . \tag{24}$$

Thus, $H_1(m_\rho^2,m_\rho^2,0)$ has the same form as that obtainable in the single-particle approximation, in which case the relation $C_V=g_\rho^2/m_\rho^2$ holds.

Further, using (21b) and the symmetry relations (20) we arrive at

$$H_3(m_\rho^2,m_\rho^2,0)=\frac{C_V^{-2}}{m_\rho^2}-H_1(0,m_\rho^2,m_\rho^2)\ , \tag{25}$$

from which we finally have the alternative expression for the total magnetic moment of charged $\rho$ mesons:

$$\mu_\rho=\frac{g_\rho^2C_V^{-1}}{m_\rho^2}-H_1(0,m_\rho^2,m_\rho^2)g_\rho^2C_V\ . \tag{26}$$

In the single-particle approximation all $H_i$'s are constant, in particular $H_1=-g_\rho^{-2}C_V^{-1}$, and with $C_V=g_\rho^2/m_\rho^2$, Eq. (26) reduces to the result obtained in the previous section, Eq. (14).

General considerations applied to (26) already give us more information on $\mu_\rho$. As $\rho_V(a)$ is positive definite, continuum contributions increase the value of $C_V$ [Eq. (11)] and thus decrease the first term on the right-hand side of Eq. (26). No information is obtainable on $H_1(0,m_\rho^2,m_\rho^2)$ from current algebra alone, but we expect $\mu_\rho$ to decrease when including the continuum contributions in the second term as well.

It is obviously desirable to confirm the last statement in some model which has all the current-algebra constraints in it, but is not restricted to the single-particle approximation. A model fulfilling these requirements was suggested recently by Nutbrown.[8] In his model, the amplitude $A^{abc}_{\mu\nu\lambda}(p_1,p_2,p_3)$ reads as follows:

$$A^{abc}_{\mu\nu\lambda}(p_1,p_2,p_3)=\epsilon^{abc}C_V'^{-1}\int duds\frac{\rho_V(u)\rho_V(s)}{u}\frac{g_{\mu\mu'}-(p_1)_\mu(p_1)_{\mu'}/u}{u-p_1^2}\cdot\frac{g_{\nu\nu'}-(p_2)_\nu(p_2)_{\nu'}/s}{s-p_2^2}\cdot\frac{g_{\lambda\lambda'}-(p_3)_\lambda(p_3)_{\lambda'}/s}{s-p_3^2}$$
$$\times[g_{\nu'\lambda'}(p_2-p_3)_{\mu'}-g_{\lambda'\mu'}(p_2)_{\nu'}+g_{\mu'\nu'}(p_3)_{\lambda'}]+\text{cyclic permutations}, \tag{27}$$

where

$$C_V'=\int\frac{\rho_V(s)}{s^2}ds\ . \tag{28}$$

Using representation (27), we obtain the following expression for $H_3(p_1^2,p_2^2,p_3^2)$:

$$H_3(p_1^2,p_2^2,p_3^2)=\Delta_T^{-1}(p_1)\Delta_T^{-1}(p_2)\Delta_T^{-1}(p_3)C_V'^{-1}\int\frac{2}{u(u-p_2^2)(s-p_3^2)(s-p_1^2)}\rho_V(u)\rho_V(s)duds\ . \tag{29}$$

Evaluating Eq. (29) in the limit $p_1^2=p_2^2=m_\rho^2$, $p_3^2=0$, one finds

$$H_3(m_\rho^2,m_\rho^2,0)=\frac{2}{C_VC_V'm_\rho^4}\ . \tag{30}$$

Inserting (30) in Eq. (23) we obtain for the magnetic moment

$$\mu_\rho=\frac{2g_\rho^2}{C_V'm_\rho^4}\ . \tag{31}$$

In the single-particle approximation $\rho_V(s) = g_\rho^2\delta(s-m_\rho^2)$, and we recover the value $\mu_\rho=2$. In general, however, $C_{V'} > g_\rho^2/m_\rho^4$, and therefore with the aid of Nutbrown's model we confirm our previous assertion that

$$\mu_\rho < 2 \ . \tag{32}$$

## IV. SUMMARY AND DISCUSSION

In the first part of this work we obtained several known relations, Eqs. (12)–(15) (KSRF modified relation, Weinberg's first sum rule, and magnetic moments of $A_1$ and $\rho$), from the requirement of consistency of the hard-pion amplitude with the exactly calculable low-energy limit of a radiative amplitude. Besides being free of difficulties present in various previous derivations (which were analyzed in Ref. 1), our procedure throws interesting light on the physical significance of these relations. In particular, within the framework used in Sec. II, we found that deviations from Eq. (15) are possible only from an $I=2$ component in the $\sigma$ term.

In Sec. III we pursued an analysis of the magnetic moment of charged $\rho$ mesons within the context of SU(2) current algebra, without assuming, however, the single-particle approximation for the vector spectral functions, and we obtained the general alternative expressions (23) and (26) for it. In a fairly model-independent approach, we concluded that $\mu_\rho$ would be decreased from the value in Eq. (14) by the continuum contributions $\rho_V(a)$. This conclusion is substantiated by using Nutbrown's representation,[8] which then leads to expression (31) for $\mu_\rho$.

It is also possible to obtain a lower bound for $\mu_\rho$ by considering the information on $\rho_V(a)$ obtainable from the $e^+e^- \to$ hadrons process. Let us consider the one-photon-exchange process $e^+ + e^- \to \gamma \to n$ hadrons, whose amplitude $F$ is given by

$$F = 2\pi e \bar{v}(p_2,\lambda_2)\gamma^\mu u(p_1,\lambda_1) \times D_{\mu\nu}(q)\langle 0|J^\nu_{\text{em}}(0)|n\rangle \delta(q-p_n) \ , \tag{33}$$

where $|n\rangle$ are all hadronic allowed states, containing both $I=1$ and $I=0$ states. The cross section for $e^+ + e^- \to n$ is then given by[11]

$$\sigma_n(s) = \frac{4\pi^2\alpha}{3s^2}\tilde{\rho}_V(s) \ , \tag{34}$$

where

$$\tilde{\rho}_V = (2\pi)^3 \sum_n \langle 0|J^\mu_{\text{em}}(0)|n\rangle\langle n|J^\mu_{\text{em}}(0)|0\rangle \delta^{(4)}(p_n - q) \ , \quad q^2 = s \tag{35}$$

includes summation over isoscalar and isovector states. Thus the isovector part of the spectral function, which is of interest to us, fulfills (only the isovector part enters in the expression for $\mu_\rho$)

$$_1\tilde{\rho}_V(s) < \frac{3s^2}{4\pi^2\alpha}\sigma_{e^+e^-\to n}(s) \ . \tag{36}$$

Using (36) in conjunction with (28) and (31), we obtain

$$\frac{16\pi^2\alpha^2 g_\rho^2}{m_\rho^4\int \sigma_{e^+e^-\to n}\, ds} < \mu_\rho < 2 \ . \tag{37}$$

An evaluation of the lower limit must obviously await better experimental knowledge of $\int \sigma_{e^+e^-\to n} ds$.

It is of interest to compare our result (37) with previous estimates for $\mu_\rho$. Glück and Wagner[12] review theoretical quark and composite models for $\rho$ and show that they give values in the range $1 < \mu_\rho < 4.7$. On the other hand, information on $\mu_\rho$ can be obtained[13] from an analysis of charged $\rho$ photoproduction. Levy *et al.*[14] recently analyzed the experimental data for $\gamma n \to \rho^- p$ with photon energies $1.4 < E_\gamma < 2.5$ GeV, and using a Born approximation for the calculated amplitude they obtain an estimate $\mu_\rho \simeq 2\mu_N$ for the magnetic moment of charged $\rho$ mesons. This value is slightly lower than our upper limit exhibited in (37).

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[1]P. Singer, Phys. Rev. D 1, 321 (1970).

[2]F. Low, Phys. Rev. 110, 974 (1958).

[3]H. Chew, Phys. Rev. 123, 377 (1961).

[4]J. Pestieau, Phys. Rev. 160, 1555 (1967).

[5]H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).

[6]I. S. Gerstein and H. J. Schnitzer, Phys. Rev. 170, 1638 (1968).

[7]H. Terazawa [Phys. Rev. Letters 26, 1207 (1971)] has made the related observation that Weinberg's first sum rule obtains as a consistency condition when demanding the gauge invariance of the amplitude $\gamma + \gamma \to n\pi^+ + n\pi^-$ in the soft-pion limit (see especially the remark in his footnote 16).

[8]D. A. Nutbrown, Phys. Rev. D 1, 2328 (1970).

[9]V. Glaser and B. Jaksić, Nuovo Cimento 5, 1197 (1957); L. Durand, III, Phys. Rev. 123, 1393 (1961); T. D. Lee and C. N. Yang, *ibid.* 128, 885 (1962); J. A. Young and S. A. Bludman, *ibid.* 131, 2326 (1963).

[10]R. N. Chaudhuri and R. Dutt, Phys. Rev. 177, 2337 (1969).

[11]See, e.g., R. Gatto, in *Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969*, edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 235.

[12]M. Glück and S. Wagner, Phys. Rev. D 3, 1669 (1971).

[13]S. M. Berman and S. D. Drell, Phys. Rev. 133, B791 (1964); U. Maor, *ibid.* 135, B1205 (1964).

[14]N. Levy, M. Glück, and S. Wagner, Phys. Rev. D 4, 874 (1971).