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Simultaneous Fit for Forward and Backward Pion-Nucleon Charge-Exchange Scattering

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In a previous paper, we proposed a dual resonance model for meson-baryon scattering. It is shown here that this model can account for the existing high-energy pion-nucleon charge-exchange scattering data.

I. INTRODUCTION

One of the most appealing features of the Veneziano model in its original and simplest form[1] is, in principle, the formal relation between the low-energy resonances, the high-energy forward, and the high-energy backward scattering. Obviously, this triple connection could and should be tested in simple and direct two-body scattering processes, such as meson-baryon scattering.

Unfortunately, for these processes one cannot draw a firm conclusion from various calculations[2-6] which use this model. For example, Berger and Fox[3] found it impossible to relate the magnitudes of the baryon resonance widths to the sizes of the forward and backward differential cross sections. Similarly, Fenster and Wali[4] have shown that if one uses the model to fit the widths of the baryon resonances in a limited low-energy region and the forward high-energy scattering data, then the predicted backward differential cross section is 2000 times larger than the experimental data.

One could argue that introduction of enough satellite terms would improve the situation. However, beside the fact that the predictive content of the model is then weaker, the satellite terms are expected to play a role at most in the low-energy region where the unitarity corrections must also be taken into account. Since at present there is no reliable way of evaluating those unitarity corrections, in order to make a more clear-cut test of the Veneziano model for two-body scattering processes, one should adopt a less ambitious attitude, namely to confine oneself to studying the correlation between the high-energy forward and backward scattering. In the usual Regge-pole model, such a correlation is not compelling and, for instance, the meson trajectory and the baryon trajectory contributions are unrelated in the existing phenomenological fits.

In the present work, we shall show that the dual-resonance-type model that we proposed in a previous paper[7] is suitable for describing such a connection. We apply our model to pion-nucleon charge-exchange scattering in order to avoid uncertainties due to the contribution of the Pomeranchon.

II. THE INVARIANT AMPLITUDES FOR PION-NUCLEON CHARGE-EXCHANGE SCATTERING

In Ref. 7 we showed that the replacement of the usual partial-wave expansion in terms of Appell-Pochhammer polynomials (which we believe to yield a "natural" basis for the dual resonance model) leads to a Veneziano-type series for the scattering amplitude. This result was obtained by following the usual procedure used in the derivation of the Regge representation, and by requiring the general properties of the dual resonance model to be satisfied.

When only those leading contributions which are due to the degenerate (ρ, P') system, to the nucleon, and to the Δ trajectories are taken into account, the "minimal" form for the $A^{(-)}$ and $B^{(-)}$ invariant amplitudes can be written as

$$A_\rho^{(-)}=\gamma_A^{\rho N}C_{\rho N}^{-}(1,\tfrac{3}{2})+\gamma_A^{\rho\Delta}C_{\rho\Delta}^{-}(1,\tfrac{3}{2}) , \qquad (2.1)$$

$$B_\rho^{(-)}=\gamma_B^{\rho N}B_{\rho N}^{+}(1,\tfrac{1}{2}) +[(\gamma_B^{\rho\Delta})_1+t(\gamma_B^{\rho\Delta})_2]B_{\rho\Delta}^{+}(1,\tfrac{3}{2}) , \qquad (2.2)$$

where

$$B^+_{\rho N}(m,n)=\frac{\Gamma(m-\alpha_\rho(t))\,\Gamma(n-\alpha_N(s))}{\Gamma(m+n-\alpha_\rho(t)-\alpha_N(s))}+\frac{\Gamma(m-\alpha_\rho(t))\,\Gamma(n-\alpha_N(u))}{\Gamma(m+n-\alpha_\rho(t)-\alpha_N(u))}\,, \tag{2.3}$$

$$C^-_{\rho N}(m,n)=\frac{\Gamma(m-\alpha_\rho(t))\,\Gamma(n-\alpha_N(s))}{\Gamma(m+n-1-\alpha_\rho(t)-\alpha_N(s))}-\frac{\Gamma(m-\alpha_\rho(t))\,\Gamma(n-\alpha_N(u))}{\Gamma(m+n-1-\alpha_\rho(t)-\alpha_N(u))}\,, \tag{2.4}$$

and all γ's are constants.

The expressions for $B^+_{\rho\Delta}$ and $C^-_{\rho\Delta}$ can be deduced in a self-evident way.

Formulas (2.1)–(2.2) are obtained from formulas (3.19), (3.20), (3.34), and (3.35) of Ref. 7 by using the exchange-degeneracy property of the (ρ, P') trajectory.

The A_0 and B_0 amplitudes describing the pion-nucleon charge-exchange scattering are then given by

$$A_0\equiv A_s(\pi^-p\to\pi^0n)=-\sqrt{2}A^{(-)} \tag{2.5}$$

and

$$B_0\equiv B_s(\pi^-p\to\pi^0n)=-\sqrt{2}B^{(-)}\,. \tag{2.6}$$

In order to get a nonvanishing polarization in $\pi^-p\to\pi^0n$, in the framework of the Regge-pole model, Högaasen *et al.*[8] invoked the existence of a ρ'-meson trajectory with the same quantum numbers as the ρ trajectory. Subsequently, a ρ'-meson trajectory was advocated by several authors; for instance, by Sertorio and Toller[9] from O(4) symmetry and by Dolen, Horn, and Schmid[10] and Barger and Phillips[11] from finite-energy sum rules and high-energy fits. The ρ' contribution has been also considered by Joshi and Pagnamenta[6] in the framework of a Veneziano-type model.

For the same reasons, we included the contribution of a nonleading meson exchange-degenerate (ρ', P'') trajectory by adding to Eqs. (2.1)–(2.2) similar expressions where ρ is replaced by ρ'.

As already mentioned, we restrict ourselves to high-energy scattering. We can therefore consider only the asymptotic limits of the complete amplitude (as usual, these limits are obtained by allowing the trajectory functions to have suitable imaginary parts[12]):

1. t fixed, $s\to\infty$:

$$A_0^f/\sqrt{2}=\beta_1\Gamma(1-\alpha_\rho(t))\xi_\rho(t)(\alpha's)^{\alpha_\rho(t)}+\beta_2\Gamma(1-\alpha_{\rho'}(t))\,\xi_{\rho'}(t)(\alpha's)^{\alpha_{\rho'}(t)}, \tag{2.7}$$

$$B_0^f/\sqrt{2}=-(\beta_3+t\beta_4)\Gamma(1-\alpha_\rho(t))\,\xi_\rho(t)(\alpha's)^{\alpha_\rho(t)-1}-(\beta_5+t\beta_6)\Gamma(1-\alpha_{\rho'}(t))\,\xi_{\rho'}(t)(\alpha's)^{\alpha_{\rho'}(t)-1}\,. \tag{2.8}$$

2. u fixed, $s\to\infty$:

$$A_0^b/\sqrt{2}=\beta_7\Gamma(\tfrac{3}{2}-\alpha_\Delta(u))\,(\alpha's)^{\alpha_\Delta(u)-1/2}+(\beta_1+\beta_2-\beta_7)\Gamma(\tfrac{3}{2}-\alpha_N(u))\,(\alpha's)^{\alpha_N(u)-1/2}\,, \tag{2.9}$$

$$B_0^b/\sqrt{2}=(1/\alpha')(\beta_4+\beta_6)\Gamma(\tfrac{3}{2}-\alpha_\Delta(u))\,(\alpha's)^{\alpha_\Delta(u)-1/2}-\beta_8\Gamma(\tfrac{1}{2}-\alpha_N(u))\,(\alpha's)^{\alpha_N(u)-1/2}\,, \tag{2.10}$$

where the following notations have been introduced:

$$\begin{aligned}&\beta_1=\gamma_A^{\rho\Delta}+\gamma_A^{\rho N}, \quad \beta_2=\gamma_A^{\rho'\Delta}+\gamma_A^{\rho' N},\\&\beta_3=(\gamma_B^{\rho\Delta})_1+\gamma_B^{\rho N}, \quad \beta_4=(\gamma_B^{\rho\Delta})_2,\\&\beta_5=(\gamma_B^{\rho'\Delta})_1+\gamma_B^{\rho' N}, \quad \beta_6=(\gamma_B^{\rho'\Delta})_2\\&\beta_7=\gamma_A^{\rho\Delta}+\gamma_A^{\rho'\Delta}, \quad \beta_8=\gamma_B^{\rho N}+\gamma_B^{\rho' N}\,,\end{aligned} \tag{2.11}$$

and

$$\xi_{\rho,\rho'}=1-e^{-i\pi\alpha_{\rho,\rho'}(t)}\,. \tag{2.12}$$

Let us make the following two remarks on the form (2.7)–(2.10) of the amplitudes:

(i) As can be seen from Eqs. (2.1)–(2.2), our amplitudes contain no (s,u) terms. This is a consequence, as explained in Ref. 7, of a particular but natural way of imposing the s-u symmetry in our derivation. In the usual Veneziano-type models, the (s,u) terms vanish when $s\to\infty$ for fixed t, and give rise to the signature factor for the baryon trajectories when $s\to\infty$ for fixed u. However, these (s,u) terms imply a degeneracy for the N and Δ trajectories because of the isospin symmetry and the conditions

$$\begin{aligned}&A^{(-)}(s,t,u)=-A^{(-)}(u,t,s)\,,\\&B^{(-)}(s,t,u)=B^{(-)}(u,t,s)\,.\end{aligned} \tag{2.13}$$

Our amplitudes, since they have no (s,u) terms, do not possess the signature factor for the baryon trajectories; the difficulty of N-Δ degeneracy,

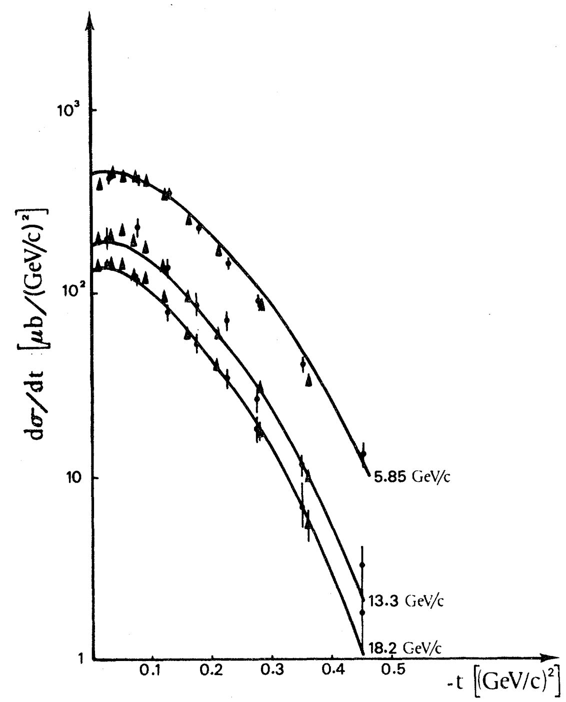

FIG. 1. Small-angle differential cross section for $\pi^- p \to \pi^0 n$ (data from Refs. 13 and 14).

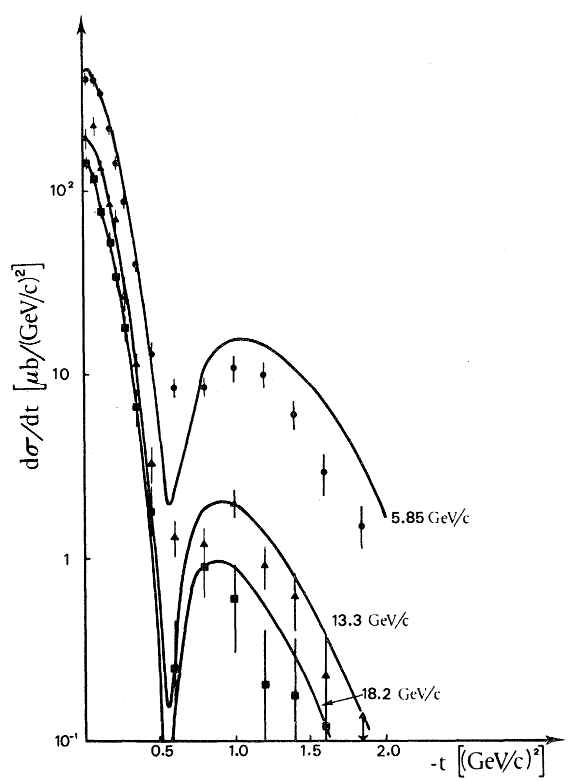

FIG. 2. Wide-angle differential cross section for $\pi^- p \to \pi^0 n$ (data from Ref. 14).

however, does not occur here. In the forward direction, they have essentially the same behavior as those considered in Refs. 2–6. Anyway, as long as we are not interested in the polarization in the backward direction, one can fit, even without the signature of the baryon trajectories, the differential cross section in the backward direction by using effective parameters for N and Δ trajectories.

(ii) Concerning the residue functions appearing in formulas (2.7)–(2.10) it can be observed that once the parameters $\beta_1, \ldots, \beta_6$ are fixed by a fit of forward data, the Δ contribution in the B_0^b amplitude is *completely* determined and the N and Δ contributions in the A_0^b amplitude are *correlated*, namely, they depend on only one free parameter.

This strong quantitative connection obtained between the forward direction and the backward direction is a reflection of the duality built into the model, in the sense that the meson and baryon Regge residues are correlated in this Veneziano-type representation, in contrast to what is expected in the nondual Regge-pole model. Also, if one considers the elastic processes $\pi^\pm p$, one has to introduce the Pomeranchon, and this would affect the backward direction. This is an additional re. son to make a fit only of the πN charge-exchange reaction and not of the elastic ones, even in the backward direction.

III. RESULTS AND DISCUSSION

Our amplitude depends on eight free parameters (the β_i's).

We first made a fit of the differential cross section and of the polarization for a large domain of t $[0<|t|<2\ (\mathrm{GeV}/c)^2]$ at $p_L = 5.85-18.2$ GeV/c (Refs. 13–17), and then of the differential cross section for a large domain of u $[-1.8<u<0.05\ (\mathrm{GeV}/c)^2]$ at $p_L = 5.9-13.8$ GeV/c (Refs. 18 and 19).

The values of the parameters, corresponding to $\chi^2 \simeq 3$/point, are the following (the values of the β parameters, when multiplied by a factor of 10^3, are in conventional units, which express all the quantities in GeV/c units):

$$\alpha' = 0.89, \quad \alpha_\rho(0) = 0.48, \quad \alpha_{\rho'}(0) = -0.18\ ,$$

$$\alpha_\Delta^{\mathrm{eff}}(0) = -0.03, \quad \alpha_N^{\mathrm{eff}}(0) = -0.13,$$

and

$$\beta_1 = 34.5, \quad \beta_2 = -21.7, \quad \beta_3 = 65.9, \quad \beta_4 = -5.6,$$

$$\beta_5 = -19.5, \quad \beta_6 = 31.5, \quad \beta_7 = -8.5, \quad \beta_8 = 37.1\ .$$

The values for the slope α' (which is here universal) and for the intercepts are in rough agreement with the ones usually used in the literature.

The quality of the fit is displayed in Figs. 1–7.

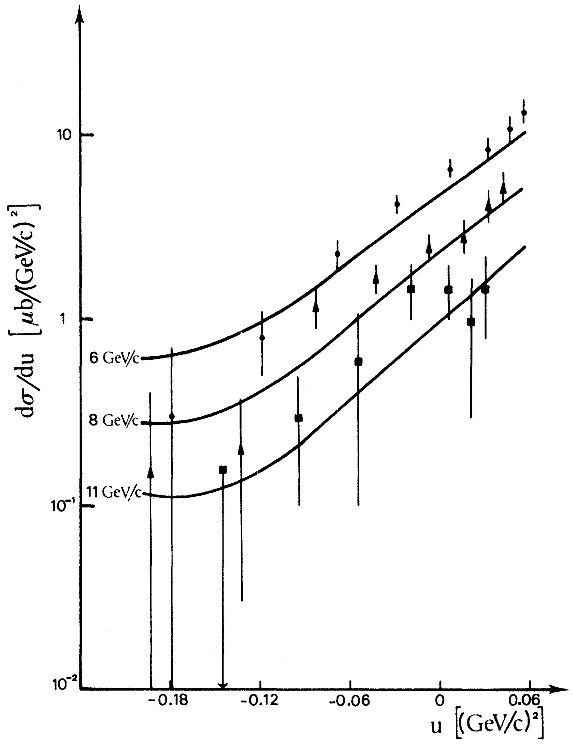

FIG. 3. Small-angle differential cross section for $\pi^- p \to n\pi^0$ (data from Ref. 18).

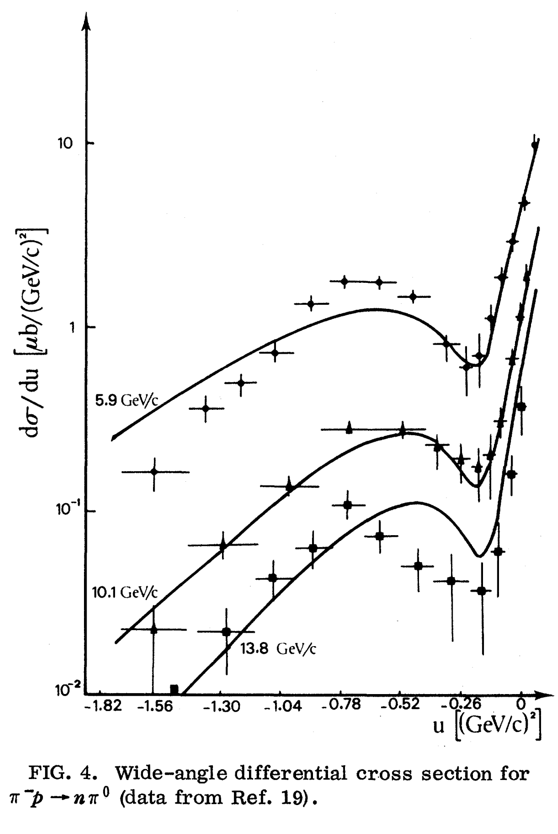

FIG. 4. Wide-angle differential cross section for $\pi^- p \to n\pi^0$ (data from Ref. 19).

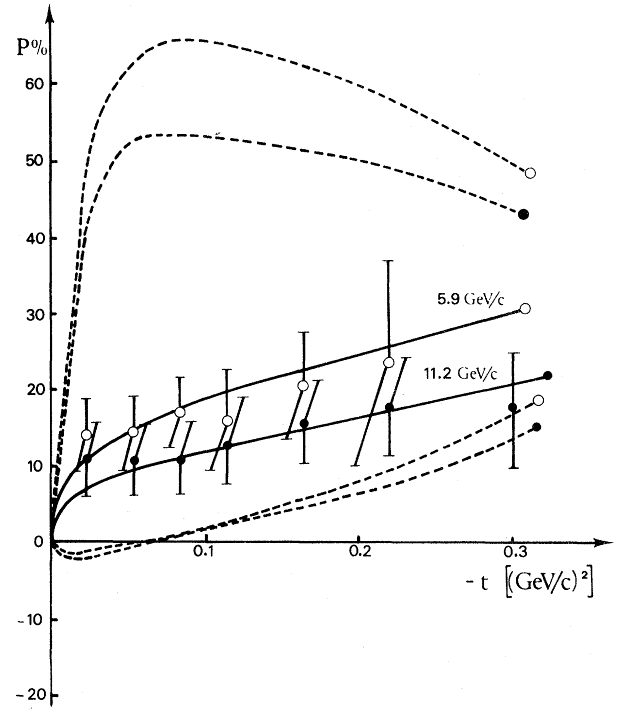

FIG. 5. Small-angle polarization for $\pi^- p \to \pi^0 n$ (data from Ref. 16). The upper dashed curves are from Ref. 5, and the lower dashed curves are from Ref. 6.

One can observe there a general agreement with the values and the structure of the experimental data.

A. Forward Differential Cross Section

The sharp dip in $d\sigma/dt$, as well as the zero in the polarization at $t \simeq -0.54$ (GeV/c)^2, are explained by the fact that the (ρ, P') exchange degeneracy imposes the Gell-Mann coupling mechanism of the ρ trajectory at $\alpha_\rho(t)=0$, in which the ρ contributions to both the helicity-flip and -nonflip amplitudes vanish.

B. Polarization

For small values of $t\,[0<-t<0.3$ (GeV/c)^2] we obtain a satisfactory fit of the polarization data.[15,16] The polarization computed previously in Veneziano-type models are either too large,[5] or too small and negative[6] (see Fig. 5). In Ref. 5 a nonzero polarization is produced without introducing a ρ' trajectory, but at the price of a rather large "background" (s, u) term. However, according to the authors, these "background" terms can sometimes be as large as the leading Regge terms, whereas they should vanish at high energy. In Ref. 6 the ρ' Regge intercept is chosen to be $\alpha_{\rho'} \simeq 0.07$, and consequently the (ρ', P'') exchange degeneracy

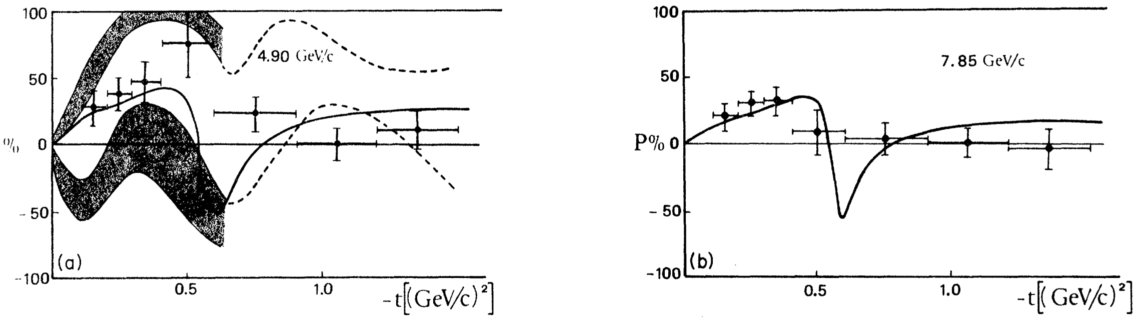

FIG. 6. (a) Wide-angle polarization for $\pi^- p \to \pi^0 n$ at 5 GeV/c (data from Ref. 17). The isospin bounds evaluated at 6 GeV/c are from Ref. 20. The shaded area corresponds to the errors on the isospin bounds, and the dashed curves are the bounds evaluated using the Barger and Phillips amplitudes (Ref. 20). (b) Wide-angle polarization for $\pi^- p \to \pi^0 n$ at 8 GeV/c (data from Ref. 17).

forces a zero in the polarization for small t.

For larger values of t, the model also fits reasonably well the recent data of Bonamy *et al.*[17]

Recently bounds on the polarization in $\pi^- p \to \pi^0 n$ in terms of experimental differential cross sections, and polarizations in elastic $\pi^\pm p \to \pi^\pm p$ scatterings were derived by Dass *et al.*[20] from isospin conservation considerations. One of the main results is that, at $p_L = 6$ GeV/c, large negative polarizations for $0.2 \lesssim |t| \lesssim 0.4$ (GeV/c)^2 are ruled out by the bounds. Comparison of our results with these bounds is shown in Fig. 6(a).

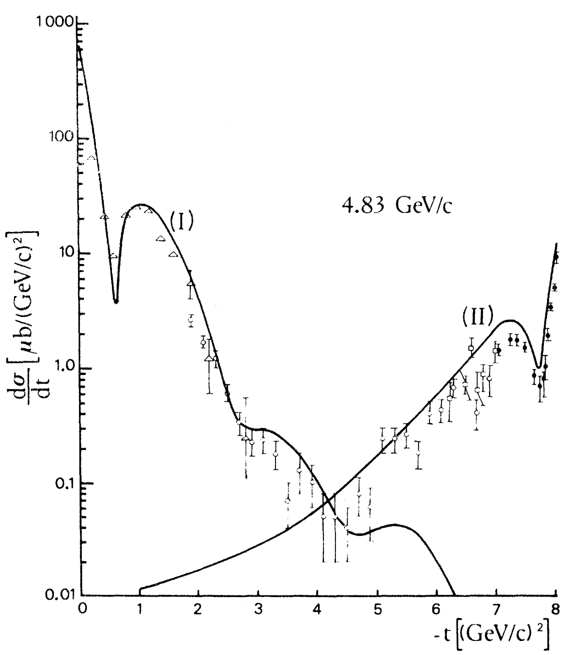

FIG. 7. Differential cross section for $\pi^- p \to \pi^0 n$ at 4.83 GeV/c (data △, ○, and □ from Ref. 23 and References cited therein; ● are data from Ref. 19 at 5.9 GeV/c). The significance of the curves (I) and (II) is explained in the text.

C. Backward Differential Cross Section

Our model also fits correctly the backward data (see Figs. 3 and 4).

The reproduction in our model of the experimental dip in the region $u \simeq -0.2$ (GeV/c)^2 is explained by a zero in $A_0'^b \simeq A_0^b + m_N B_0^b$ (which is the dominant amplitude in the backward direction) near this value. Actually, we obtain a dip which slowly moves to smaller values of $|u|$ as the energy increases, as the Cornell-group data[19] seem to indicate. Our mechanism for the dip in the backward direction is obviously different from the one which occurs in the usual Regge-pole model, where one explains the dip at $u \simeq -0.2$ by the wrong-signature nonsense zero (WSNZ), $\alpha_N(u) \simeq -\frac{1}{2}$. We have no such WSNZ simply because we have no (s, u) terms. Our results suggest that dips, in the case of baryonic exchanges, are not necessarily associated with the WSNZ. This suggestion seems to be supported by an observation of Berger *et al.*,[21,22] who note that the nucleon WSNZ dip is *absent* in some inelastic processes ($pp \to d\pi^+$, $\pi^+ p \to \rho^+ p$, $pp \to \pi^+ X$).

It is interesting to study to what extent the forward limit (2.7)–(2.8) and the backward limit (2.9)–(2.10) of the A and B amplitudes can be extrapolated to the intermediate region. With all the parameters *fixed*, as previously described, we have calculated the differential cross section at $p_L = 4.83$ GeV/c for large t and large u.

It can be seen (Fig. 7) that the forward limit of the amplitudes [curve (I)] gives a very reasonable description of the very recent data[23] not only for $0 \lesssim |t| \lesssim 2$ (GeV/c)^2, but also for $2 \lesssim |t| \lesssim 4$ (GeV/c)^2, where the cuts are expected to dominate. The fact that the data are compatible with the ρ trajectory dominance even for large t strongly suggests that, at least at the energy corresponding to $p_L \simeq 5$ GeV/c, the necessity of introducing cuts is not yet compelling. This gives further support for using a $\rho+\rho'$ model.

On the other hand, it can be also seen in Fig. 7 that the backward limit of the amplitudes [curve (II)] gives also a reasonable description of the data[23] for large u.

This approximate analysis of the data for large t and u seems to show that our Veneziano-type amplitudes given by (2.1) and (2.2) could describe satisfactorily the whole region between the forward direction and the backward direction. However, in attempting such a description in the intermediate region, one needs an explicit expression for the imaginary parts of the trajectories, which, at the present stage of the theory, are arbitrary functions, specified only by their asymptotic behavior (12).

As a concluding remark, we could say that, although very little freedom is allowed in the parametrization, our model does account for the existing high-energy data for $\pi^- p \to \pi^0 n$ scattering. Perhaps this might mean that the phenomenological capabilities of even the simplest form of the Veneziano-type model for two-body reactions are not yet totally explored.

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