Resonance Decays Involving Vector Mesons*

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(Received 28 August 1972)

A picture of resonance decays using only very general features of the quark model is applied to the case of $\frac{1}{2}^+1^-$ final states. Recent experiments on $\pi N \to N\rho$ provide a crucial test of the model, which predicts correctly the dominant spin of the $N\rho$ system in several cases.

The search for symmetries of decay matrix elements beyond SU(3) has been a part of resonance physics for nearly a decade. Only recently, however, has it become clear that certain relatively modest extensions of SU(3) describe a large body of data fairly well.[1-4]

The picture to which we refer makes some minimal use of the quark model.[5] A hadron is assumed to decay by the creation out of the vacuum of a $q\bar{q}$ pair with $J^{PC}=0^{++}$, i.e., a $^3P_0$ unitary singlet pair. Each final hadron contains one member of this pair. Such a picture[2,6] arises naturally when duality graphs[7,6] are endowed with spin.

The above model has $SU(6)_W$ as a limit corresponding to $L_z=0$ for the initial hadron[8] (the z axis is defined by the final particles in the c.m. system). Physically, this corresponds to the assumption that quarks have no transverse momentum inside a hadron. This assumption appears to contradict experimental data.

When an admixture of $L_z\neq 0$ in the initial hadron is allowed, $SU(6)_W$ is broken in a natural way. The quark L of the initial hadron and that of the $^3P_0$ pair ($L=1$) couple to various values of final orbital angular momentum l. For final hadron pairs with no internal L, parity conservation requires $l=L\pm 1$. The amplitudes corresponding to these two values are related in $SU(6)_W$,[9] but freed from one another when both $L_z=0$ and $L_z\neq 0$ decays are allowed. We shall call such a scheme "l-broken $SU(6)_W$."[10]

The study of baryon resonance decays to $\frac{1}{2}^+1^-$ final states is a particularly interesting application of l-broken $SU(6)_W$. Three amplitudes contribute to these decays: two with one l (spin $\frac{1}{2}$ and $\frac{3}{2}$) and one with the other l (spin $\frac{3}{2}$). In $SU(6)_W$ all three of these are related to one another. In l-broken $SU(6)_W$ the relations between spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ amplitudes for the l that has both of them *remain valid*. These are easy predictions to test. They do not rely on assumptions about centrifugal barriers, in contrast to most of the previous cases where l-broken $SU(6)_W$ was discussed.[1-3,10]

The recent availability of data on $\pi N\to\pi\pi N$,[11] allowing a phase-shift analysis of $\pi N\to\rho N$, makes our predictions of particular relevance at present.

Should the following relations be found violated, the usefulness of the quark model for describing resonance decays[1,3,12] would be brought into serious question.

Define σ to be the natural parity of the initial baryon: $\sigma=\tau P$, $\tau=(-)^{J-1/2}=$ signature, $P=$ parity. Then the following partial waves contribute to $\frac{1}{2}^+1^-$ decays:

$$\sigma=+\begin{cases} l_-\equiv J-\frac{3}{2}, & S=\frac{3}{2}, \\ l_+\equiv J+\frac{1}{2}, & S=\frac{1}{2},\frac{3}{2}, \end{cases} \tag{1}$$

$$\sigma=-\begin{cases} l_-\equiv J-\frac{1}{2}, & S=\frac{1}{2},\frac{3}{2}, \\ l_+\equiv J+\frac{3}{2}, & S=\frac{3}{2}, \end{cases} \tag{2}$$

where S is the spin of the $\frac{1}{2}^+1^-$ system. Barrier effects will usually suppress the higher partial wave, so that only $\sigma=-$states will involve detectable $S=\frac{1}{2}$ and $S=\frac{3}{2}$ admixtures. Exceptions are the cases $J^P=\frac{1}{2}^-$, for which only $S=\frac{1}{2}$ is possible in $l=l_-=0$, and $J^P=\frac{1}{2}^+$, for which l_- is unphysical and l_+ should be observable, with both spins possible. To summarize, we expect useful spin-ratio predictions for $\frac{1}{2}^+1^-$ decays of resonances with

$$J^P=\tfrac{1}{2}^+;\ \tfrac{3}{2}^+,\ \tfrac{5}{2}^-,\ \tfrac{7}{2}^+,\ \ldots\ . \tag{3}$$

We define decay amplitudes $\mathfrak{M}_\lambda$ with λ the vector-meson helicity (and the baryon helicity taken $+\frac{1}{2}$). The total $\frac{1}{2}^+1^-$ width is

$$\Gamma=\frac{P}{M^2}\frac{2}{2J+1}\sum_\lambda|\mathfrak{M}_\lambda|^2 \tag{4}$$

in our normalization. Various choices of multiplet assignment usually are possible for a resonance. For each choice, the amplitudes $\mathfrak{M}_\lambda$ may be calculated by standard $SU(6)_W$ methods.[3,4] The ratios $\mathfrak{M}_1:\mathfrak{M}_0:\mathfrak{M}_{-1}$ are well defined in each case.

Next one expands $\mathfrak{M}_\lambda$ in partial-wave amplitudes $[l]_S$:

$$\mathfrak{M}_\lambda=\sum_{l,S}[l]_S(\tfrac{1}{2},\tfrac{1}{2};1,-\lambda|S,\tfrac{1}{2}-\lambda)(S,\tfrac{1}{2}-\lambda;l,0|J,\tfrac{1}{2}-\lambda)\ . \tag{5}$$

TABLE I. Fractional contributions of different partial waves to $N\rho$ decay predicted by $SU(6)_W$. Pairs of underlined numbers are expected to preserve their ratio in the presence of $SU(6)_W$ breaking. The numbers in the second column refer to SU(6) representation, SU(3)×SU(2) representation, and quark orbital angular momentum, respectively.

Resonance	Assignment	$\Gamma[l_-]_{1/2}/\Gamma$	$\Gamma[l_-]_{3/2}/\Gamma$	$\Gamma[l_+]_{1/2}/\Gamma$	$\Gamma[l_+]_{3/2}/\Gamma$ [a]
$\Delta(1910)$: $\frac{1}{2}^+$	$\underline{56}$, (10, 4), $L=2$	0	0	$\underline{0.67}$	$\underline{0.33}$
	$\underline{70}$, (10, 2), $L=0$	0	0	$\underline{0.76}$	$\underline{0.24}$
$N(1860)$: $\frac{3}{2}^+$	$\underline{56}$, (8, 2), $L=2$	$\underline{0.28}$	$\underline{0.07}$	0	0.65
	$\underline{70}$, (8, 2), $L=2$	$\underline{0.02}$	$\underline{0.10}$	0	0.88
	$\underline{70}$, (8, 4), $L=2$	$\underline{0.17}$	$\underline{0.53}$	0	0.30
	$\underline{70}$, (8, 4), $L=0$	$\underline{0.17}$	$\underline{0.83}$	0	0
$\Delta(1950)$: $\frac{7}{2}^+$	$\underline{56}$, (10, 4), $L=2$	$\underline{0.25}$	$\underline{0.75}$	0	0

[a] The definitions of $l_\pm$ are given in Eqs. (1) and (2).

The ratios of the three $[l]_S$ amplitudes are thus specified in $SU(6)_W$. In l-broken $SU(6)_W$, the ratios $[l]_{S_1}/[l]_{S_2}$ for the l with two possible spin values *continue to hold*.

The following $N\rho$ amplitudes in (3) are important experimentally[11] between 1700 and 2000 MeV:

$$\begin{aligned} &\tfrac{1}{2}^+, \quad I=\tfrac{3}{2}, \quad l=1, \quad S=\tfrac{1}{2} \quad [\Delta(1910)],\\ &\tfrac{3}{2}^+, \quad I=\tfrac{1}{2}, \quad l=1, \quad S=\tfrac{1}{2} \quad [N(1860)],\\ &\tfrac{7}{2}^+, \quad I=\tfrac{3}{2}, \quad l=3, \quad S=\tfrac{3}{2} \quad [\Delta(1950)]. \end{aligned} \tag{6}$$

In each case a *particular spin* predominates though both are allowed.

Table I summarizes the predictions of $SU(6)_W$ for fractional contributions of each partial wave to the total $N\rho$ partial widths of the states in Eq. (6). It has some interesting features in comparison with the data:

a. Predominance of $S=\frac{1}{2}$ for $\Delta(1910)$. No choice between $\underline{56}$, (10, 4), $L=2$ and $\underline{70}$, (10, 2), $L=0$ is possible purely on the basis of the experimental dominance of spin $\frac{1}{2}$. Both assignments are consistent with this result. The influence of the predicted $S=\frac{3}{2}$ admixture is very different in the two cases, however. For the former assignment, $SU(6)_W$ predicts $\mathfrak{M}_0=0$, while for the latter one expects $\mathfrak{M}_0/\mathfrak{M}_1=-3/\sqrt{2}$. The neglect of the spin-$\frac{3}{2}$ amplitude[12] thus throws away considerable information in this case.

b. Favored assignment for N(1860). The experimental dominance of spin $\frac{1}{2}$ in Eq. (7) is consistent with only one assignment of this resonance to a pure state: It must belong to $\underline{56}$, (8, 2), $L=2$. This assignment, interestingly enough, may help to explain the observed suppression of $N(1860)\to\Delta\pi$, as (by similar methods) it predicts $\Gamma_P[\Delta\pi]/\Gamma[\Delta\pi]=\frac{1}{10}$, $\Gamma_F[\Delta\pi]/\Gamma[\Delta\pi]=\frac{9}{10}$ for this state. Even with the expected suppression of F waves, in fact, one might expect some F-wave $\Delta\pi$ decay of this state. The fact that none is seen[11] may indicate that small admixtures of other $SU(6)\otimes O(3)$ representations are present in $N(1860)$.

c. Predominance of $S=\frac{3}{2}$ for $\Delta(1950)$. The assignment shown is the only reasonable one. The

TABLE II. Predictions analogous to those of Table I but for as yet unobserved states.

Resonance; Decay mode	Assignment	$\Gamma[l_-]_{1/2}/\Gamma$	$\Gamma[l_-]_{3/2}/\Gamma$	$\Gamma[l_+]_{1/2}/\Gamma$	$\Gamma[l_+]_{3/2}/\Gamma$ [a]
$N(1780)$; $\frac{1}{2}^+$					
$\to N\rho$	$\underline{56}$, (8, 2), $L=0$	0	0	$\underline{0.28}$	$\underline{0.72}$
$\to N\omega$		0	0	$\underline{0.03}$	$\underline{0.97}$
$\to N\rho$	$\underline{70}$, (8, 2), $L=0$	0	0	$\underline{0.02}$	$\underline{0.98}$
$\to N\omega$		0	0	$\underline{0.67}$	$\underline{0.33}$
$N(1860)$: $\frac{3}{2}^+$	$\underline{56}$, (8, 2), $L=2$	$\underline{0.03}$	$\underline{0.10}$	0	0.87
$\to N\omega$	$\underline{70}$, (8, 2), $L=2$	$\underline{0.67}$	$\underline{0.03}$	0	0.30
$N(1990)$: $\frac{7}{2}^+$					
$\to N\rho$	$\underline{70}$, (8, 4), $L=2$	$\underline{0.25}$	$\underline{0.75}$	0	0
$\to N\omega$		$\underline{0.25}$	$\underline{0.75}$	0	0

[a] The definitions of $l_\pm$ are given in Eqs. (1) and (2).

TABLE III. Comparison of relations for resonance photoproduction amplitudes on neutrons $[\mathfrak{M}_\lambda(n)]$ in terms of those on protons $[\mathfrak{M}_\lambda(p)]$. Note the existence of one linear relation common to both approaches.

Resonance	Refs. 12 and 14	Present work (assuming vector dominance)
$D_{13}(1520)$	$M_1(n)=-M_1(p)/3-2M_{-1}(p)/3\sqrt{3}$	$M_1(n)=-M_1(p)/3-M_{-1}(p)/3\sqrt{3}$
	$M_{-1}(n)=-M_{-1}(p)$	$M_{-1}(n)=-\frac{2}{3}M_{-1}(p)$
	$M_1(n)-\frac{1}{\sqrt{3}}M_{-1}(n)=-\frac{1}{3}\left(M_1(p)-\frac{1}{\sqrt{3}}M_{-1}(p)\right)$	
$F_{15}(1690)$	$M_1(n)=-\frac{2}{3}M_1(p)+\sqrt{2}\,M_{-1}(p)/3$	$M_1(n)=-\frac{2}{3}M_1(p)+M_{-1}(p)/3\sqrt{2}$
	$M_{-1}(n)=0$	$M_{-1}(n)=-\frac{1}{3}M_{-1}(p)$
	$M_1(n)-\frac{1}{\sqrt{2}}M_{-1}(n)=-\frac{2}{3}\left(M_1(p)-\frac{1}{\sqrt{2}}M_{-1}(p)\right)$	

result of Table I follows from two conditions which are expected to hold in general in the present model: $l=5$ decays are absent, and $\mathfrak{M}_0=0$. As in the case of $\Delta(1910)$, information on the relative phase of the $S=\frac{1}{2}$ and the (smaller) $S=\frac{3}{2}$ amplitude would be helpful here, to see if $\mathfrak{M}_0$ really vanishes.

Further applications of the present method could be envisioned if data were good enough to see the amplitudes neglected in Ref. 11. We envision small contributions from the states listed in Table II. Also presented are predictions for $N\omega$ decays. Since the ω contains a unitary singlet piece, these predictions are based on the ansatz $N \not\to N\phi$. Phase-shift analyses of the reaction $\pi^- p \to \omega n$, for example, with detection of the $\pi^0\gamma$ decay of the ω, would thus provide a useful complement to the analysis of Ref. 11. The prediction of dominance of $S=\frac{3}{2}$ for $N(1860)\to(N\omega)_{l=1}$, given our favored assignment, is one amusing result; recall that $S=\frac{1}{2}$ was found dominant for $N\rho$ decay.

We close with some brief remarks relating our work to that of others.

A recent bootstrap calculation[13] predicts the dominance of $S=\frac{3}{2}$ in all the cases of Eq. (6). Hence, although our quark-model results may well follow from other "nonquark" arguments (and we would welcome such alternative descriptions) the correct prediction of spin ratios in such schemes is apparently not automatic.

The present method, when combined with the vector-dominance model (VDM), predicts relations among photoproduction amplitudes similar but not identical to those of Refs. 12 and 14. The results of the two approaches are compared in Table III for photoproduction of $D_{13}(1520)$ and $F_{15}(1690)$. One linear relation is common to both approaches. In fact, we suspect that the relations of Refs. 12 and 14 may be correct at $q^2=0$ while our relations should hold only at $q^2=m_\rho^2\simeq m_\omega^2$, where q^2 is the virtual photon mass. Presumably a smooth interpolation holds between the two points. This conjecture would provide a basis for predictions of resonance electroproduction, but requires accurate neutron data for its verification.

We wish to thank Dr. Roger Cashmore for helpful discussions.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-1764.

†Alfred P. Sloan Research Fellow, 1971–1973.

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PHYSICAL REVIEW D VOLUME 7, NUMBER 3 1 FEBRUARY 1973

Linear Regge Trajectories with a Left-Hand Cut

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The properties of boson Regge trajectories $\alpha(s)$ with a left-hand cut for $s<0$ are studied, with the following assumptions: (a) $\text{Re}\alpha(s)$ is linear in the physical regions of the s and t channels. (b) The imaginary part is asymptotically smaller than the real part. (c) At $s=0$, $\alpha(s)$ has a branch point due to the existence of a Regge cut $\alpha_c(s)$ in the angular momentum plane, with $\alpha_c(0)=\alpha(0)$. The branch point is attributed to the collision of the physical pole $\alpha(s)$ with a second-sheet pole $\tilde{\alpha}(s)$, at $s=0$. (d) The function $\alpha(s)$ is analytic in the s plane. Under these assumptions the imaginary part for each boson trajectory is obtained in terms of a parameter A and a parameter-free universal function determined by solving an integral equation numerically. The parameter A is determined by requiring that the first meson of each trajectory have the observed width. The model gives imaginary parts increasing almost linearly with s for large $|s|$. The widths of the recurrences increase linearly with their mass. The results support exchange degeneracy ρ-f, K^*-K_N, ω-A_2. The imaginary part, for $s<0$, is compared with the phenomenological results of other authors.

I. INTRODUCTION

The possibility that Regge poles $\alpha(s)$ are complex for $s<0$ has been investigated recently.[1-3] This means that Regge trajectories develop a left-hand cut for $s<0$. The mechanism of production of this cut is the collision of two Regge poles.[1,2] In relativistic scattering the mechanism of generating complex poles is strongly related to the existence of cuts in the angular momentum plane. The colliding poles are expected, on physical grounds, to lie on different Riemann sheets of the angular momentum plane for $s>0$, whereas for $s<0$ they are complex conjugates of each other on the same sheet, which is assumed in this paper to be the physical (first) sheet. There is therefore a strong pole-cut relationship based on analyticity in the angular momentum plane.[2] Hence the study of complex poles is connected to the investigation of the properties of the Regge cuts. In particular, it turns out that the effect of Regge cuts can be simulated by a pair of complex poles in a satisfactory way.[1]

The new quantity entering into the Regge-pole description of the high-energy two-body reactions in the complex-pole model is the imaginary part of the trajectory $\alpha_I(s)$, along the left-hand cut ($s<0$). The form of the function $\alpha_I(s)$ is unknown and only a crude phenomenological determination can be achieved. On the other hand, since the nature of the cuts in the angular momentum plane is not yet known in detail, any information for $\alpha_I(s)$ from the pole-cut relationship is necessarily model-dependent.

In the present work an integral equation for the discontinuity function has been established by the use of analyticity in the s plane together with some general properties of Regge poles and cuts. The solution of this equation is then determined numerically and the results are compared with the widths of the boson recurrences and with the phenomenological left-hand absorptive part.

In Sec. II the assumptions are stated and the equations of the model are derived and discussed. In Sec. III our results are compared with the experimental data and the phenomenological results