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Tests of Hadronic Bremsstrahlung Production at Large Nucleon Momentum Transfer*

H. M. Fried

Department of Physics, Brown University, Providence, Rhode Island 02912

and

T. K. Gaisser

Bartol Research Foundation of the Franklin Institute, Swarthmore, Pennsylvania 19081

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It is suggested that hadronic bremsstrahlung may become the dominant production process in nucleon-nucleon reactions at large momentum transfer to the final nucleons. Correlation tests are proposed to distinguish the model from other models of inclusive processes.

As the multiplicity of mesons produced in inelastic hadron reactions continues to increase with incident particle energy, it becomes possible to distinguish between competing theories of such production.[1] Specific tests involve the measurement of correlations, or fluctuations, in the inclusive probability distributions for a finite number of measured particles, with different models generating different forms for the energy dependence of the emitted particles.[2] The purpose of this note is to remark that there exist elementary correlation tests for the validity of the hadronic bremsstrahlung model,[3] a model which may become important in inelastic proton-proton collisions at large nucleon momentum transfers. Here, pions are produced by the decay of soft, neutral vector mesons (SNVM) which are shaken off by the nucleons as they scatter. One neglects finite-width corrections of the NVM-pion decay in order to avoid introducing correlations between decay pions of like charge.[4]

This simple picture of SVM bremsstrahlung is quite similar in principle, if not in detail, to well-known "limited correlation" models,[5] and permits a certain theoretical ease in deriving distributions which may be compared to incipient bubble-chamber and hybrid-system data.[6] Just as wide-angle elastic pp scattering may be qualitatively understood on the basis of a generalized eikonal model,[7] so may one intuitively expect the bremsstrahlung mechanism to become important when nucleon momentum transfers become large. The SVM bremsstrahlung mechanism is insufficient, by itself, to describe the majority of low-momentum-transfer events, since the single-SVM distribution function vanishes at $X = P_{\parallel}^{\text{c.m.}}/P_{\parallel\,\max}^{\text{c.m.}} = 0$ for small $p_{\perp}$.[4] The low-momentum-transfer events are presumably due to a peripheral production mechanism (diffraction dissociation and/or multiperipheral production) and are apparently characterized by a Gaussian transverse-momentum dependence.[8] In contrast, hadronic bremsstrahlung is characterized by an inverse power dependence on momentum transfer to the final nucleons. Hadronic bremsstrahlung may therefore eventually become the dominant production mechanism outside the peripheral region.[9] The transition between the two regimes may be expected to occur for final nucleon transverse momenta squared in the region 2.5 $-$ 3.5 $(\text{GeV}/c)^2$, as we show below in our concluding remarks. To test the model in its proper region of applicability, we suggest the following analysis of these anticipated National Accelerator Laboratory (NAL) experiments.

From the array of final charged-particle tracks in a typical pp bubble-chamber picture, one selects those with two outgoing protons only,[10] corresponding to a fixed set of (six) invariants constructed from the proton 4-momenta. One attempts to pair each of the charged-pion tracks with another of opposite charge, and determines in this way which pairs may be considered to have originated in the decay of a ρ^0. Those pions which cannot be so identified must arise from ω (or other resonance) decay, or from nonresonance multiperipheral mechanisms. If the π^0 from ω decay could be identified, the number of such NVM's measured would be considerably higher, but if only charged pions are to be measured, this analysis picks out the ρ^0 decays only. If the bremsstrahlung mechanism is valid, one expects the relative number of such observed ρ^0 to increase with nucleon momentum transfer,[3] as shown in Fig. 1. (A less definitive test would be simply to measure correlations between pions of like charge at large values of nucleon momentum transfer.)

Each pair of so-labeled pions defines a parent ρ^0 of appropriate 4-momentum k; a sufficient number of such events permits the introduction of one- and two-particle inclusive distribution functions, $\rho(k)$ and $\rho(k_1, k_2)$, constructed at fixed nucleon momenta. These functions may then be compared

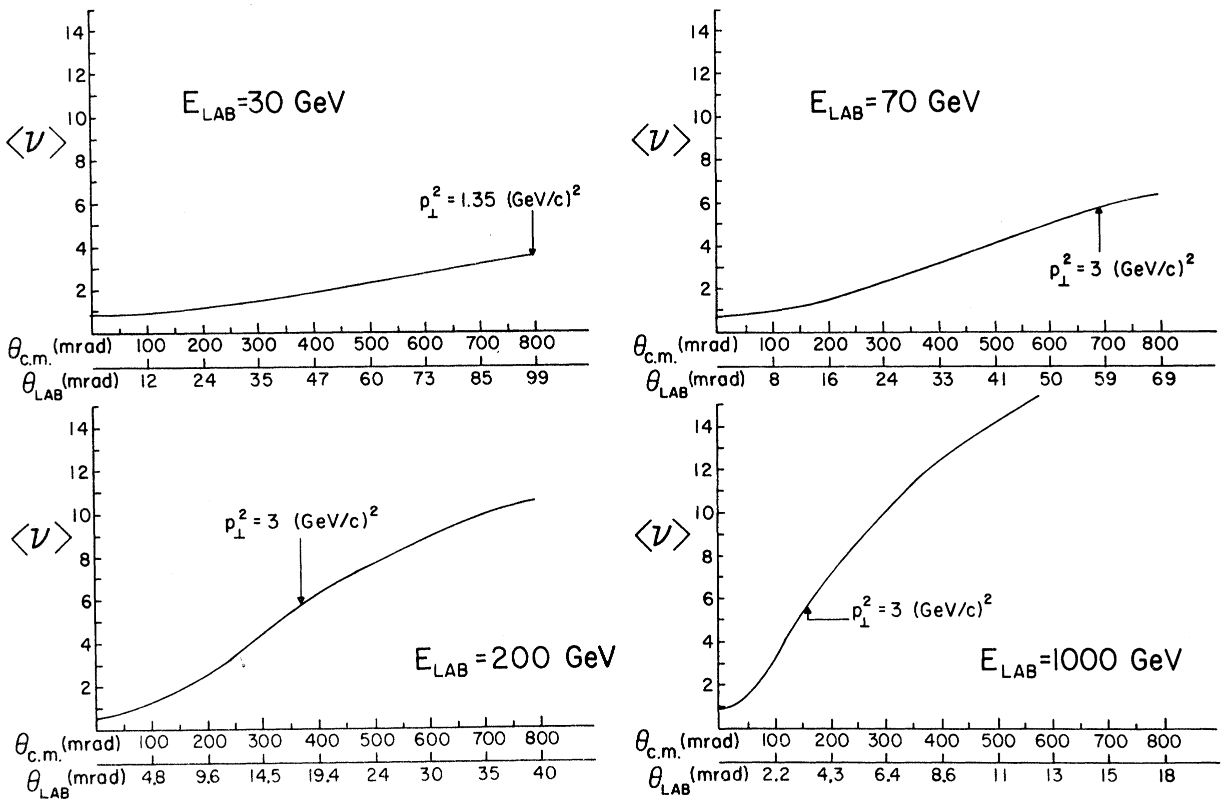

FIG. 1. Differential SNVM multiplicity as a function of final nucleon angle at various incident energies.

with approximate theoretical distributions suggested by the bremsstrahlung model, below. The most elegant way of derivation makes use of techniques recently described by deGroot, and by Bassetto, Toller, and Sertorio, in their discussions of the uncorrelated jet model,[5] producing approximate results resembling earlier results of Van Hove.[11] The "total" cross section (a two-proton inclusive distribution) summed over all emitted mesons for fixed final nucleon momenta p_3, p_4 is given by

$$\tau(p,p') = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\!\!^{-}k_1 \cdots \int d\!\!^{-}k_n |T(p_1,p_2;p_3,p_4,k_1\cdots k_n)|^2 \delta^{(4)}\Big(Q - \sum_{i=1}^{n} k_i\Big) , \tag{1}$$

where $d\!\!^{-}k = d^3k/\omega$, and $Q \equiv p_1 + p_2 - p_3 - p_4$. The class of limited correlation models may be defined by the replacement

$$|T(p_1,p_2;p_3,p_4,k_1\cdots k_n)|^2 \to |T_{el}(p_1,p_2;p_3,p_4)|^2 \times f(k_1)\cdots f(k_n) , \tag{2}$$

where $f(k)$ implicitly depends on p_1,p_2,p_3,p_4 in some model-dependent way; e.g., deGroot uses (in the c.m. system) $f = f(k_\perp, k_\parallel/p_0)$, while the bremsstrahlung model may be defined by

$$f = \frac{g^2}{(2\pi)^3}\left(\frac{p_1}{k\cdot p_1} + \frac{p_2}{k\cdot p_2} - \frac{p_3}{k\cdot p_3} - \frac{p_4}{k\cdot p_4}\right)^2 .$$

The "limited" correlations of such models are those due only to energy-momentum conservation.

Inclusive meson distributions are most conveniently written in terms of the "grand partition function"

$$C(Q) = (2\pi)^{-4} \int d^4z\, e^{iQ\cdot z} \exp\left[\int d\!\!^{-}k\, f(k) e^{-ik\cdot z}\right] , \tag{3}$$

where the implicit p_1,p_2,p_3,p_4 dependence of $f(k)$ is also understood to be present in $C(Q)$. For fixed nucleon momenta, and hence fixed Q, one has

$$\frac{1}{\tau}\frac{d\tau}{d\!\!\!{}^{-}k} \equiv \rho(k) = f(k)C(Q-k)/C(Q) \tag{4}$$

and

$$\frac{1}{\tau}\frac{d^2\tau}{d\!\!\!{}^{-}k_1 d\!\!\!{}^{-}k_2} \equiv \rho(k_1, k_2) = f(k_1)f(k_2)C(Q-k_1-k_2)/C(Q)\,. \tag{5}$$

By using (4) to eliminate the $f(k)$ dependence of (5), there results

$$\frac{\rho(k_1,k_2)}{\rho(k_1)\rho(k_2)} = \frac{C(Q)C(Q-k_1-k_2)}{C(Q-k_1)C(Q-k_2)}\,, \tag{6}$$

in which it is assumed that the $C(K)$ are themselves nonsingular distributions, so that the ratios (4) and (6) are sensible.

Equation (6), which is valid for all limited correlation models of this sort, suggests a simple test of the bremsstrahlung model, in which the arithmetic forms correspond to the emission of relatively soft (in the c.m. system) NVM's. An analytic evaluation of (3) is quite difficult, but the physical picture suggests the use of an approximation which generalizes a previous "dipole" approximation employed in wide-angle estimates[7] of elastic reactions: For "large Q" and "small k," only small values of $k \cdot z$ should be important, and one expands the exponential factor of (3) up to and including its quadratic z dependence,

$$C(Q) \sim (2\pi)^{-4} e^{\Lambda} \int d^4z \exp[i(Q-k_0)\cdot z - \tfrac{1}{2} z_\mu \Omega_{\mu\nu} z_\nu]\,, \tag{7}$$

where $\Lambda = \int d\!\!\!{}^{-}k\, f(k)|_{\mu_c}$, $k_0^\mu = \int d\!\!\!{}^{-}k\, k_\mu f(k)|_{\mu_c}$, and $\Omega_{\mu\nu} = \int d\!\!\!{}^{-}k\, f(k) k_\mu k_\nu|_{\mu_c}$. In each integrand, a cutoff μ_c has been introduced in the manner of previous computations,[3] while the Gaussian integral of (7) exists and may easily be computed:

$$C(Q) \sim (2\pi)^{-2}[\det\Omega]^{-1/2} \exp[-\tfrac{1}{2} Q\cdot\Omega^{-1}\cdot Q]\,. \tag{8}$$

All Λ and k_0^μ dependence, which drops out of the ratio (6), has been suppressed. Equation (8) is equivalent to the results obtained by the use of the central limit theorem, as in Ref. 11, and is valid in this model context when the multiplicities are very large. With (8), one immediately finds correlations given by

$$\frac{\rho(k_1,k_2)}{\rho(k_1)\rho(k_2)} = \exp[-k_1\cdot\Omega^{-1}\cdot k_2]\,, \tag{9}$$

where the $(\Omega^{-1})_{\mu\nu}$ components depend upon fixed nucleon momenta.

Equation (9) can readily be tested by observing those ρ_0 of small transverse momenta, $k_\perp \sim 0$, whereupon (9) reduces to

$$\exp[-a k_1^{\parallel} k_2^{\parallel} + \tfrac{1}{2} b (k_1^{\parallel}\omega_2 + k_2^{\parallel}\omega_1) - c\omega_1\omega_2]\,, \tag{10}$$

with a, b, c three constants (functions of nucleon momenta). A very crude but very simple estimate of Ω^{-1} may be obtained by replacing the $k_\mu k_\nu$ factor in the integrand of $\Omega_{\mu\nu}$ by $\frac{1}{4}\delta_{\mu\nu}k^2$. Then

$$\Omega_{\mu\nu} \sim -\tfrac{1}{4}\mu^2\delta_{\mu\nu} \int d\!\!\!{}^{-}k\, f(k)\Big|_{\mu_c} \sim -\tfrac{1}{4}\mu^2\delta_{\mu\nu}\langle\nu\rangle\,, \tag{11}$$

where $\langle\nu\rangle$ represents the "differential multiplicity" for these fixed-proton-momentum reactions.[3] Then (9) becomes

$$\exp[4k_1\cdot k_2/\mu^2\langle\nu\rangle] = \exp[-2(s_{12}-\mu^2)/\mu^2\langle\nu\rangle]\,,$$

where s_{12} denotes the (positive) invariant $-(k_1 + k_2)^2$. For the special case of $\vec{k}_{1,2}^{(T)} = 0$, and sizable longitudinal momenta, one has more negative correlation when k_1 and k_2 are in opposite hemispheres ($2k_1\cdot k_2 \sim -4\omega_1\omega_2$) than when they are in the same hemisphere [$2k_1\cdot k_2 \sim -\mu^2(\omega_2/\omega_1 + \omega_1/\omega_2)$]. In this aspect the correlations are similar to those of Ref. 2, but their detailed forms are quite different.

The estimate (11) replaces the ratio (9) by a number which never exceeds unity, thereby emphasizing an expected lack of positive correlation in this bremsstrahlung model. If large nucleon momentum transfer experiments do not display the large positive correlations suggested in another context in Ref. 2, it would be worthwhile to examine the data according to (10). If (10) appears to be roughly correct, separate measurements of (4) and (5), along with more careful estimates of $C(Q)$, would be in order.

To conclude, we summarize various predictions of the hadronic bremsstrahlung mechanism of particle production, and estimate the region in which it may be the dominant process of meson production. The specific picture we use is that, at high energies and large momentum transfers to the nucleons, the dominant process is single-hard-meson exchange accompanied by SNVM bremsstrahlung. This generalizes to inelastic pp scattering the eikonal picture of Ref. 7.

We emphasize that the prediction for the SVM differential multiplicity $\langle\nu\rangle$ (the quantity corresponding to the pion differential multiplicity $\bar{n}$ of Anderson and Collins[9]) does not depend on the details of the hard-meson exchange, since this cancels out in the calculation of $\langle\nu\rangle$. The differential multiplicity is, however, proportional to γ_{inel}, the parameter which combines the effects of momentum cutoff μ_c, SVM-nucleon coupling g, and our approximate way of enforcing energy-momentum conservation. Since the corresponding quantity γ_{el} that enters into the elastic pp calculation depends only on μ_c and g (energy-momentum conservation is exact here), there is no *a priori* reason to have $\gamma_{el}=\gamma_{inel}$. In Ref. 3, γ_{inel} was chosen to satisfy $\gamma_{inel}\ll\gamma_{el}$ in order to obtain a coefficient of $\ln s$ for the total multiplicity that was in accord with the experimental value. In view of the fact that hadronic bremsstrahlung is not the dominant contribution to particle production at small values of momentum transfer to the nucleons,[4] it is more natural to choose $\gamma_{inel}\lesssim\gamma_{el}$. The total multiplicity due to the hadronic bremsstrahlung is small (negligible compared to production via a peripheral mechanism) and is either constant or logarithmically increasing with energy, depending on the details of the hard-meson-exchange amplitude at large t, as well as on the precise relation between γ_{el} and γ_{inel}.[3]

It will be argued below that

$$\gamma_{inel}=(1-\eta)\gamma_{el}\ , \tag{12}$$

with η the (c.m.) nucleon elasticity, is a simple and reasonable choice for γ_{inel} which incorporates energy-momentum conservation in a consistent way and leads to constant total SVM multiplicity. In Fig. 1 we plot the differential SVM multiplicity for various primary energies with this choice of γ_{inel} and $\eta=\frac{1}{2}$.[12] The value of $p_\perp{}^2\sim 3\ (\text{GeV}/c)^2$, above which the hadronic bremsstrahlung process is expected to dominate, is indicated in the figure. It is outside the accessible kinematic region at $\eta=\frac{1}{2}$ and $E_{lab}=30$ GeV. We have made the assumption that most events correspond to the final nucleons having zero total momentum in the c.m. system. The last assumption was also made by Anderson and Collins[9] in deriving values for the differential multiplicity to compare with the 30-GeV pp experiment. Furthermore, the data they use have inelasticities in the range 0.25–0.8, so that $\frac{1}{2}$ is an appropriate elasticity for comparison. The differential multiplicity was compared with experiment in Ref. 9 by a model-dependent calculation that involved evaluating the phase space for $\bar{n}_\pi$ pions and comparing differential phase-space shapes with single proton cross sections at various angles to determine $\bar{n}_\pi$ as a function of proton angle. In general, and in the present model, $\langle\nu\rangle$ also depends on inelasticity. Furthermore, comparison between our prediction for $\langle\nu\rangle$ and that of Anderson and Collins for $\bar{n}_\pi$ is to be treated with caution because (i) their phase-space integrals are evaluated for pions, and (ii) at 30-GeV lab energy and inelasticity $\sim\frac{1}{2}$ the maximum number of ρ's that can be produced is four or five, so that kinematic limits are important here. Furthermore at $\eta=\frac{1}{2}$, the range of $p_\perp$ values at which hadronic bremsstrahlung may begin to dominate is possibly beyond the kinematic limit. Nevertheless, the similarity between the two results for differential multiplicity is encouraging.

To estimate the value of final nucleon transverse momentum for which the hadronic bremsstrahlung mechanism may begin to dominate peripheral production processes we make a specific assumption for the single-hard-meson exchange that gives rise to the large-angle scattering, and calculate the resulting single-proton distribution. The transverse momentum dependence calculated from hadronic bremsstrahlung can then be compared with the experimentally observed Gaussian falloff of the peripheral process to locate the critical value of $p_\perp$.

The bremsstrahlung-model expression for the single-nucleon inclusive distribution is [see Eq. (7) of Ref. 4]

$$E'\frac{d^3\sigma}{d^3p'}=\frac{m^4}{E_1E_2|v_{12}|}\int\frac{d^3p_4}{E_4}\int\frac{d^4x}{(2\pi)^4}\,e^{i(p_1+p_2-p_4-p')\cdot x}\,\frac{|\bar{M}_0|^2}{4\pi^2}\exp[K(x)]\ , \tag{13}$$

where

$$K(x)\equiv\int\frac{d^3k}{2\omega}\,e^{-ik\cdot x}f(k,p_4,p',s) \tag{14}$$

and f is the factor associated with the emission of an SVM.[4] The differential multiplicity is related to K by

$$K(0)=\langle\nu\rangle=-2\gamma_{inel}[F(t_{13})+F(t_{24})+F(u_{14})+F(u_{23})-F(4m^2-s)-F(4m^2-s_{34})]\ , \tag{15}$$

where $s=-(p_1+p_2)^2$, $s_{34}=-(p_3+p_4)^2$, $t_{13}=-(p_1-p_3)^2$, $u_{23}=-(p_2-p_3)^2$, etc. The quantity $|\bar{M}_0|^2$ is the spin-averaged "elastic" amplitude squared for $p_1+p_2\to p_3+p_4$, with $s_{34}\leq s$.

We approximate Eq. (13) as in Ref. 4 by neglecting the x dependence inside $K(x)$ and assuming that the dominant contribution is for $(\vec{p}_3+\vec{p}_4)_{c.m.}=0$ so that $t_{13}=t_{24}\equiv t$ and $u_{23}=u_{14}\equiv u$. Thus (from now on all energies and momenta are expressed in the c.m. system)

$$E'\frac{d^3\sigma}{d^3p'}=\frac{m^4}{E^3E'|v_{12}|}P\left(\frac{E'}{E}\right)\frac{|\bar{M}_0|^2}{8\pi^2}e^{K(0)}, \tag{16}$$

where $P(\eta)$ is the relative probability of producing elasticity $=\eta$. It is normalized to unity. When we now split the "elastic" amplitude into a factor M_H, containing the single hard-meson exchange, multiplying the elastic eikonal factor[7] we have

$$E'\frac{d^3\sigma}{d^3p'}=\frac{m^4}{16E^2E'\pi^2|p|}P\left(\frac{E'}{E}\right)\frac{1}{4}\sum_{\text{spins}}|M_H|^2\exp\{2(\gamma_{el}-\gamma_{inel})[2F(t)+2F(u)-F(4m^2-s)-F(4m^2-s_{34})]\}. \tag{17}$$

In general, $\gamma_{inel}=\gamma(\eta)$ must satisfy $\gamma_{inel}(1)=0$, and it must be small enough so that the differential SVM multiplicity does not violate energy conservation. It is otherwise unknown. For simplicity we assume the form stated in Eq. (12). With $\gamma_{el}\approx 2\frac{1}{4}$ (taken from the fit to elastic pp scattering in Ref. 7) this choice enforces energy conservation well enough for the present calculation for energies as low as $E_{lab}\approx 30$ GeV.[13]

For single-pion exchange the quantity $\frac{1}{4}\sum|M_H|^2$ is independent of s for large s, so that the corresponding $E'd^3\sigma/d^3p'$ vanishes as $s\to\infty$. We therefore use single-vector-meson exchange for M_H and find

$$\frac{1}{4}\sum_{\text{spins}}|M_H|^2=\frac{g_{\rho NN}{}^4}{8m^4}\left[\frac{4tm^2+(s-2m^2)(s_{34}-2m^2)+(u-2m^2)^2}{(t-m_\rho{}^2)^2}+2\frac{(s-2m^2)(s_{34}-2m^2)+2m^2(t+u)-m^2(s+s_{34})}{(u-m_\rho{}^2)(t-m_\rho{}^2)}+\frac{4um^2+(s-2m^2)(s_{34}-2m^2)+(t-2m^2)^2}{(u-m_\rho{}^2)^2}\right]. \tag{18}$$

This expression, together with Eq. (17), constitutes the model approximation for the single-nucleon distribution outside the peripheral region. For $m^2\ll p_\perp{}^2\ll p_\parallel{}^2$ Eq. (18) simplifies considerably to

$$\frac{1}{4}\sum_{\text{spins}}|M_H|^2\approx\frac{g_{\rho NN}{}^4}{m^4}\frac{4(E')^4}{p_\perp{}^4}. \tag{19}$$

In the same limit

$$2F(t)+2F(u)-F(4m^2-s)-F(4m^2-s_{34})\approx -2\ln\left(0.4\frac{E}{E'}p_\perp{}^2\right), \tag{20}$$

so that we have

$$E'\frac{d^3\sigma}{d^3p'}\approx\left(\frac{g_{\rho NN}{}^2}{4\pi}\right)^2\eta^3P(\eta)\frac{4}{p_\perp{}^4}\times\exp\left[-4\eta\gamma_{el}\ln\left(\frac{0.4}{\eta}p_\perp{}^2\right)\right], \tag{21}$$

a scaling form for the inclusive nucleon cross section in this region.

To estimate numerically the range of $p_\perp{}^2$ for which hadronic bremsstrahlung dominates peripheral production processes we take[7] $\gamma_{el}\approx 2\frac{1}{4}$, $g_{\rho NN}{}^2/4\pi\approx 1$, and $P(\eta)\approx 1$. Then

$$E'\frac{d^3\sigma}{d^3p'}\approx C'\eta^3(2.5\eta)^{9\eta}\left(\frac{1}{p_\perp{}^2}\right)^{9\eta+2},$$

where $C'\approx 1.6$ mb/(GeV$^2/c^3$) if $p_\perp$ is expressed in GeV/c. This function is compared in Fig. 2 with the Gaussian falloff of the peripheral process observed in Ref. 8. The transition takes place in the range 2.5 (GeV/c)$^2\lesssim p_\perp{}^2\lesssim 3.5$ (GeV/c)^2.[14]

It is interesting to note the qualitative resem-

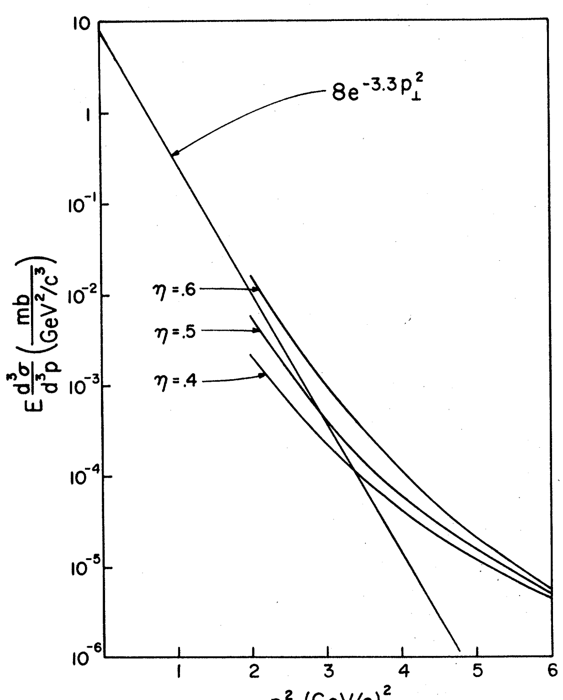

FIG. 2. Transverse-momentum dependence of the single-proton inclusive cross section: Comparison of hadronic bremsstrahlung model prediction with Gaussian falloff at large $p_\perp{}^2$.

blance of Fig. 2 to the deviation from Gaussian transverse momentum dependence observed by Marmer *et al.*[15] in the process $pp \to p$ + anything. The beam momentum in this experiment (12.5 GeV/c) is, however, so low that the resemblance is inconclusive.

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[9]E. W. Anderson and G. B. Collins [Phys. Rev. Letters 19, 201 (1967)] suggested some time ago that a bremsstrahlung picture could account for the features of meson production in pp interactions outside the peripheral region. The relation of the present model to their analysis will be discussed below.

[10]Extraction of this information from the data entails far more complicated procedures than are suggested by the cavalier description given here. Typically, one expects to see one slow and one fast proton in the midst of a pion spray. If the slow proton has a momentum less than ~1.5 GeV/c it can be distinguished as a proton, while the fast proton is frequently assumed to be the most energetic of the outgoing cluster of charged particles. This discussion assumes that, at energies high enough to produce large numbers of pions, there will be an arbitrarily large number of data, from which the small fraction of processes at large nucleon momentum transfer are selected and used to construct ρ^0 distributions.

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[12]The expression for the differential multiplicity used in Eq. (13) of Ref. 3 must be modified for the case in which $\theta_{\text{c.m.}}$ is held fixed as $s \to \infty$. The correct expression, when the final protons have equal and opposite center-of-mass momenta and when $s, s_{34} \gg m^2$, is $\langle \nu \rangle \approx -4\gamma_{\text{inel}} F(t) + 8\gamma_{\text{inel}} \ln|\frac{1}{2}(\cos\theta_{\text{c.m.}})|$, where $F(t) \approx -\ln(1+0.4|t|)$ for $t<0$, and t is expressed in GeV^2. Note that with the choice (12) for γ_{inel} the SVM differential multiplicities of Fig. 1 are now about an order of magnitude larger than in Ref. 3.

[13]By determining $\gamma_{\text{el}} = 2\frac{1}{4}$ from the elastic calculation and taking the simple form (12) for the relation between γ_{el} and γ_{inel} energy conservation is violated marginally ($\lesssim 1$ excess SVM), near $\theta_{\text{c.m.}} \sim 90°$ only, for $E_{\text{lab}} \lesssim 200$ GeV. This is an artificial consequence of the simplifications inherent in the way energy-momentum conservation is enforced, and could be removed by a more complicated relation between γ_{inel} and γ_{el} than Eq. (12).

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