PHYSICAL REVIEW D VOLUME 7, NUMBER 3 1 FEBRUARY 1973

Interaction Lengths of Energetic Pions and Protons in Iron*

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(Received 19 June 1972)

The interaction lengths of pions and protons in iron have been measured using an ionization spectrometer composed of alternating layers of iron and plastic scintillator. These measurements cover an energy range from 9.3 to 18 GeV. The interaction lengths were determined by accurate statistical analyses of the experimental data using the maximum-likelihood method. The dependence of the interaction length on the parameters used to define an interaction was studied, and the results reported employ parameters chosen to minimize the percentage uncertainty in the interaction length. The mean interaction length of pions was found to be approximately 20% greater than that of protons. No significant dependence on the energy of the incident primary was found in the range measured. Consistency was found with the great variety of accelerator measurements made to date, all of which report greater interaction lengths than those reported by the cosmic-ray measurements.

INTRODUCTION

As energetic charged particles traverse matter they lose energy in small amounts via an almost continuous process of excitation and ionization of the atoms and molecules of the absorber material. However, strongly interacting particles (hadrons) also have a probability of interacting with *in situ* nuclei. The strong interaction is characterized by the production of one or more additional strongly interacting particles (usually π or K mesons) and hence constitutes a much larger and discrete energy loss by the incident hadron than is typical of the more frequent atomic collisions.

The interaction probability of hadrons in hydrogen has been studied extensively, both as a function of the incident energy and the type of incident hadron. The probability for interactions in materials other than hydrogen has been less extensively studied. In this paper we report the results of measurements on the probability for strong interactions of incident pions and protons in iron. Similar measurements have been reported previously,[1-9] but these earlier works suffered from large uncertainties in the incident energy or from less accurate methods of identifying the strong interaction. In this experiment, momentum-analyzed beams of pions and protons produced by the Alternating Gradient Synchrotron were employed. The detector was composed of modules consisting of alternating layers of iron and plastic scintillator. The output signal from each module was digitized and recorded on magnetic tape, so that each event could be reconstructed and analyzed in terms of a variety of parameters.

In the present experiment, as well as in most of the previous work, the parameter that is measured is the module number of the detector in which the interaction took place. If the probability of an interaction is independent of the particle energy, then the probability of an interaction per unit thickness of absorber is independent of the depth in the material. Hence the probability of a particle traversing a depth l without interacting is given by

$$P(l)=e^{-l/\lambda}. \qquad (1)$$

The constant λ is the mean interaction length, and is related to the total interaction cross section, σ, by

$$\lambda=\frac{A}{\sigma A_{v}\rho}, \qquad (2)$$

where A and ρ are respectively the average atomic mass and the density of the target material, and A_v is Avogadro's number. The parameter λ is determined from the distribution of interactions in the detector.

Details of our experimental technique, procedures for data reduction and analysis, and comparisons with previously reported results are described in the following sections.

APPARATUS

The cosmic-ray detector used in this experiment is designed to measure the energy spectra, charge composition, and arrival directions of cosmic rays in the energy range from 10^{10} to 10^{14} eV. The data for this experiment were accumulated during a calibration run performed in the $G10$, $+4.7°$ test beam at the Alternating Gradient Synchrotron at Brookhaven National Laboratory. The instrument, shown schematically in Fig. 1, contains a particle identification section which serves to define the charge and trajectory of the incident particle and an ionization spectrometer to determine the energy of the incident particle.

The spectrometer consists of twelve tungsten modules and seven iron modules. For a portion of the present measurements, four additional iron modules were incorporated in the spectrometer, using electronics from four tungsten modules. The spectrometer is designed to permit unambiguous identification of incident electrons through the rapid development of electromagnetic cascades in the tungsten portion of the detector. Each tungsten module consists of a tungsten plate 6.3 g/cm^2 thick (0.32 cm or approximately one radiation length) and a scintillating plastic sheet 0.65 g/cm^2 thick (0.64 cm). Each of these modules is viewed separately by a pair of photomultiplier tubes.

The spectrometer is also designed to measure the energy of primary cosmic-ray protons, principally through the response of the iron modules to energy deposited by nuclear-electromagnetic cascades. In the detector used i.. the present experiment, the arrangement of absorber material (iron) and active elements (plastic scintillator) is such as to optimize the energy resolution of the spectrometer by minimizing fluctuations in its energy response. Conventional ionization spectrometers used in cosmic-ray experiments exhibit an energy resolution which is dominated by two major sources of fluctuations. One is caused by uncertainty in the location of the first hadronic interaction, which leads directly to an uncertainty in what fraction of the primary energy escapes out the bottom of a finite-depth detector. The other fluctuation results from infrequent sampling of the energy deposited in electromagnetic cascades which are initiated by the neutral pions produced in high-energy hadronic interactions. In the present experiment, each iron module consists of 66.4 ± 0.7 g/cm^2 (approximately one half of a nuclear interaction length) of iron, and is viewed by a pair of

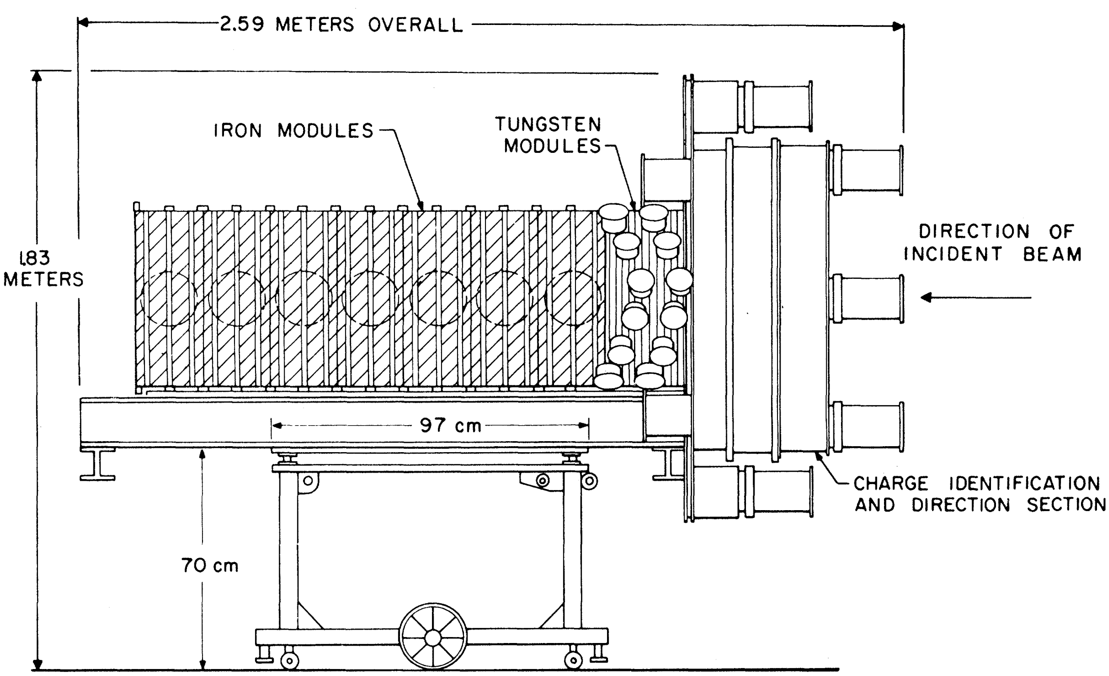

FIG. 1. A schematic view of the high-energy cosmic-ray ionization spectrometer used in this experiment.

photomultiplier tubes. This enables location of hadronic interactions to within one half of a nuclear interaction length. The iron in each module is arranged in layers interspersed with three 0.64-cm-thick plastic scintillator sheets. The active elements are thus located approximately every 1.5 radiation lengths of absorber material so as to thoroughly sample electromagnetic cascades. This spectrometer design has greatly improved the energy resolution over previously reported results.[10]

The output signal of each of the tungsten and iron modules was calibrated using cosmic-ray muons, and noninteracting protons produced by the accelerator. Calibration runs were taken both before and after the proton and pion data were accumulated. The response of each of the modules to equivalent single relativistic muons was determined by a pulse-height analysis of events in which energetic cosmic-ray muons were incident on the detector. Such events were selected by demanding that alternate modules situated before and after each of the modules being analyzed have pulses corresponding to the passage of a single relativistic particle.

ANALYSIS OF DATA

In the approximately four nuclear interaction lengths contained in the ionization spectrometer (six interaction lengths after the additional modules were added) almost all incident hadrons participated in a nuclear interaction. The problem was to determine reliably in which module the interaction occurred. It was assumed that at least one additional high-energy particle (usually a pion) is created in any nuclear interaction. Thus the modules located at greater depth than the interaction show an increase in signal associated with the passage of more than one relativistic particle. When a neutral pion is created in the nuclear interaction, the resulting decay to two γ rays and the subsequent electromagnetic cascade produce many ionizing particles in the following modules.

In order to determine the module in which the nuclear interaction took place, the pulse height from each module is divided by the average equivalent muon pulse height. The numbers thus obtained, which are assumed to represent the number of ionizing particles in each module, are used to determine the interaction location.

To illustrate the nature of the problems encountered, plots of the number of ionizing particles as a function of module number are shown in Figs. 2(a) and 2(b). Figure 2(a) illustrates a case which is easy to analyze. A single incident particle passed through all 12 tungsten modules and two iron modules. The outputs of these modules show only the usual statistical variations associated with the passage of one relativistic particle. In Iron Module 3 (Fe3) an interaction occurred which produced one additional particle. The subsequent nuclear and electromagnetic cascade is seen to build up in the deeper modules. Figure 2(b) shows a more typical case and is clearly more difficult to analyze unambiguously. In this case two isolated modules, Tungsten 1 and 11 (T1, T11) record pulse heights consistent with the passage of several relativistic ionizing particles but are followed by modules with outputs corresponding to one relativistic particle. It is assumed that these isolated high-output signals are due to stochastic variations in the number of scintillation photons

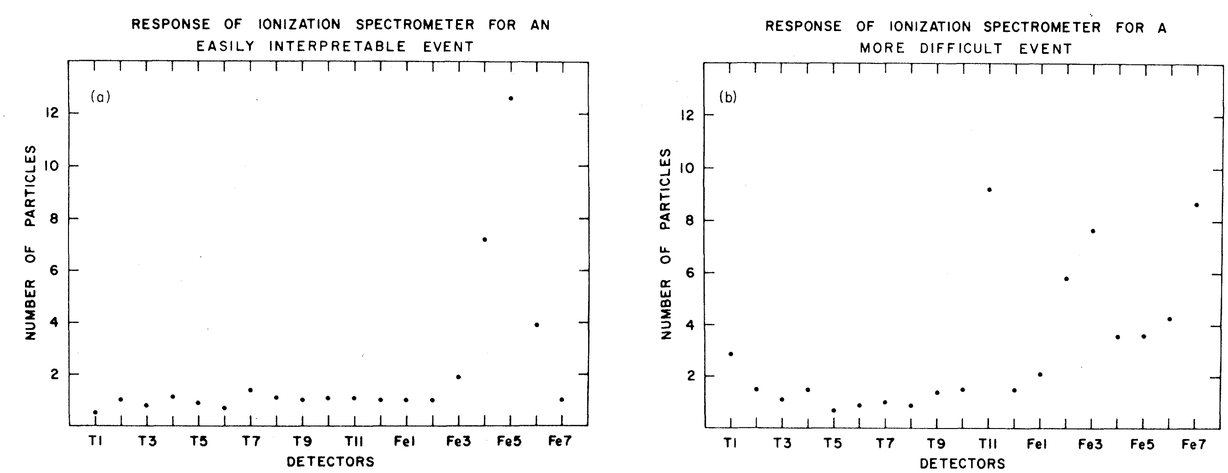

FIG. 2. (a) Plot of pulse height as a function of module number. In this case the module in which the interaction occurred is clearly Fe3. (b) Plot of pulse height as a function of module number. This plot shows the difficulty of making a unique selection of the interaction module.

detected, photomultiplier tube noise, or the production of δ rays that stop in the detector. This conclusion is supported by the observation that these "spuriously" high outputs appear with the same frequency even when the incident particles are muons. In the Fe modules the effect of Landau fluctuations is greatly reduced by the averaging of the pulse height from the three scintillators. In all of the analyses reported here, at least two consecutive modules indicating more than one relativistic ionizing particle were required to define an interaction. This effectively requires that charged pions be produced with a range of more than 66 g/cm^2 in iron or that a π^0 be produced with enough energy (~600 MeV) so that the associated electromagnetic cascade penetrates more than 9 radiation lengths of iron.

In order to analyze the 5×10^4 events recorded during the course of this experiment, a computer program was written to process the data. The data recorded from each incident particle were searched for two or more consecutive modules with outputs above a threshold level. The interaction was defined to have started in the first of the modules in which the threshold was exceeded. Both the value of the threshold and the number of consecutive modules required to exceed this value were adjustable parameters. After reduction of the data for a particular run, a summary of the information for each module was printed. This summary included the total number of events analyzed, the number of times a single particle passed through each module, the number of times the signal was spurious (i.e., the signal exceeded the threshold value but the signal in the next module did not), and the number of times a particle interacted in each module.

In some instances spurious effects generate a high signal in the module just before the one in which an interaction occurred. This spurious signal causes the wrong interaction module to be selected. If all modules were spuriously high the same percentage of the time, the slope of the interaction curve would not be affected. However, this was not the case; the spurious rate varied from much less than 1% to slightly more than 5%. The number of interactions found in each plate was therefore corrected, as follows, to account for the effects of spuriously high events.

If module i produced a spuriously high signal a fraction of the time S_i, then the observed number of events in module i, N_i, should be reduced by $S_i N_i$ because of the spurious signals. However, the number of events in module i should be increased by the number of events falsely assigned to the $i-1$ module. Thus the correct number of events n_i in module i is given by

$$n_i = N_i(1 - S_i) + S_{i-1} N_{i-1}\,. \tag{3}$$

The mean interaction length was found by determining the slope of the exponential decay of the number of interactions in each of the iron modules. In all of these determinations the interactions in both the first and last iron modules were ignored. The first iron module was ignored because of predicted backscattering effects,[11] and more importantly because this module is not symmetrically embedded in iron. The showers created by interactions in the deeper portions of this module are detected by only one or two scintillator sections, and are hence seen less efficiently than interactions occurring in the front of the module. Such low-efficiency events will often be registered as having started in the subsequent iron module, but there is no corresponding module in front of the first one to add to events detected as starting in the first module. Thus the efficiency of the first module is predictably too low. The interactions detected in the last module were also ignored because of a lack of symmetry in the backscattering and because there was no effective way to determine the number of times the last detector was spuriously high.

STATISTICAL ANALYSIS OF THE EXPERIMENTAL DATA

The processes by which energetic hadrons interact in an ionization spectrometer are stochastic, and the relationships describing the expected response of the detectors are statistical. In the present work, the method of maximum likelihood was used to determine interaction lengths from distributions of interactions in an ionization spectrometer. This method is readily applicable because each particle has a well-defined *a priori* probability of interacting as a function of detector module number, independent of the interaction probabilities of each of the other particles.

Another approach, sometimes used to determine the interaction length, employs a less cumbersome technique. The logarithm of the observed interaction distribution is fitted by a weighted least-squares method to a straight line. The applicability of the least-squares method relies on the independence of the numbers of particles observed to interact in each detector module and the appropriate choice of the weighting function. In any attenuation experiment such as reported here, the numbers of particles interacting in the different detector modules are not independent, and are in fact related by the expression

$$N = \sum_{j=1}^{k} n_j\,, \tag{4}$$

where N is the total number of interacting particles, k is the number of detector modules, and the n_j are the numbers of particles observed to interact in each of the modules. That is, the probability of interacting in module i is coupled with the probability of not interacting in the $i-1$ preceding modules. In the present experiment, the total detector thickness is large, approximately 4 interaction lengths of absorbing material. Therefore, the total number of interacting particles is a large fraction of the total flux through the detector. Fluctuations in the number of particles interacting in any module propagate as fluctuations in the flux through succeeding detector modules. Thus, the approximation that the n_j are independent cannot be justified. Its use would lead to results which are influenced by coupled statistical fluctuations in the numbers of interacting particles in successive detector modules.

For each data set in the present experiment, an interaction length λ was determined by maximizing the likelihood, as a function of λ, given by the product of all the *a priori* probabilities for each event in the data set.

The probability that a particle, present at $l=0$, will arrive at l is $e^{-l/\lambda}$, where λ is the nuclear interaction length. The probability that a particle at l will interact between l and $l+dl$ is dl/λ. Since these probabilities are independent, the probability that a particle present at $l=0$ will arrive at l and will interact between l and $l+\Delta l$ is given by

$$P(l, \Delta l, \lambda) = \left| \int_l^{l+\Delta l} e^{-l/\lambda} dl/\lambda , \right. \tag{5}$$

or

$$P(l, \Delta l, \lambda) = e^{-l/\lambda}(1 - e^{-\Delta l/\lambda}), \tag{6}$$

where Δl is a module thickness, and l is the total thickness of modules up to the one in which the particle interacts. However, in this experiment, the *only* particles selected are those which interact between $l=0$ and $l=L$, where L is the total thickness of all the modules. Thus, the probability of interacting in the whole detector, treating it as *one* module, is (from Eq. 6)

$$P(0, L, \lambda) = 1 - e^{-L/\lambda}. \tag{7}$$

So the probability of interacting between l and $l+\Delta l$ for all particles which arrive at $l=0$ and which do interact between $l=0$ and $l=L$ is given by

$$P(l, \Delta l, L, \lambda) = e^{-l/\lambda} \frac{1 - e^{-\Delta l/\lambda}}{1 - e^{-L/\lambda}}. \tag{8}$$

An analogous expression is commonly used in bubble-chamber experiments to determine particle lifetimes.[12]

For large L/λ and small $\Delta l/\lambda$, Eq. (8) reduces to the well-known limiting expression

$$\lim_{L/\lambda \to \infty;\ \Delta l/\lambda \to 0} P(l, \Delta l, L, \lambda) = \frac{\Delta l}{\lambda} e^{-l/\lambda}. \tag{9}$$

However, since the limit criteria are not satisfied in the present work, Eq. (8) is the appropriate expression for the *a priori* probability, and is used to calculate the likelihood function.

The likelihood function is given by

$$\mathcal{L} = \prod_{i=1}^{N} P_i(l, \Delta l, L, \lambda). \tag{10}$$

Using P_i from Eq. (8) gives

$$\mathcal{L} = \prod_{i=1}^{N} e^{-l_i/\lambda} \frac{1 - e^{-\Delta l/\lambda}}{1 - e^{-L/\lambda}}, \tag{11}$$

where i indexes each of the N independent events in the data set. The interaction length, to be determined from the data set, is that value of the quantity λ for which the likelihood $\mathcal{L}$ is a maximum. Since $\mathcal{L}$ is a positive definite quantity, it is a maximum when

$$\ln(\mathcal{L}) = \sum_{i=1}^{N} \left[\frac{-l_i}{\lambda} + \ln(1 - e^{-\Delta l/\lambda}) - \ln(1 - e^{-L/\lambda}) \right] \tag{12}$$

is a maximum. Analytically, $\ln(\mathcal{L})$ can be maximized by setting its first derivative with respect to λ equal to zero. The resulting transcendental expression for λ must be solved by numerical techniques. In the present work values of $\ln(\mathcal{L})$ were computed for each data set for a range of values of λ. $\ln(\mathcal{L})$ was found to be nearly normally distributed in reciprocal λ, $1/\lambda$, for all data sets, independent of size. For data sets containing more than 2500 events, $\ln(\mathcal{L})$ was found to be normally distributed in λ as well. The observed behavior of the likelihood function in λ and reciprocal λ has been predicted for this type of data in the review of particle properties by the Particle Data Group,[13] and can be derived from the analytical behavior of Eq. (12).

The measured uncertainties were determined from the numerical calculations from that change in λ, $\delta\lambda$, for which $\ln(\mathcal{L})$ decreases by the amount 0.5 from its maximum value. This corresponds to the one-standard-deviation points of a normal distribution.[14] A limit on the best accuracy obtainable in λ for a total number of events, N, for a small $\Delta l/\lambda$ and for a finite L/λ has been presented in Ref. 13. In the present work, in which both $\Delta l/\lambda$ and L/λ are finite, the minimum uncertainty in λ can be shown to be

$$\delta\lambda_{\min} = \frac{\lambda}{\sqrt{N}}\left[e^{\Delta l/\lambda}\left(\frac{\Delta l/\lambda}{e^{\Delta l/\lambda}-1}\right)^2 - e^{L/\lambda}\left(\frac{L/\lambda}{e^{L/\lambda}-1}\right)^2\right]^{-1/2}. \tag{13}$$

All standard deviations determined in the present work slightly exceed $\delta\lambda_{\min}$. The values of λ and $\delta\lambda$ determined from the analyses just described are presented in the next section.

RESULTS

Because the analysis program was written in a general manner, it was relatively easy to determine both the slope of the interaction curve and the quality of the fit as a function of numerous parameters. The data were analyzed as a function of the number of consecutive "high" modules required to define an interaction, as a function of the number of equivalent muons needed to define an interaction, and as a function of different numbers of muon equivalent particles in the first and second interaction modules.

The slope of the interaction curve was calculated for several different runs using the requirement of outputs exceeding a threshold from two or three consecutive modules to define an interaction. For each run the slopes determined using the requirement of two consecutive modules agreed well, within the experimental uncertainties, with the slopes determined using the requirement of three consecutive modules. However, the statistical accuracy associated with the three-module measurements was somewhat less. The data from one less module could be used, since the number of interactions occurring in the next-to-last module obviously could not be determined with the same requirements placed on the other modules. The loss of information which might have been provided by this one additional module increases the statistical uncertainty. Since the determination of the slope was independent of whether the two- or the three-consecutive-module requirement was used, and because greater accuracy was achieved with the two-module requirement, all succeeding determinations were made with the requirement that two consecutive modules had outputs which exceeded the selected thresholds.

The threshold chosen to define an interaction selects the physical processes to be observed. As the threshold for determining an interaction is raised, interactions which have low multiplicity and also fail to develop a significant shower are not detected. However, interactions with high multiplicity will continue to be detected. Most interactions in which one or more π^0's are produced will also be detected due to the rapid buildup of the electromagnetic shower following the decay of the π^0 into two γ rays. At the energies investigated in this work, higher-multiplicity events have lower probabilities. Since increasing the threshold requirement will create a bias against some low-multiplicity, higher-probability events while continuing to select the high-multiplicity, lower-probability events, the interaction length is expected to rise as a function of increasing threshold requirement. Figure 3 shows a plot of the measured in-

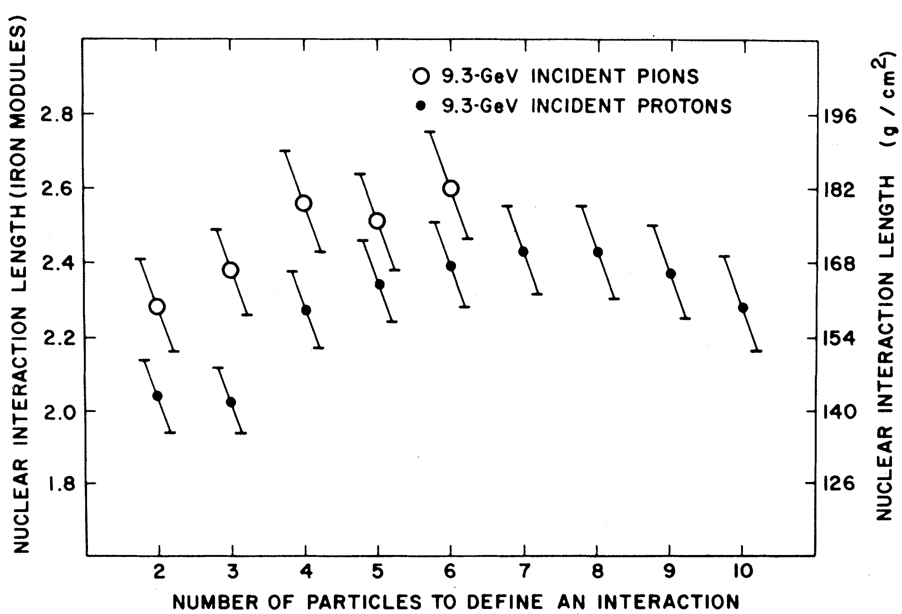

FIG. 3. Plot of the measured interaction length as a function of threshold requirements.

teraction length for both 9.3-GeV protons and positive pions as a function of the threshold. The interaction length is seen to increase by about 20% as the threshold requirement is raised from 2 to 7 single-particle equivalents. The measured interaction length seems to remain roughly constant or decrease slightly for still higher thresholds. However, the statistical accuracy for the higher threshold requirements rapidly becomes worse because not many events satisfy the rather unusual threshold requirements. For very low threshold requirements the statistical uncertainty in the determination of the slope increases for similar reasons. For example, as the threshold is lowered to one ionizing particle, the interaction will almost always be detected in the first few tungsten modules, and no useful data for the determination of the slope in the iron modules will be available.

Not many data are available for the particle multiplicities to be expected from nuclear interactions in iron at the energies investigated in the present work. Thus the interaction mean free path was determined for a range of values of the thresholds. Figure 4 shows a plot of the statistical uncertainty in the mean free path as a function of threshold. For these data there is a very clear minimum in the percentage uncertainty for a threshold requirement of 3.0 muon equivalent particles. The other data (13.8- and 17.8-GeV protons and 9.3-GeV pions) also show a minimum in the uncertainty at a threshold value of 3.0 muon equivalent particles, although in these latter cases the minimum is not so pronounced. This minimum in the uncertainty occurs even though the total number of interactions found in the iron modules is a monotonically increasing function of the threshold value until the threshold is approximately 8 muon equivalent particles. The uncertainty in the determination of the slope depends both upon the statistical accuracy of the data and upon how well the data fit the assumed exponential decay function. The fact that for values of the threshold greater than 3 the quality of the fit becomes worse while the total number of interactions detected increases indicates that data for these higher thresholds do not fit the hypothesis of a simple exponential decay so well. The threshold which resulted in the minimum statistical uncertainty in λ was chosen.

Once criteria for the number of modules (2) and the threshold value (3 muon equivalent particles) had been determined, the effect of adjusting the threshold of the second module while holding the threshold in the first at 3 was investigated. For some cases, particularly for interactions which initiate electromagnetic cascades, the signal level in the second module is higher than in the first. It was thought that a higher threshold for detection in the second module would more nearly represent true events and be a better criterion for event selection. The data for 9.3-GeV protons were analyzed by varying the threshold for the second module from 1 to 5 muon equivalent particles while the threshold for the first module was held constant at 3 muon equivalent particles. As can be seen in Fig. 5, the minimum uncertainty in the determination of the slope occurs with the same threshold

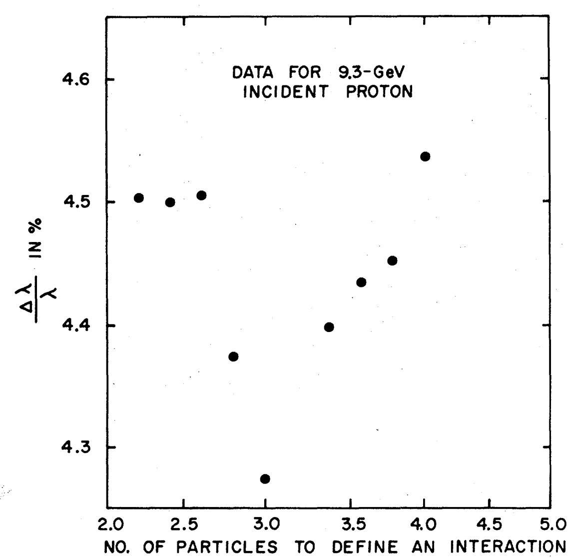

FIG. 4. Plot of statistical uncertainty in the determination of the mean interaction length as a function of threshold requirement.

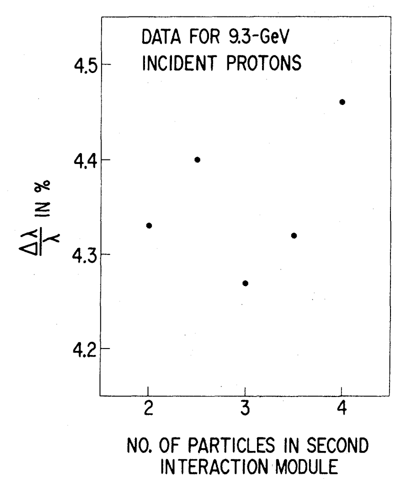

FIG. 5. Plot of statistical uncertainty in the determination of the mean interaction length as a function of threshold requirement in the second interaction module with the first held fixed at three equivalent particles.

TABLE I. Mean interaction length of protons in iron.

Incident energy (GeV)	Number of iron modules	Total number of detected interactions	λ (g/cm^2)	$\Delta\lambda/\lambda$ (%) Measured	Minimum
9.3	5	2942	$150.8^{+4.8}_{-4.7}$	±3.2	±3.0
13.8	5	2804	$151.4^{+5.1}_{-4.7}$	±3.2	±3.0
17.8	5	3187	139.0 ± 4.1	±2.9	±2.8
17.8	9	2683	$140.2^{+3.2}_{-3.0}$	±2.3	±2.2

for both the first and second modules. While adjusting the threshold value in the second module changed the accuracy of the slope determination, no appreciable change in the value of the interaction length was detected. Thus all final results on the determination of interaction lengths presented here are based on the requirement of two successive modules with a signal output above 3 times the single-particle level to define an interaction.

The results of the determination of mean interaction lengths for protons and pions are presented in Tables I and II. The number of modules employed, the total number of interactions analyzed, and the measured mean nuclear interaction length are given for each incident energy. The uncertainty in the interaction length determined by the maximum-likelihood method is also given, along with the theoretical minimum statistical uncertainty based on the total number of interactions detected. Each of the experimentally determined interaction lengths contains an additional uncertainty of ±1% due to the uncertainty in the mean density of the detector material.

It needs to be strongly stressed that in Tables I and II the values of the interaction lengths and the associated uncertainties are given for our best-fit definition of an interaction (3 muon equivalent particles in each of two consecutive modules). As has been discussed previously, the value of the interaction length can be changed considerably outside the stated errors by appropriate changes in the definition of an interaction.

The mean interaction lengths are observed to be independent of the number of iron modules employed to within the statistical uncertainties in the measurement. No significant dependence of the interaction length on energy is detected. On the hypothesis that the interaction length is independent of energy, $P=0.09$ for protons and $P=0.95$ for pions. The mean interaction length for pions is approximately 20% greater than that for protons.

Some measure of the over-all accuracy of the determination of the mean interaction length can be obtained from a comparison of the experimentally determined uncertainties with the theoretical minimum uncertainties predicted from Eq. (13). The fact that the experimental uncertainties are only slightly larger than the theoretical minimum indicates that the assumption of a single exponential decay function is reasonable. Departures of the data from the assumed exponential decay could be caused by variation of the interaction length with energy, unequal efficiencies of the detector modules, and contamination in the beam.

The effect of energy dependence on the determination of the mean interaction length is expected to be very small. The incident particles lose less than 10% of their energy in traversing the entire detector, even for the lowest-energy (9.3-GeV) protons. Since the interaction probability is at worst a very slowly varying function of the energy, the small change in incident energy should not appreciably affect the determination of the mean interaction length.

Contamination of the proton beam by other hadrons was small. Efficiency for rejection of π and K mesons by the gas Čerenkov detector was measured to be better than 98%. Since the flux of pro-

TABLE II. Mean interaction lengths of pions in iron.

Incident energy (GeV)	Number of iron modules	Total number of detected interactions	λ in (g/cm^2) Uncorrected for K^+ contamination	Corrected for K^+ contamination	$\Delta\lambda/\lambda$ (%) Measured	Minimum
9.3	5	1291	166.8 ± 8.5	164.5 ± 8.5	±5.1	±4.4
17.8	9	550	167.4 ± 9.1	163.8 ± 9.0	±5.3	±4.9

tons always represented more than half of the total flux, the contamination in the proton beam was always less than 2%.

There was, however, some K-meson contamination in the π-meson beam. At both 9.3 and 17.8 GeV the ratio of K^+ to π^+ at the production target was 12%. Because of the long flight path (57.35 m) and the lower energy of the K^+, about half of the K^+ mesons decayed before reaching the detector. The ratio of K^+ to π^+ incident upon the detector was 5.1% and 7.8% at 9.3 and 17.8 GeV, respectively. The correction for the K^+ contamination in the beam has been made assuming the ratio of the cross sections $\sigma(K^+, \mathrm{Fe})$ to $\sigma(\pi^+, \mathrm{Fe})$ will be the same as the ratio of the cross sections $\sigma(K^+, \mathrm{D})$ to $\sigma(\pi^+, \mathrm{D})$ of approximately 0.71 given by Denisov *et al.*[15] The correction for the K-meson contamination has been included in the results presented in Table II. The corrected value for the π^+ interaction length in iron at 17.8 GeV of 164 ± 9 g/cm^2 is in excellent agreement with the value of the π interaction length measured at 20 GeV of 162 ± 2 g/cm^2, obtained by interpolating between the values given for Al and Cu by Allaby *et al.*[16] Harris *et al.*[17] have reported a mean interaction length of 130 g/cm^2 for 2.9-GeV $\pi^{\pm}$ in iron. This determination is based on the interaction criterion of scattering of the incident pion, so that the much higher interaction cross section may be due to an effectively lower-threshold criterion.

The effect of detector efficiencies upon the determination of the mean interaction length has been investigated. First, as a check for variations in the efficiencies for detection of an interaction, the total number of interactions detected in each module for all proton runs was plotted as a function of module number. For each module included in the data analysis the sum of the interactions differed by no more than 2% from the expected fit to the data. We thus conclude that all detectors have the same efficiency for the detection of an interaction, to within a few percent. Secondly, the muon equivalent particle calibration for the first useful iron module was arbitrarily changed by 10%, a number several times the assigned uncertainty. The value of the mean interaction length changed by less than 1%, but the quality of the fit became worse.

CONCLUSIONS

The mean interaction lengths for 9.3-, 13.8-, and 17.8-GeV protons and 9.3- and 17.8-GeV positive pions in iron have been determined. The value obtained depends somewhat upon the signature required to define an interaction. Exact comparison with the other determinations of the mean interaction length is not possible because other experiments have slightly different criteria to define an interaction. The results of this experiment and those of previous workers are shown in Table III. Because of the accurately determined incident energy and the large number of events, the precision of the present results is generally greater.

No statistically significant variation of the mean interaction length for protons or pions as a function of energy was observed in this work. However, with only two exceptions, the data presented in Table III show a systematic 5% difference between measurements of the mean-interaction length made with cosmic rays and those made with accelerator-produced protons. A correction for pion contamination in the cosmic-ray flux would lower the value obtained in uncorrected cosmic-ray experiments by about 5%, thereby increasing the discrepancy.

All of the recent accelerator experiments have employed both a similar detector, constructed of alternating layers of iron and scintillator, and similar criteria for defining an interaction. Thus the agreement among these experiments is not unexpected. The analysis of the data in the present work has shown that the value of the mean interaction length is dependent upon the criteria used to define an interaction. However, most of the changes in criteria from the ones finally selected for our analysis tend to raise the value of the mean interaction length. None of the various interaction criteria that were tried gave a value as small as with those reported by the cosmic-ray experiments. The difference may be due to the effectively higher-threshold criteria in the cosmic-ray experiments resulting from less frequent sampling and greater module thickness in their detectors.

The determinations of the interaction length using cosmic rays included only particles with energies above 50 GeV. The observed differences in the mean interaction length could be explained by an energy dependence of the interaction cross section. Such an energy dependence has been sought by Jones *et al.*[9] over the energy range 70–800 GeV, but no statistically significant effect was observed. The accuracy of the investigations was, however, not sufficient to rule out an energy dependence as the cause of the observed discrepancy.

It is always difficult to make comparisons between experimental measurements in which different techniques, that may affect the results, have been employed. It will be interesting to repeat the mean-interaction-length measurements using an iron-scintillator spectrometer with cosmic rays as a source of protons.

TABLE III. Comparison of experimental determinations of mean interaction lengths of protons in iron.

Experimenters	Incident energy (GeV)	Source of protons	λ (g/cm^2)	Comments
Brenner and Williams[a] (1957)	50	Cosmic rays	152 ±7	Integrated all hadrons in cosmic-ray flux.[b]
Ashmore *et al.*[c] (1960)	24.2	Accelerator	143 ±8	Value interpolated from measurements on Al and Cu.
Dobrotin *et al.*[d] (1965)	20–600	Cosmic rays	125 ±16	Integrated all hadrons in cosmic-ray flux.[b]
Bellettini *et al.*[e] (1966)	19.2	Accelerator	119 ±3	Value interpolated from measurements on C, Al, Cu, and Pb.
Andronikashvili *et al*.[f] (1968)	50–1000	Cosmic rays	130 ±6	Integrated all hadrons in cosmic-ray flux.[b]
Schmidt[g] (1968)	28	Accelerator	143 ±3	Minimum statistical error ±3.3 g/cm^2.
W. V. Jones *et al.*[h] (1970)	20.5–28	Accelerator	139^{+5}_{-9}	Combined 20.5- and 28-GeV data.
Bashindjhagyan *et al.*[i] (1971)	400	Cosmic rays	132 ±5	Corrected for pions in cosmic-ray flux.
Kurz *et al.*[j]	18	Accelerator	135.3±3.4	Experimental equipment and data analysis similar to present experiment.
L. W. Jones *et al.*[k] (1972)	70–800	Cosmic rays	132.4±2	Corrected for pions in cosmic-ray flux.
Present experiment	17.8	Accelerator	139.9±2.5	Demands 3 particles in 2 successive modules.

[a]Ref. 1
[b]Correcting for pion contamination in the cosmic-ray flux would lower the value of the mean interaction length by about 5%.
[c]Ref. 2.
[d]Ref. 3.
[e]Ref. 4.
[f]Ref. 5.
[g]Ref. 6.
[h]Ref. 7.
[i]Ref. 8.
[j]Private communication. We are indebted to Dr. Kurz for permission to use these data prior to publication.
[k]Ref. 9.

ACKNOWLEDGMENTS

Thanks go to J. Tanguay and the crew of the AGS for their assistance. Appreciation is expressed for the cooperation of the other experimenters with whom we shared the beam line. In particular we are indebted to R. Lanou and L. Rosenson for the use of the Čerenkov counter, and to S. Siegler for the use of the beam telescope.

R. Cunningham, D. Dellatorre, A. Peterson, J. Reynolds, R. Silverberg, and S. Withers assisted in obtaining the data. Mr. Dellatorre also assisted with much of the data handling, and was responsible for an enormous job of cataloging the results.

We especially want to thank Dr. R. Kurz for very useful information about his AGS calibration, for permission to use his data prior to publication, and for many very helpful discussions.

Finally, a special word of appreciation is in order for the technical staff from Goddard Space Flight Center, including R. Greer, J. Laws, M. Powers, and L. Stonebraker, each of whom put in many long hours ensuring that the experimental apparatus would operate properly.

*Work supported in part by NASA under Grants No. NGR-09-005-007 and NGR-09-050-001.

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