Study of $\pi\pi$ Scattering in the Isotopic-Spin-2 Channel*

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(Received 8 September 1972)

We have carried out a study of $\pi^-\pi^-$ scattering using the reaction $\pi^-d \to pp\pi^-\pi^-$ at an incident π^- momentum of 7 GeV/c. We have extracted the isotopic-spin-2 $\pi\pi$ phase shifts and elastic cross section using a modified Chew-Low extrapolation. We obtain a value of $-7.7° \pm 1.2°$ for the s-wave phase shift at the K^0 mass. Our measurements are in good agreement with current-algebra predictions.

I. INTRODUCTION

A number of different reactions have been analyzed for the purpose of determining the isotopic-spin-2 ($I=2$) $\pi\pi$ cross section and phase shifts.[1-7] The impetus for these studies has arisen from the fact that the determination of the $I=0$ and $I=1$ $\pi\pi$ phase shifts, as well as the parametrization of the 2π decay of the K_S^0-K_L^0 system, relies on our knowledge of $\pi\pi$ scattering in the $I=2$ channel.

Heretofore, $\pi\pi$ scattering in the $I=2$ channel has been analyzed using the reactions

$\pi^-p \to \pi^-\pi^0 p$ (Refs. 2, 3, 4),

$\pi^-p \to \pi^-\pi^-\Delta^{++}$ (Refs. 5, 6),

and

$\pi^+p \to \pi^+\pi^+ n$ (Ref. 7).

All these analyses have been troubled to varying degrees by background problems which can cause non-negligible systematic errors in the determination of cross sections and phase shifts. In this experiment we study the $I=2$ $\pi\pi$ system using the highly kinematically constrained reaction

$$\pi^-d \to p_s p\pi^-\pi^-, \qquad (1)$$

where p_s refers to the spectator proton.

We first attempt to isolate the contribution of the one-pion exchange (OPE) diagram shown in Fig. 1(a), and then proceed to obtain the $\pi^-\pi^-$ elastic cross section by means of a modified Chew-Low[8] extrapolation to the pion pole. The procedure consists of modifying the OPE formula as given by Chew and Low by an arbitrary multiplicative function (to account for off-shell effects), which has the effect of forcing the model into better agreement with the observed experimental distributions. This procedure reduces the complexity of the required extrapolating function. We obtain the inelastic $\pi^-\pi^-$ cross section from the following reactions:

$$\pi^-d \to p_s p\pi^+\pi^-\pi^-\pi^- \qquad (2)$$

and

$$\pi^-d \to p_s p\pi^-\pi^-\pi^0\pi^0. \qquad (3)$$

Using the elastic and inelastic cross sections, along with the distribution of the scattering angle in the $\pi\pi$ center-of-mass system, we compute the $I=2$ phase shifts. These results are then compared with other measurements of $I=2$ $\pi\pi$ scattering parameters. In the closing section of this paper we discuss how the cross-section determination is influenced by the modification of the Chew-Low extrapolation formula. We also examine the question of using extrapolated rather than off-mass-shell moments in the determination of the phase shifts.

II. EXPERIMENTAL PROCEDURE

The data for reaction (1) are from a 7-event/μb exposure of the Brookhaven National Laboratory 80-in. deuterium-filled bubble chamber to a beam of 6.96-GeV/c π^- mesons, and from a 10-event/μb sample of an exposure of the Stanford Linear Accelerator Center 82-in. deuterium-filled chamber to a beam of 6.71-GeV/c π^- mesons. Both exposures were scanned for events of the 3-pronged and 4-pronged topologies for which all positive tracks were heavily ionizing (at least two times minimum). This scan yielded a sample of events enriched in interactions containing two low-momentum protons ($\lesssim 1$ GeV/c) in the final state. A total of 5157 events were obtained belonging to reaction (1) for which the square of the four-momentum transfer from the deuteron to the two outgoing protons was $\lesssim 0.7$ GeV^2. The event sample for reaction (2) was obtained from a similar scan of 5-pronged and 6-pronged topologies, with the requirement that there be at least one or two heavily ionizing tracks for the 5- and 6-pronged events, respectively. Reconstruction and kinematic fitting were carried out using the TVGP-SQUAW programs.[9] For further details concerning the data analysis for this experiment the reader is referred to Ref. 10.

To study $\pi^-\pi^-$ scattering we wish to isolate the contribution of the OPE diagram shown in Fig. 1(a). We define t to be the square of the four-momentum transfer from the incident π^- to the outgoing $\pi^-\pi^-$ system (t is negative in the physical region). We make a peripheral cut and only consider events with $|t|$ less than $11\mu^2$, where μ is the π mass. We also do not consider events with $|t|<3\mu^2$ due to experimental scanning losses in this region. The losses at small t arise from the fact that events with small momentum transfer give rise to recoil protons with very low momentum, which are difficult to see and measure accurately in the bubble chamber.

We have also removed from our sample events with spectator momenta in excess of 0.25 GeV/c; we have chosen the spectator to be the proton with the lower momentum. The fact that about 12% of the events have a spectator momentum greater than 0.25 GeV/c, in contrast to the 3% one would expect assuming the impulse approximation and a Hulthén wave function for deuterium, indicates that most such events involve rescattering of the spectator proton and cannot therefore be considered as interactions occurring on a free neutron.[11] This excess of events with large values of spectator momentum is known to occur generally in deuteron experiments.[12]

In addition to the above, we have also ignored in our analysis events with $\pi\pi$ mass (m) greater than 1.48 GeV. The minimum momentum transfers required to produce such events are large, and consequently t is relatively far from the π pole, thus making extrapolation of data in this mass region unreliable. We also do not expect OPE to be the dominant process involved in the production of such massive $\pi\pi$ systems at our incident energy.[13]

Applying the following selections to data from reaction (1):

$$3\mu^2<|t|<11\mu^2,\quad p_s<0.25\ \text{GeV}/c,\quad m<1.48\ \text{GeV} \tag{4}$$

we obtain a final sample of 1002 events. Figures 2(a) and 2(b) show the momentum-transfer distribution and the $\pi\pi$ mass spectrum, respectively, for this event sample. In Fig. 2(c) we display the distribution in the Treiman-Yang angle (ϕ) for these data. The isotropy observed in the Treiman-Yang angle is consistent with the expected dominance of OPE. The observed isotropy is independent of both m and t. Our data do not appear to show substantial resonance production in the π^-p system [Fig. 2(d)], and consequently we believe the present sample to be relatively free of background.

A question might be raised concerning the effect on our analysis of the presence of a possible contribution from the diffractive dissociation of the neutron, as depicted in Fig. 1(b). [The low-mass enhancement in Fig. 2(d) is presumably due to this process.] As was pointed out by Trilling,[14] the contribution of the diagram shown in Fig. 1(b) forms an essential part of the $\pi\pi$ scattering process we are studying and is contained in the diagram depicted in Fig 1(a). Trilling uses duality arguments to point out that the amplitude for neutron diffraction dissociation is an integral part of the

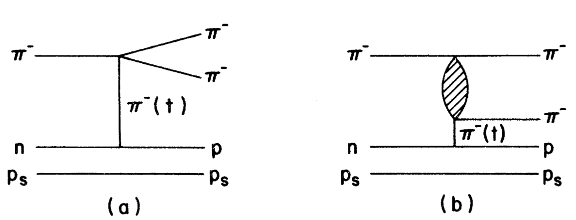

FIG. 1. Processes pertinent to $\pi^-\pi^-$ elastic scattering: (a) the one-pion-exchange process; (b) the diffractive dissociation of the neutron. The connection between these two diagrams is discussed in the text.

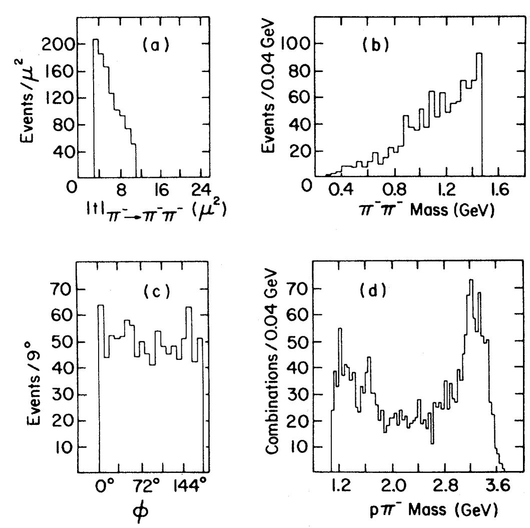

FIG. 2. Experimental distributions for reaction (1) after applying the cuts described in the text: (a) distribution in the square of the momentum transfer from the incident π^- to the outgoing $\pi^-\pi^-$; (b) $\pi^-\pi^-$ mass spectrum; (c) distribution in the Treiman-Yang angle; (d) $p\pi^-$ mass spectrum.

over-all $\pi\pi$ scattering problem, and the events corresponding to this region of kinematics must consequently be retained in any study of $\pi\pi$ scattering.

III. EXTRAPOLATION PROCEDURE

In order to obtain the elastic $\pi\pi$ cross section we must first choose a function for the purpose of extrapolating the measured off-shell cross sections to the pion pole. This function is not unique. An extrapolating function, such as the one originally derived by Chew and Low,[8] can be modified by any smooth function of t and m which reduces to unity at the pion pole. It has been shown, however, that with presently available data samples different forms for the extrapolating function yield quite different cross sections upon extrapolation to the pion pole. In particular, Ma *et al.*[15] have attempted to extract the known π^+p elastic cross section in the region of the Δ^{++} using the reaction $pp \to p\pi^+n$ at an incident beam momentum of 6.6 GeV/c. They found that the conventional Chew-Low extrapolation procedure yielded results which were not in good agreement with the measured π^+p cross section. However, upon modifying the Chew-Low formula by the Dürr-Pilkuhn[16] (DP) vertex correction factors, which relate off-shell to on-shell scattering, they obtained results in excellent agreement with the expected values. In addition, Wolf[17] has shown that, using the Benecke-Dürr[18] (BD) parametrization for the vertex factors (which for $|t|<0.3$ GeV^2 yields the same results as the DP parametrization), one could describe the experimental mass and momentum-transfer distributions for a number of reactions over a wide range of beam momenta.

Because of the above-mentioned successes, we have also chosen to use a modified version of the OPE formula in extracting the $\pi\pi$ scattering parameters. Our decision is based on the fact that by choosing a form which agrees well with data in the physical region we reduce the complexity of the extrapolating function. For the $NN\pi$ vertex we use the DP factor, since the BD parametrization leads to unphysical expressions.[17] For the $\pi^-\pi^-$ scattering vertex we use the BD parametrization as obtained by Wolf. We also use a first-order value for the ratio of s-wave to d-wave scattering cross sections,[19] which is required in the calculation of the BD correction factor.[20] We found that the BD factor for the $\pi^-\pi^-$ vertex affects our calculation by $\lesssim 1\%$, and, consequently, we have chosen to ignore this correction term. The OPE formula we use is thus given by

$$\frac{d^2\sigma}{dtdm}=\frac{f^2}{\pi\mu^2p^2}\left[1-\tfrac{1}{2}H(\sqrt{|t|})\right]\times m^2(\tfrac{1}{4}m^2-\mu^2)^{1/2}\sigma_{el}(m)\frac{|t|}{(|t|+\mu^2)^2}F_{NN\pi}{}^2(t)\,, \tag{5}$$

where, as defined previously, t is the square of the momentum transfer from the incident π^- to the outgoing $\pi^-\pi^-$ system, m is the mass of the outgoing $\pi^-\pi^-$ system, and μ is the mass of the π^-; f^2 is the πNN coupling constant (we assume that $f^2=0.162$; i.e., it is independent of the presence of the spectator proton), p is the beam momentum in the laboratory system, and $\sigma_{el}(m)$ is the on-mass-shell elastic $\pi^-\pi^-$ cross section. $F_{NN\pi}(t)$ is the Dürr-Pilkuhn factor for the $NN\pi$ vertex given by

$$F_{NN\pi}{}^2(t)=\frac{1+R_N{}^2q_N{}^2}{1+R_N{}^2q_{N_t}{}^2}\,, \tag{6}$$

where q_{N_t} is the momentum of the neutron evaluated in the center-of-mass frame of the scattering between the virtual pion of mass $\sqrt{t}$ and the "free" neutron, and q_N is q_{N_t} evaluated for an on-shell pion. For the parameter R_N we use Wolf's value of 2.66 GeV^{-1}.

The term $1-\frac{1}{2}H(\sqrt{|t|})$ in Eq. (5) is a correction for losses due to the Pauli exclusion principle. Since the two protons in the final state are identical fermions, states which are symmetric under the interchange of the two protons are forbidden to occur. This effect leads to a reduction in the differential cross section at small t which depends on the amount of spin flip present at the πNN vertex. When there is no spin flip the reduction is $1-H(\sqrt{|t|})$, while for total spin flip the correction is $1-\frac{1}{3}H(\sqrt{|t|})$, where $H(\sqrt{|t|})$ is the deuteron form factor.[21] We have used $1-\frac{1}{2}H(\sqrt{|t|})$ to correct for losses due to the Pauli principle. Over the range of t values we have considered, our results are relatively insensitive to the amount of spin flip assumed.

Although the predictions of DP-OPE have been shown to be in good agreement with existing experimental data, we expect Eq. (5) to hold precisely only in the limit as we approach the pion pole. Thus, to extract the $\pi^-\pi^-$ elastic cross section from our data, we extrapolate Eq. (5) to the pion pole:

$$\sigma_{el}(m)=\lim_{|t|\to-\mu^2}\frac{\pi\mu^2p^2}{f^2}\frac{1}{1-\frac{1}{2}H(\sqrt{|t|})}\frac{1}{m^2(\frac{1}{4}m^2-\mu^2)^{1/2}}\times\frac{(|t|+\mu^2)^2}{|t|}\frac{1}{F_{NN\pi}{}^2(t)}\left(\frac{d^2\sigma}{dtdm}\right)_{\rm exp}\,, \tag{7}$$

where $(d^2\sigma/dtdm)_{\rm exp}$ is now the experimentally mea-

TABLE I. Results of fits of the "σ" data points obtained using DP-OPE to the form $a+bt$.

$\pi^-\pi^-$ mass (GeV)	a (mb)	b (mb/μ^2)	Probability (%)	σ_{el} (mb)
0.28–0.48	2.5± 5.6	0.94± 0.98	48	1.6± 6.5
0.48–0.68	3.8± 3.7	0.93± 0.62	88	2.9± 4.3
0.68–0.88	5.5± 1.7	−0.01± 0.26	24	5.5± 2.0
0.88–1.08	8.0± 1.5	−0.12± 0.22	76	8.1± 1.7
1.08–1.28	5.1± 1.0	0.06± 0.15	78	5.1± 1.1
1.28–1.48	5.5± 0.7	−0.19± 0.10	49	5.6± 0.8

sured differential cross section for reaction (1).

From Fig. 2(b) we see that the $\pi^-\pi^-$ mass spectrum is relatively smooth, with no apparent resonancelike structure. We therefore expect σ_{el} to be a slowly varying function of the $\pi\pi$ mass, and we have consequently divided the data into six mass intervals, each having a width of 0.2 GeV, starting at the two-pion threshold of 0.28 GeV. We compute the average value of σ_{el} for each of these mass intervals. We do this by first dividing the data in each mass interval into eight t regions, each having a width of μ^2 (from $3\mu^2$ to $11\mu^2$); we then compute the average of the right-hand side of Eq. (7), a quantity which we denote by "σ", for each of these t regions.[22] The results of this computation are shown in Fig. 3.

Following the above computation we perform a least-squares fit to the data for each mass interval. We fit the "σ" points to a function of the form $a+bt$, and then extrapolate this function and evaluate it at the pion pole to obtain the elastic $\pi^-\pi^-$ cross section. By allowing the extrapolating function to depend on t we allow for deviations of the data from the DP-OPE model. (An exact description of the data by the model would make a t-dependent term superfluous.) The fits are superimposed on the data of Fig. 3. Table I contains the confidence levels of the fits and values of the parameters a and b along with the extrapolated $\pi^-\pi^-$ elastic cross section. All of the fits can be seen to yield very acceptable confidence levels. (We also tried quadratic fits to the data of the form "σ" $=a+bt+ct^2$. Such fits had little effect on the confidence levels, but yielded wildly fluctuating cross sections with extremely large errors.)

In Fig. 4(a) we plot the $\pi^-\pi^-$ elastic cross section obtained from the above extrapolation. The curve is a second-order polynomial fit to our results, drawn so as to smooth out experimental fluctuations. It is constrained to go through zero at the 2-pion threshold of 0.28 GeV. Figure 4(b) contains the results of previous measurements of the $I=2$ $\pi\pi$ elastic cross section. Our curve is superimposed on it for purposes of comparison.

As can be seen from Table I, the value of the parameter b in the fit to "σ" $=a+bt$ is within one standard deviation of zero for most of the $\pi\pi$ mass intervals considered. (It was within two standard deviations of zero for the entire mass range.) This indicates that DP-OPE describes the t depen-

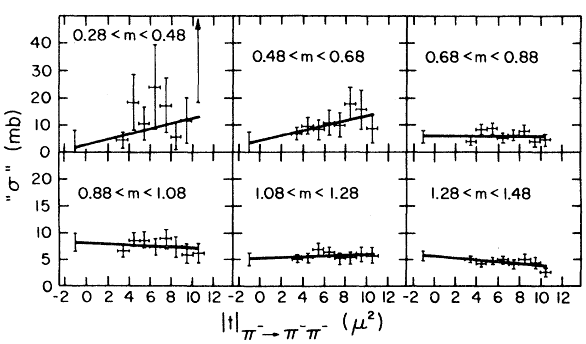

FIG. 3. Experimental "σ" values for different ranges of $\pi\pi$ mass. For the definition of "σ" refer to the text. Superimposed are the results of fitting these points to a function of the form $a+bt$.

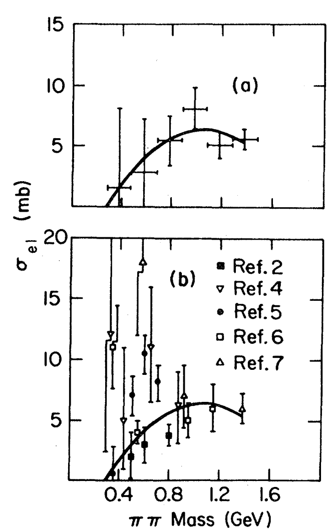

FIG. 4. Isotopic-spin-2 $\pi\pi$ elastic cross section: (a) the cross section obtained from our extrapolation with a second-order polynomial fit drawn on it; (b) previously measured values with the same curve drawn on them.

dence of our data extremely well. In order to better illustrate the agreement between our data and the DP-OPE model we display in Fig. 5(a) the $d\sigma/dt$ distribution; the prediction of the model is also shown on the figure (solid curve). This comparison yields a confidence level of 40%, indicating excellent agreement. We also show in Fig. 5(b) the $d\sigma/dm$ distribution along with the prediction of the model. The model appears to give an adequate description of the distribution, although discrepancies between the model and our data are noticeable at low and high values of $\pi\pi$ mass.

IV. PHASE-SHIFT DETERMINATION

From our results for the $I=2$ $\pi\pi$ elastic cross section we have proceeded to compute the $I=2$ phase shifts using a partial-wave expansion for the differential cross section. The relevant formulas for the scattering of identical spinless particles are[23]

$$\frac{d\sigma_{el}}{d\Omega}=\frac{1}{k^2}\left|\sum_{l\ \text{even}}(2l+1)P_l(\cos\theta)(\eta_l e^{2i\delta_l}-1)\right|^2,$$

$$\sigma_{el}=\frac{2\pi}{k^2}\sum_{l\ \text{even}}(2l+1)(1-2\eta_l\cos 2\delta_l+\eta_l^2), \qquad (8)$$

$$\sigma_{inel}=\frac{2\pi}{k^2}\sum_{l\ \text{even}}(2l+1)(1-\eta_l^2),$$

where σ_{el} and σ_{inel} refer to the elastic and inelastic

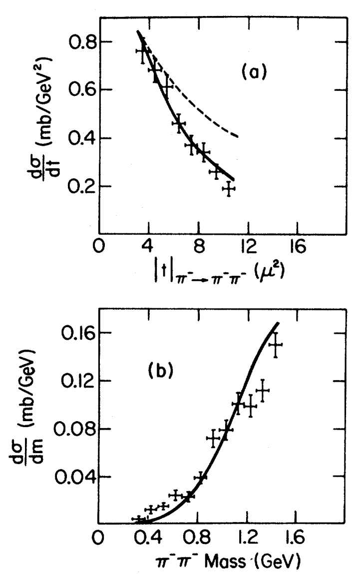

FIG. 5. Distributions for reaction (1) after applying the cuts described in the text: (a) $d\sigma/dt$ (the smooth curve is the prediction of the DP-OPE model, while the dashed curve is the prediction of pure OPE; the dashed curve was normalized to the first point); (b) $d\sigma/dm$ (the smooth curve is the prediction of DP-OPE).

$\pi\pi$ cross sections, respectively; P_l is the lth-order Legendre polynomial; η_l is the lth-wave inelasticity; δ_l is the lth-wave phase shift; k is the momentum in the $\pi\pi$ center-of-mass frame; and θ is the scattering angle in the $\pi\pi$ center-of-mass frame.

To obtain σ_{inel} we investigated reactions (2) and (3) using an analysis similar to the one previously described for reaction (1). We obtain from data belonging to reactions (2) and (3) the cross sections for the reactions

$$\pi^-\pi^-\to\pi^+\pi^-\pi^-\pi^-$$

and (9)

$$\pi^-\pi^-\to\pi^-\pi^-\pi^0\pi^0.$$

The relevant OPE diagrams are shown in Fig. 6. We assume that the sum of the cross sections for reactions (9) make up the total $\pi^-\pi^-$ inelastic cross section (see below).

Reaction (3) is kinematically under-constrained in the bubble chamber. We assigned to this reaction all events that have three or four visible outgoing prongs, and have a missing mass of more than 0.3 GeV recoiling from the charged tracks. (We did not include events that are consistent with the one-constraint missing π^0 hypothesis.) In addition, we considered only those events from reactions (2) and (3) that satisfy conditions (4), where now m refers to the four-pion mass and t to the square of the four-momentum transfer from the incident π^- to the outgoing four-pion system. Due to insufficient statistics for these reactions we did not perform an extrapolation to the pion pole to determine σ_{inel}, but rather assumed that the extrapolating function would be the same as in the determination of σ_{el}. We therefore equated σ_{inel} for a particular mass interval to the product of σ_{el} and the ratio of the number of events in the elastic channel to the number of events in the inelastic channel, each computed for the same mass interval. It should be pointed out that we found no events consistent with the inelastic production of six pions in our chosen m and t range. Our values for σ_{inel} are shown in Fig. 7.

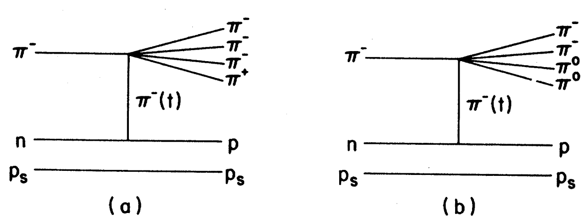

FIG. 6. One-pion-exchange diagrams pertinent to $\pi^-\pi^-$ inelastic interactions: (a) refers to reaction (2), and (b) to reaction (3).

To obtain the $\pi\pi$ phase shifts we examined for each mass interval the distribution in the $\pi\pi$ center-of-mass scattering angle (θ). Any function of θ can be represented in terms of an expansion of Legendre polynomials:

$$W(\cos\theta)=\tfrac{1}{2}\left[1+\sum_{l=1}^{\infty}a_l P_l(\cos\theta)\right],$$

where the Legendre polynomial moments a_l are given by

$$a_l=(2l+1)\int_{-1}^{1}W(\cos\theta)P_l(\cos\theta)\,d(\cos\theta)\,.$$

The experimental differential cross section can be written in a similar fashion:

$$\left(\frac{d\sigma_{\rm el}}{d\Omega}\right)_{\rm exp}=\frac{\sigma_{\rm el}}{4\pi}\left[1+\sum_{l=1}^{\infty}a_l P_l(\cos\theta)\right], \qquad (10)$$

where the a_l's are the calculated moments of the Legendre polynomials; the values of these moments, up to $l=8$, are given in Fig. 7.[24] Using Eqs. (8) and (10), along with our values for the elastic and inelastic cross sections[25] and the analytic constraint that the inelasticities lie between 0 and 1, we have performed a fit to the data to obtain the phase shifts and inelasticities. For $\pi\pi$ masses below 1.08 GeV the moments for $l>4$ are consistent with zero. We have consequently constrained our fit for $m<1.08$ GeV to contain only s-wave and d-wave scattering terms. For $\pi\pi$ masses greater than 1.08 GeV we note the presence of a significant sixth-order moment and therefore allow the presence of g waves. The results of our analysis are shown in Fig. 8.

In our calculation we are only sensitive to the relative sign of the phase shifts. The sign of the s-wave phase shift was previously determined to be negative using data pertinent to $\pi^-\pi^0$ scattering.[2-4] The s-wave phase shift can be seen to start out close to 0° near threshold and fall to about −30° at 1.4 GeV. The d-wave phase shift is considerably smaller in magnitude throughout, being about −5° at 1400 MeV, while the g-wave phase shift near 1400 MeV is about −2°. We found the d and g waves to be totally elastic throughout our mass range ($\eta_2=\eta_4=1$), while the s wave be-

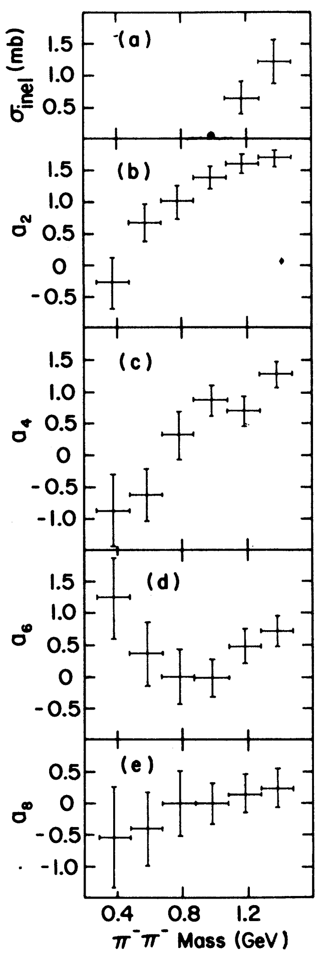

FIG. 7. (a) The inelastic $\pi^-\pi^-$ cross section as a function of $\pi^-\pi^-$ mass; (b)–(e) the second-, fourth-, sixth-, and eighth-order Legendre polynomial moments, respectively, as functions of $\pi^-\pi^-$ mass.

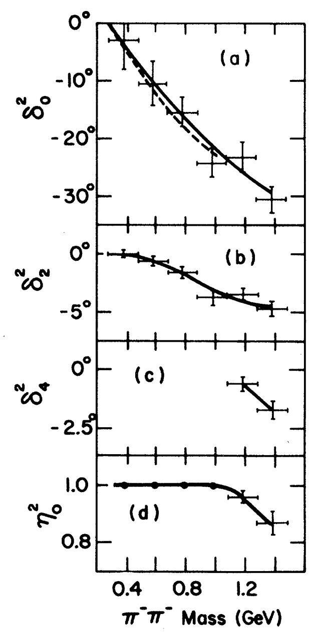

FIG. 8. (a)–(c) The s-wave, d-wave, and g-wave phase shifts, respectively, as functions of $\pi^-\pi^-$ mass; (d) the s-wave inelasticity. The d-wave and g-wave inelasticities are 1. The solid curves are results of polynomial fits to the measured values. The dashed curve in (a) is a current-algebra prediction of Arnowitt (Ref. 26).

came inelastic at about 1.1 GeV.

The smooth curves shown in Fig. 8 are results of polynomial fits to the measured parameters as a function of m. The fits to the phase shifts were constrained to go through 0° at threshold. Our rationale for such fitting of the data is based on our desire to obtain analytical representations of the phase shifts so as to allow us to interpolate between measured points. From these fits we determine our value of the s-wave phase shift at the K^0 mass to be $-7.7° \pm 1.2°$. The dashed curve drawn on the s-wave phase-shift distribution in Fig. 8 is a current-algebra prediction of Arnowitt[26]; it is in excellent agreement with our data.

In addition to the above polynomial fit we have also performed a fit to the s-wave phase shifts using an expression involving the effective-range formalism:

$$k\cot\delta_0(k) = \frac{1}{a_0} + \tfrac{1}{2} r_0 k^2 .$$

The results of this fit were compatible with our polynomial fit in m. We have obtained a value of -0.15 ± 0.03 F for the s-wave scattering length (a_0), and a value of 0.13 ± 0.26 F for the s-wave effective range (r_0). Because of the paucity of events at low m values, we do not expect our data to provide very reliable measurements of these threshold parameters.

V. DISCUSSION

In determining our values for the $I=2$ $\pi\pi$ phase shifts we have made use of the off-mass-shell Legendre polynomial moments of the $\pi\pi$ scattering angular distribution. We have also determined the phase shifts using the on-shell moments, obtained through a linear extrapolation of the off-shell moments to the pion pole. Because of poor statistics, the errors in the extrapolated moments were rather large, but, for virtually all of the moments, the extrapolation yielded results which were consistent with the off-shell moments. The phase shifts obtained using the extrapolated on-shell moments showed no systematic differences from the results presented earlier, and were found to be within one standard deviation of the presented results.

In order to ascertain the degree of dependence of our $\pi\pi$ cross section on the method we used for the extrapolation, we have repeated the procedure described in Sec. III, but have not included the DP correction factor for the πNN vertex. Our extrapolation equation is now given by Eq. (7) with $F_{NN\pi}^2(t)$ set equal to unity. Table II contains the confidence levels for these fits, along with the parameters a and b described previously, and the $\pi^-\pi^-$ elastic cross section. This table is to be compared with Table I. We see that the confidence levels for the two methods are roughly the same, but that the extrapolated cross sections are systematically lower in Table II. Also, the parameter b required by the fit in the latter case deviates from zero considerably more than it did for the case where we included the DP factor. This indicates that OPE without the DP factor does not describe the t dependence of our data very well. To better illustrate this we have drawn on the $d\sigma/dt$ distribution of Fig. 5(a) the predictions of OPE without the DP factor (the dashed curve). The agreement is very poor. The shape of the $d\sigma/dm$ distribution was found to be relatively insensitive to the presence of the form factor. Also, whereas the normalization using DP-OPE was in agreement with the data in the physical region, the normalization of the pure OPE was not. It is clear that DP-OPE is in much better agreement with the data than OPE without the DP factor, and that by using it we have reduced the complexity of the extrapolating function, thereby obtaining a more reliable cross-section measurement.[27]

ACKNOWLEDGMENTS

We thank Dr. W. Katz for his help in the early stages of this experiment. We also thank Dr. R. Arnowitt, Dr. E. Colton, Dr. M. Ross, and Dr. P. K. Williams for helpful suggestions.

TABLE II. Results of fits of the "σ" data points obtained using pure OPE to the form $a + bt$.

$\pi^-\pi^-$ mass (GeV)	a (mb)	b (mb/μ^2)	Probability (%)	σ_{el} (mb)
0.28–0.48	3.3 ± 3.0	0.08 ± 0.45	49	3.2 ± 3.4
0.48–0.68	4.0 ± 1.8	0.05 ± 0.26	87	4.0 ± 2.0
0.68–0.88	4.0 ± 0.9	-0.22 ± 0.11	33	4.2 ± 1.0
0.88–1.08	5.4 ± 0.7	-0.31 ± 0.10	89	5.7 ± 0.8
1.08–1.28	3.7 ± 0.5	-0.17 ± 0.07	83	3.8 ± 0.5
1.28–1.48	3.5 ± 0.3	-0.23 ± 0.04	65	3.7 ± 0.4

*Research sponsored by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-3065. Computer analysis supported through funds provided by the University of Rochester.

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[1]For a review of the experimental situation at that time see *Proceedings of the Conference on ππ and Kπ Interactions, Argonne National Laboratory, 1969,* edited by F. Loeffler and E. D. Malamud (Argonne National Laboratory, Argonne, Ill., 1969).

[2]J. P. Baton, G. Laurens, and J. Reignier, Nucl. Phys. B3, 349 (1967).

[3]J. P. Baton, G. Laurens, and J. Reignier, Phys. Letters 33B, 528 (1970).

[4]W. D. Walker *et al.*, Phys. Rev. Letters 18, 630 (1967).

[5]E. Colton *et al.*, Phys. Rev. D 3, 2028 (1971).

[6]Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento 35, 1 (1965).

[7]Aachen-Berlin-Birmingham-Bonn-Hamburg-London-München Collaboration, Phys. Rev. 138, B897 (1965).

[8]G. F. Chew and F. E. Low, Phys. Rev. 113, 1610 (1959).

[9]T. B. Day, University of Maryland Technical Report No. 649 (unpublished).

[10]W. Katz, Ph.D. thesis, University of Rochester (unpublished).

[11]After the peripheral t cut, the spectator cut resulted in a further removal of ~12% of the events; consequently, this cut should not have substantially influenced our results.

[12]B. Musgrave, in *Phenomenology in Particle Physics, 1971,* edited by C. Chiu, G. Fox, and A. J. G. Hey (Caltech, Pasadena, Calif., 1971), p. 467.

[13]An attempt to extrapolate to the pion pole for $\pi\pi$ masses much greater than 1.48 GeV resulted in unphysical $\pi\pi$ elastic cross sections.

[14]G. H. Trilling, in *Particles and Fields–1971,* proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Melissinos and P. F. Slattery (A.I.P., New York, 1971), p. 1.

[15]Z. Ming Ma, G. A. Smith, and R. J. Sprafka, Phys. Rev. Letters 23, 342 (1969).

[16]H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40, 899 (1965).

[17]G. Wolf, Phys. Rev. 182, 1538 (1969); Phys. Rev. Letters 19, 925 (1967).

[18]J. Benecke and H. P. Dürr, Nuovo Cimento 56, 269 (1968).

[19]W. Katz *et al.*, in *Proceedings of the Conference on ππ and Kπ Interactions, Argonne National Laboratory, 1969,* edited by F. Loeffler and E. D. Malamud (Ref. 1), p. 300. It is assumed for the purposes of this estimation that all higher-order waves are negligible.

[20]For the method of handling vertices with several partial waves see Ref. 17.

[21]G. Chew and H. Lewis, Phys. Rev. 84, 779 (1951).

$$H(q)=\frac{2\alpha\beta(\alpha+\beta)}{(\alpha-\beta)^2}\times\frac{1}{q}\left[\tan^{-1}\left(\frac{q}{2\alpha}\right)+\tan^{-1}\left(\frac{q}{2\beta}\right)-2\tan^{-1}\left(\frac{q}{\alpha+\beta}\right)\right],$$

where $q=\sqrt{|t|}$ and we have used $\alpha=45.7$ MeV, $\beta=6.2\alpha$. See also Ref. 12.

[22]We use the expression

$$``\sigma"(m,t)=\frac{\pi\mu^2}{f^2}\frac{N}{\Delta m\Delta t}\times\sum_j\frac{p_j^2}{[1-\frac{1}{2}H(\sqrt{|t_j|})]}\frac{1}{m_j^2(\frac{1}{4}m_j^2-\mu^2)^{1/2}}\times\frac{(|t_j|+\mu^2)^2}{|t_j|}\frac{1}{F_{NN\pi}^2(t_j)}$$

to evaluate "σ" for a given $\Delta m\Delta t$ interval. N is the μb equivalent for our event sample ($N=0.075$ μb/event); the sum is carried out over all events in the given interval.

[23]See for example L. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1949).

[24]For our experimental data we have

$$a_l=\frac{(2l+1)}{N}\sum_{i=1}^{N}P_l(\cos\theta_i),$$

where the sum is over the N events in a given mass range. We have folded the angular distributions about 90° as required by the indistinguishability of the two outgoing π^- mesons.

[25]As input to the fitting program we use $k^2\sigma_{el}$ and $k^2\times\sigma_{inel}$. Since k^2 depends on m we extrapolated k^2"σ" instead of simply "σ" in calculating the phase shifts.

[26]R. Arnowitt, in *Proceedings of the Conference on ππ and Kπ Interactions, Argonne National Laboratory, 1969,* edited by F. Loeffler and E. D. Malamud (Ref. 1), p. 619.

[27]Further refinements have been suggested for the extraction of the $\pi\pi$ scattering parameters. See, for example, G. L. Kane and M. Ross, Phys. Rev. 177, 2353 (1969); P. K. Williams, Phys. Rev. D 1, 1312 (1970); J. D. Kimel and E. Reya, Florida State University Report No. 72-4-3 (unpublished). We do not believe these ideas need be pursued in the $I=2$ channel at our level of statistical accuracy.