

Angular Correlation Between Pions in Six-Prong  $K^+p$  Interactions at 12.7 GeV/c

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The angular correlations between pions have been observed in the six-prong interactions of 12.7-GeV/c  $K^+$  with protons. These effects have been compared with the other pion data produced with different primary particles at different energies. The influence of the production of  $N^*$  and  $K^*$  resonances on these angular correlations is investigated.

Goldhaber *et al.*<sup>1</sup> observed a significant difference in the distribution of the c.m. system angle between the pairs of pions of the like and unlike charges in  $p\bar{p}$  annihilation at 1.05 GeV/c. Since that time similar differences have been observed at various momenta of the incoming  $\bar{p}$  and for different pion multiplicities<sup>2-4</sup> in the final state. Similar effects have been observed in  $\pi^+p$  interactions at 4 GeV/c,<sup>5</sup> 5 GeV/c,<sup>6</sup> and 8 GeV/c;<sup>7</sup>  $\pi^-p$  interactions at 11 GeV/c,<sup>8</sup> and 16 GeV/c;<sup>9</sup> in  $pp$  interactions<sup>10</sup> at 10 GeV/c; and in  $K^+p$  interactions<sup>11</sup> at 5 GeV/c. Goldhaber *et al.*<sup>1</sup> observed that the average angle for pairs of unlike pions is greater than that for pairs of like pions and they explained this (GGLP effect) as due to the effect of the symmetrization of the pion wave function required by Bose-Einstein statistics. They proposed to investigate the energy dependence of the effect and later it was found<sup>12</sup> to be in strong disagreement with the predictions of Goldhaber *et al.*<sup>1</sup>

The observed effect depends rather strongly on the details of the interaction mechanism. In the calculation of Goldhaber *et al.*,<sup>1</sup> resonance produc-

tion was not taken into account and it was observed<sup>12</sup> that in low-multiplicity events the  $\rho$  meson strongly affects the observed difference of  $\gamma^{+-}$  and  $\gamma^{\pm\pm}$  in  $p\bar{p}$  annihilation where  $\gamma$  is equal to the ratio of the number of pairs with opening angle greater than  $90^\circ$  to that with opening angle smaller than  $90^\circ$  in the center-of-mass system. For pairs of pions with like and unlike charges we write  $\gamma^{++}$ ,  $\gamma^{--}$ ,  $\gamma^{+-}$ , respectively. It has been found that (i)  $\gamma^{++} > \gamma^{--}$  for primary  $\pi^+$  and (ii)  $\gamma^{--} > \gamma^{++}$  for primary  $\pi^-$ . The effect has been attributed possibly to the "leading pion" which plays an important role when the incident energy increases.

To the best of our knowledge, the  $\pi\pi$  angular correlation has not been observed with primary  $K^+$  mesons in its interaction with protons at energies greater than 8 GeV/c. We present here evidence for such a correlation in  $K^+p$  interactions at 12.7 GeV/c with six prongs, i.e.,  $K^+p \rightarrow K^+p2\pi^+2\pi^-$ . All events used here made a 4C (4-constraint) fit. Those events in which more than one hypothesis made a fit were identified either on the basis of ionization measurements (whenever it

TABLE I. Values of the  $\gamma$  parameter for different pairs of particles in the reaction  $K^+p \rightarrow K^+p2\pi^+2\pi^-$ .

| Frame of reference         | Target-projectile c.m. system |                             |                            |   |
|----------------------------|-------------------------------|-----------------------------|----------------------------|---|
|                            | Resonances not removed        | Resonance $N^{*++}$ removed | Resonance $K^{*0}$ removed | Resonances $N^{*++}$ and $K^{*0}$ removed |
| $\gamma = N_A/N_B$         |                               |                             |                            |   |
| $\gamma(\pi^+\pi^+)$       | $1.95 \pm 0.42$               | $1.62 \pm 0.48$             | $1.72 \pm 0.42$            | $1.19 \pm 0.41$                           |
| $\gamma(\pi^-\pi^-)$       | $1.23 \pm 0.25$               | $1.02 \pm 0.29$             | $1.23 \pm 0.29$            | $1.06 \pm 0.36$                           |
| $\gamma^L(\pi^\pm\pi^\pm)$ | $1.54 \pm 0.23$               | $1.28 \pm 0.20$             | $1.45 \pm 0.25$            | $1.12 \pm 0.27$                           |
| $\gamma^U(\pi^+\pi^-)$     | $1.22 \pm 0.12$               | $1.10 \pm 0.16$             | $1.19 \pm 0.14$            | $1.0 \pm 0.17$                            |
| $\gamma(K^+\pi^-)$         | $1.19 \pm 0.14$               | $0.99 \pm 0.23$             | $1.09 \pm 0.36$            | $1.02 \pm 0.22$                           |
| $\gamma(K^+\pi^+)$         | $1.47 \pm 0.22$               | $1.20 \pm 0.28$             | $1.43 \pm 0.33$            | $1.60 \pm 0.39$                           |
| $\gamma(p\pi^+)$           | $1.17 \pm 0.17$               | $3.53 \pm 0.99$             | $1.19 \pm 0.27$            | $2.09 \pm 0.53$                           |
| $\gamma(p\pi^-)$           | $1.92 \pm 0.29$               | $1.95 \pm 0.51$             | $1.51 \pm 0.35$            | $2.23 \pm 0.57$                           |

was possible) or on the basis of the hypothesis which gave the lowest  $\chi^2$  fit values (generally  $\chi^2 \leq 15$ ). There were approximately 8% misidentified events due to the difficulty in distinguishing between  $K^+$  and  $\pi^+$  tracks. The  $K^+p$  reaction is particularly suitable for a study of produced pions and the distribution is free from the "leading-par-

ticle" effects. In this paper, we present evidence for such a correlation, with six-particle production from 12.7-GeV/c  $K^+p$  interactions in the 80-in. BNL hydrogen bubble chamber with 80 000 pictures.<sup>13</sup>

In Figs. 1(a)–1(c) are shown the distributions of the c.m. opening angle between the momenta of like ( $\pi^+\pi^+$ ,  $\pi^-\pi^-$ ) and unlike ( $\pi^+\pi^-$ ) pions, respectively, in the reaction  $K^+p \rightarrow K^+p2\pi^+2\pi^-$ . The theoretical curves are due to the predictions of the multi-Regge model by Chan *et al.*<sup>14</sup> (C/A model—solid line). The values of the parameter have been calculated for the pairs of pions with like charges ( $\pi^+\pi^+$ ,  $\pi^-\pi^-$ ), i.e.,  $\gamma^L$  is defined as

$$\gamma^L = N_A^L / N_B^L$$

and for pairs of pions with unlike charges ( $\pi^+\pi^-$ ), i.e.,

$$\gamma^U = N_A^U / N_B^U,$$

where  $N_A$  and  $N_B$  are the number of pairs with an opening angle  $\theta_{\pi\pi}$  greater than  $90^\circ$  and less than

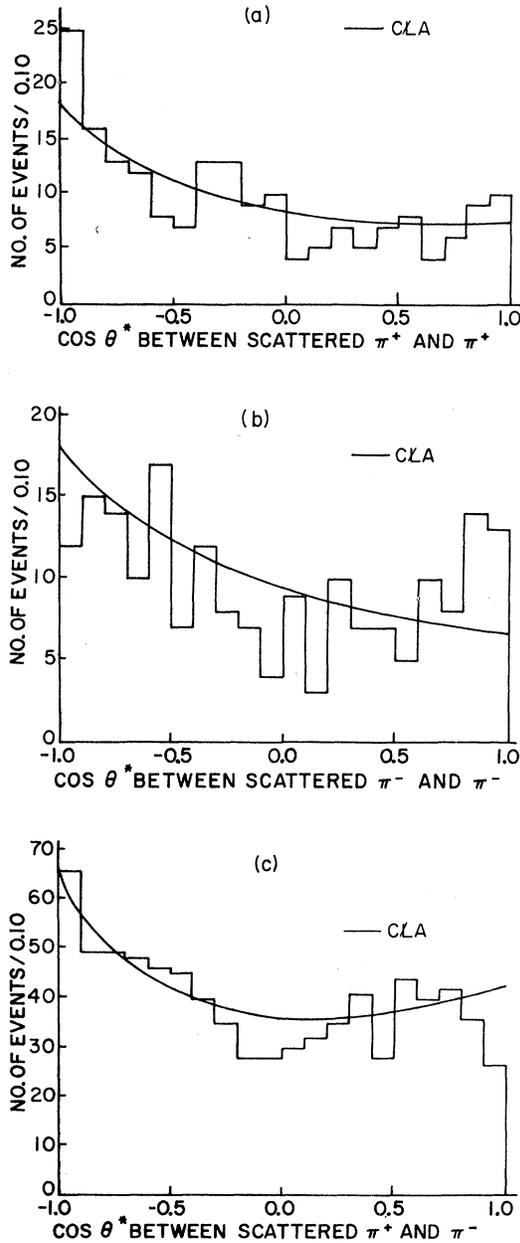


FIG. 1. Angular distribution in the c.m. system between (a) two positive pions, (b) two negative pions, and (c) positive and negative pions. The solid curves are the predictions of the C/A model.

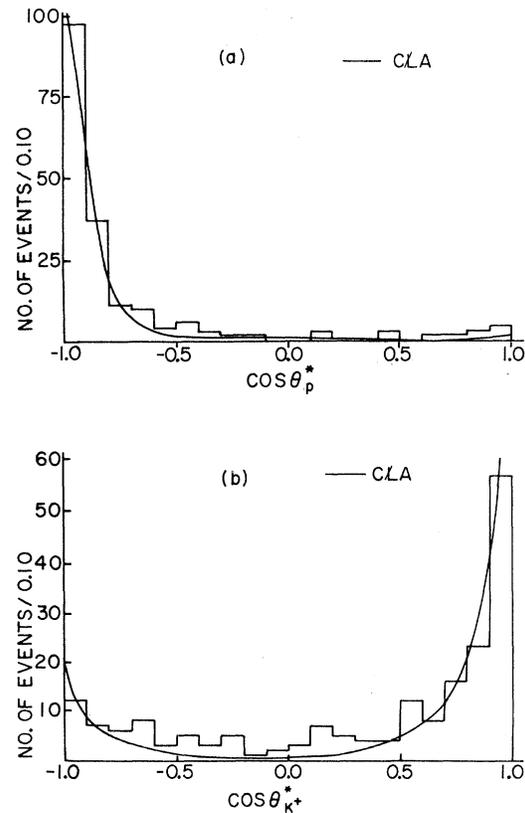


FIG. 2. Distribution of the production angle in the c.m. system for the (a) outgoing proton, and (b) outgoing kaon. The solid curves are the predictions of the C/A model.

$90^\circ$ , respectively. The  $\gamma$  values are given in Table I. The distribution for like positive pions ( $\pi^+\pi^+$ ) tends to be more peaked in the backward hemisphere. This feature is different from what has been observed in other multi-pion final states in antiproton-proton annihilations and in large multiplicities of pion-nucleon interactions.

In order to investigate the possible source of the effect, we study the peripheral production which is strongly indicated by the angular distributions of the outgoing proton and kaon. In Figs. 2(a) and 2(b) are shown the angular distributions of the proton and the kaon in the c.m. system. The distributions are strongly peaked forward and backward for kaons and protons, respectively, at small scattering angles. The proton and kaon exhibit the well-known leading-particle feature of high-multiplicity events. The multiperipheral Reggeized model (CLA) qualitatively reproduces the angular distribution of the proton and  $K^+$ . The angular distributions for the pions exhibit both forward and backward peaking in the c.m. system which is shown in Figs. 3(a) and 3(b) for  $\pi^-$  and  $\pi^+$ , respectively. A major cause for this

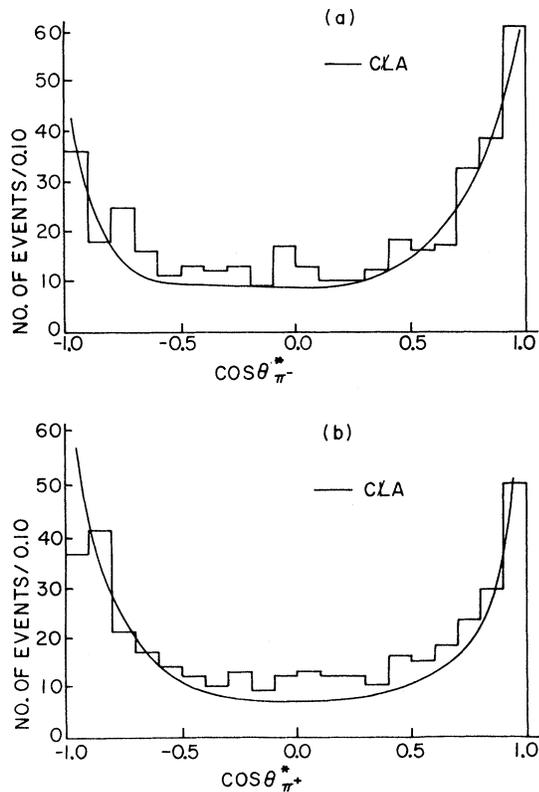


FIG. 3. The angular distribution of (a)  $\pi^-$  and (b)  $\pi^+$  in the c.m. system.

peaking is the abundant resonance production of  $N^{*++}$  and  $K^{*0}$  which<sup>15</sup> are peripherally produced and yield decay products which result in strong forward and backward peaks. They may thus influence considerably the distribution of the angles between the pion momenta. This we can see in the angular correlation between kaons and pions, between protons and pions, and between pions and pions with and without the presence of the resonance as shown in Table I. We can see in this table that the  $\gamma$  values for  $K^+\pi^-$  and  $p\pi^+$  are affected when  $K^{*0}$  and  $N^{*++}$  are removed, respectively. However, the values of  $\gamma$  for  $K^+\pi^+$  and  $p\pi^-$  remain the same when  $K^{*0}$  and  $N^{*++}$  are removed, respectively. The mass cuts used for resonances  $N^{*++}$  and  $K^{*0}$  were  $1.12 \leq M(p\pi^+) \leq 1.36$  GeV and  $0.84 \leq M(K^+\pi^-) \leq 0.94$  GeV, respectively. In the present reaction, since the percentage of  $N^{*++}$  production is larger than the  $K^{*0}$ , this effect can

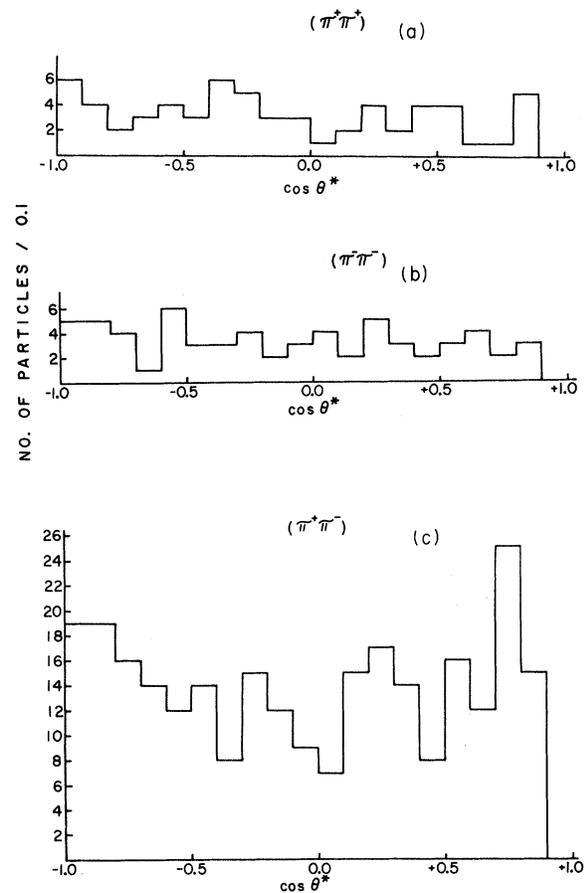


FIG. 4. Angular distribution in the c.m. system between two pions after the removal of resonances: (a) two positive pions, (b) two negative pions, and (c) positive and negative pions.

TABLE II. The values of the parameter  $A$  for different particles at different primary energies.

| Lab momentum<br>(GeV/c) | Reaction  | $A^{++}$         | $A^{--}$        | $A^{+-}=A^U$     | $A^L$           | $A^U - A^L$      | Reference |
|-------------------------|---|------------------|-----------------|------------------|-----------------|------------------|-----------|
| 4                       | $\pi^+p$  | $0.008 \pm 0.04$ | $-0.11 \pm 0.7$ | $0.23 \pm 0.03$  | $0.04 \pm 0.04$ | $0.19 \pm 0.05$  | 5         |
| 5                       | $\pi^+p$  | $0.12 \pm 0.02$  | $0.00 \pm 0.03$ | $0.26 \pm 0.01$  | $0.09 \pm 0.02$ | $0.17 \pm 0.02$  | 6         |
| 8                       | $\pi^+p$  |                  |                 | $0.18 \pm 0.03$  | $0.05 \pm 0.04$ | $0.13 \pm 0.05$  | 7         |
| 11                      | $\pi^-p$  | $0.03 \pm 0.09$  | $0.15 \pm 0.06$ | $0.10 \pm 0.04$  |                 |                  | 8         |
| 16                      | $\pi^-p$  | $0.00 \pm 0.06$  | $0.08 \pm 0.03$ | $-0.02 \pm 0.02$ | $0.06 \pm 0.03$ | $-0.08 \pm 0.04$ | 9         |
| 10                      | $pp$  |                  |                 | $0.10 \pm 0.02$  | $0.08 \pm 0.02$ | $0.02 \pm 0.03$  | 10        |
| 1.6                     | $p\bar{p}$  |                  |                 | $0.38 \pm 0.04$  | $0.09 \pm 0.05$ | $0.29 \pm 0.06$  | 2         |
| 3.25                    | $p\bar{p}$  |                  |                 | $0.39 \pm 0.13$  | $0.44 \pm 0.20$ | $-0.05 \pm 0.24$ | 3         |
| 3.5                     | $p\bar{p}$  |                  |                 | $0.44 \pm 0.08$  | $0.38 \pm 0.01$ | $0.06 \pm 0.08$  | 3         |
| 5.7                     | $p\bar{p}$  |                  |                 | $0.36 \pm 0.07$  | $0.39 \pm 0.07$ | $-0.03 \pm 0.10$ | 4         |
| 7.0                     | $p\bar{p}$  |                  |                 | $0.20 \pm 0.08$  | $0.38 \pm 0.13$ | $-0.18 \pm 0.15$ | 12        |
| 5.0                     | $K^+p$  | $0.08 \pm 0.04$  | $0.04 \pm 0.04$ | $0.20 \pm 0.02$  | $0.06 \pm 0.04$ | $0.14 \pm 0.04$  | 11        |
| 12.7                    | $K^+p$  | $0.32 \pm 0.19$  | $0.10 \pm 0.12$ | $0.10 \pm 0.06$  | $0.21 \pm 0.11$ | $-0.11 \pm 0.12$ |           |
| 12.7                    | $K^+p$<br>(resonances<br>$N^{*++}$ , $K^{*0}$<br>removed) | $0.10 \pm 0.21$  | $0.03 \pm 0.18$ | $-0.01 \pm 0.08$ | $0.06 \pm 0.13$ | $0.07 \pm 0.15$  |           |

be seen in the corresponding correlations with and without the presence of  $N^{*++}$  resonance. If we remove the events with  $N^{*++}$  and  $K^{*0}$ , we see in Figs. 4(a)–4(c) that the angular correlation between  $\pi^+\pi^+$ ,  $\pi^-\pi^-$ , and  $\pi^+\pi^-$  becomes more isotropic. The effect of the background which is removed with the resonances is negligible on the angular correlation. The corresponding values of  $\gamma$ 's are also given in Table I. We find that the resonances have a very dominating effect upon the angular correlations and after the removal of these resonances, the values of  $\gamma^L$  and  $\gamma^U$  for pions are very close to one another within the statistics. The estimated 8% of the events in which  $K^+$  and  $\pi^+$  were misidentified would not affect the conclusions drawn from Table I.

We introduce an asymmetry parameter  $A$  defined<sup>16</sup> by  $A = (N_A - N_B)/(N_A + N_B) = (\gamma - 1)/(\gamma + 1)$  and  $A^L$  and  $A^U$  refer to the angular distribution of pion

pairs of like and unlike charges, respectively. In Table II, we have summarized the values of the parameter  $A^{++}$ ,  $A^{--}$ , and  $A^{+-}$  for  $\pi^+p$ ,  $\bar{p}p$ ,  $pp$ , and  $K^+p$  interactions at different primary energies without the elimination of the resonances except in the  $K^+p$  interaction at 12.7 GeV/c. We find that the difference  $A^U - A^L$  decreases when c.m. energy increases, in disagreement with GGLP predictions.<sup>1</sup> In the present experiment, the difference  $A^U - A^L$  is practically zero for events without the presence of resonances.

We also looked at the correlation angle between the pions without the removal of the resonances as a function of the difference ( $\Delta p$ ) between the momenta of the pion pair. We find that the angular correlation is very small for pairs when both pions have similar values of momenta, and relatively it increases with the increase in the relative-momentum ( $\Delta p$ ) values which is in disagree-

TABLE III. Values of the  $\gamma$  parameter in the rest frame of all pions.

| $\gamma = N_A/N_B$   | Resonances<br>not<br>removed | Resonance<br>$N^{*++}$<br>removed | Resonance<br>$K^{*0}$<br>removed | Resonances<br>$N^{*++}$ and $K^{*0}$<br>removed |
|----------------------|------------------------------|-----------------------------------|----------------------------------|---|
| $\gamma(\pi^+\pi^+)$ | $2.65 \pm 0.33$              | $2.05 \pm 0.44$                   | $2.12 \pm 0.38$                  | $2.10 \pm 0.47$                                 |
| $\gamma(\pi^+\pi^-)$ | $2.01 \pm 0.19$              | $1.90 \pm 0.29$                   | $1.83 \pm 0.22$                  | $1.99 \pm 0.34$                                 |

ment with the predictions of the statistical model as proposed by Goldhaber *et al.*<sup>1</sup> For isolating a possible influence on the  $\pi\pi$  correlation of peripheral emission of secondary particles in resonance production one should investigate the experimental distribution of the pion-pair angles not only in the over-all c.m. system, but also in the c.m. system of the four pions where the resonance effect is better seen. These values are shown in Table III with and without the presence of resonances. The values without the presence of resonances are close to one another, which shows that resonances are responsible for the angular correlation between the pions.

Our results in Tables I, II, and III clearly exhibit the effect of the presence of resonances in the angular correlation between the like and un-

like pions. The influence of Bose-Einstein statistics plays a negligible role in the interaction studied here as there is no difference between the distribution of c.m. opening angle for like ( $\pi^+\pi^+$ ) and unlike ( $\pi^+\pi^-$ ) pairs when the resonance effect is taken into consideration.

Thus we see that a careful examination of the influence of resonance production and kinematic effect are necessary when studying pion-pion correlation in hadron-hadron interactions.

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<sup>13</sup>W. M. Labuda, Ph.D. thesis, State University of New York at Buffalo, Buffalo, New York, 1971 (unpublished). We have used the same set of parameters as given by O. Czyżewski, in *Proceedings of the Topical Conference on High Energy Collisions of Hadrons, CERN 1968* (CERN, Geneva, 1968). However,  $b_1$  now refers to the incident  $K^+$  vertex. On account of the large number of diagrams possible, certain simplifying assumptions were made in applying the model. It was assumed that the experimental distribution could be produced if average trajectories and associated average coupling constants were used instead of exact values for the exchanged Reggions. Possible exchanges for  $K^+p$  interactions include nucleon, hyperon, kaon, Pomeranchukon, and meson. We used for nucleon, hyperon, kaon, Pomeranchukon, and meson the values for  $\alpha$  (intercept of trajectory) and  $g$  (relative coupling constant) as  $-0.35$ ,  $-0.35$ ,  $0.30$ ,  $1.0$ ,  $0.5$ , and  $1.4$ ,  $1.4$ ,  $1.0$ ,  $0.7$ ,  $1.0$ , respectively. Resonance effects have also been included in the model.

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<sup>15</sup>The production of  $\rho$  mesons is very small compared to  $N^{*++}$  and  $K^{*0}$ .

<sup>16</sup>Authors of Ref. 12 have used a parameter  $C$  where  $C = N_A^U - N_B^L$ . The total number of pairs in each distribution is normalized to one,  $N_A + N_B = 1$ . The parameter  $C$  is related to  $\gamma^L$  and  $\gamma^U$  by  $C = (\gamma^U - \gamma^L) / (1 + \gamma^U)(1 + \gamma^L)$  and to  $A^U$  and  $A^L$  by  $C = \frac{1}{2}(A^U - A^L)$ .