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$$M_W \cong 37.5\,(1-\tfrac{5}{4}\theta^2)\text{ GeV for } |\theta| \ll 1,$$

$$M_W \cong 150\,|\tfrac{1}{4}\pi-\theta|\text{ GeV for } |\tfrac{1}{4}\pi-\theta| \ll 1.$$

Note that $M_W(\theta)=M_W(-\theta)=M_W(\tfrac{1}{2}\pi-\theta)$.

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First-Order Non-Abelian Compton Effect and Generalized Cabibbo-Radicati Theorem*

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It is shown that the first-order isospin-antisymmetric amplitude for the scattering of isovector photons on a spin $J \le \frac{3}{2}$ target is determined by the isovector charge radius, magnetic moment, and quadrupole moment of the system.

Bég[1] has studied the non-Abelian Compton effect on nucleons to first order in the frequency ω of the incident photon and showed in particular that the well-known Cabibbo-Radicati sum rule follows from the obtained first-order theorems.

In this paper we extend these theorems for $J \le \frac{3}{2}$ and obtain a generalized expression for the Cabibbo-Radicati theorem which relates the first-order isospin-antisymmetric spin-independent amplitude to the isovector charge mean-square radius of the system. On the other hand, the spin-dependent part is shown to be given by the isovector magnetic moment and quadrupole moment of the system.

Our starting point is the relation

$$k'_m E^{\alpha\beta}_{mn} k_n = \omega\omega' E^{\alpha\beta}_{00} + \omega\omega' U^{\alpha\beta}_{00} - k'_m U^{\alpha\beta}_{mn} k_n + iV(E_{p'}/m)^{1/2}\epsilon^{\alpha\beta\gamma}\langle \vec{p}\,' \,|\,[\tfrac{1}{2}(\omega+\omega')J^\gamma_0(0) + \tfrac{1}{2}(k'_m+k_m)J^\gamma_m(0)]\,|\,\vec{0}\rangle\,, \tag{1}$$

where $U^{\alpha\beta}_{\mu\nu}$ and $E^{\alpha\beta}_{\mu\nu}$ are, respectively, the unexcited- and excited-state contributions to the T-matrix amplitude $T^{\alpha\beta}_{\mu\nu}$, where

$$(2\pi)^4\delta(p'+k'-p-k)\left(\frac{m}{V^2E_{p'}}\right)^{1/2} T^{\alpha\beta}_{\mu\nu} = i\int d^4x d^4y e^{-ik'x+iky}\langle \vec{p}\,' \,|\,[T(J^\alpha_\mu(x), J^\beta_\nu(y)) - i\rho^{\alpha\beta}_{\mu\nu}(x)\delta^4(x-y)]\,|\,\vec{0}\rangle\,. \tag{2}$$

Equation (1) is a consequence of current conservation and the basic equal-time commutation relations[1] of the current operators J^α_μ. In these equations α and β are isotopic-spin indices, k' and k (p' and p) are outgoing and incident "photon" (target) momenta with $\vec{p}=0$.

As is well known, $E^{\alpha\beta}_{00}$ is of order ω^2 and therefore it cannot compete for the determination of $E^{\alpha\beta}_{ij}$ to first order in ω.

To calculate the known part on the right-hand side of Eq. (1) we need the $J=\frac{3}{2}$ isovector current matrix element,[2]

$$\langle p'|J_\mu^\alpha(0)|p\rangle = i\left(\frac{M^2}{V^2E_pE_{p'}}\right)\bar{u}_\rho(p')\left\{\left[F_1^\alpha(q^2)\delta_{\rho\sigma}+\frac{F_3^\alpha(q^2)}{2M^2}q_\rho q_\sigma\right]\gamma_\mu+\frac{i}{4M}[\gamma_\mu,\gamma\cdot q]\left[F_2^\alpha(q^2)\delta_{\rho\sigma}+\frac{F_4^\alpha(q^2)}{2M^2}q_\rho q_\sigma\right]\right\}u_\sigma(p), \tag{3}$$

where $q=p'-p$ and $F_i^\alpha(q^2)=F_i^V(q^2)I^\alpha$ are the isovector form factors: $F_1^V(0)=1$, $F_1^V(0)+F_2^V(0)=\mu^V$ is the isovector magnetic moment in units of $1/2M$, $F_1^V(0)+F_3^V(0)=Q^V$ is the isovector quadrupole moment in units of $1/M^2$, and $F_4(0)$ contains these and the octupole magnetic moment of the system. One obtains for the isospin-antisymmetric part of Eq. (1)

$$k'_m E_{mn}^{[\alpha\beta]}k_n=\left[\left(\frac{2\mu^V-1}{4M^2}-2F_1^{V\prime}-\frac{3Q^V}{4M^2}\right)\vec{k}\cdot\vec{k}'\omega-\frac{\mu^V}{3M}i\vec{J}\cdot(\vec{k}'\times\vec{k})+\frac{Q^V\omega}{6M^2}\{\vec{J}\cdot\vec{k},\vec{J}\cdot\vec{k}'\}+O(\omega^4)\right][I^\alpha,I^\beta], \tag{4}$$

where we have already written the right-hand side in J space with the help of known relations.[3] The isospin-symmetric part has not been written down since it gives a trivial extension of the general results obtained by Pais[4] for physical Compton scattering. We write now the tensor decomposition,

$$E_{mn}^{[\alpha\beta]}=[I^\alpha,I^\beta](A_1\delta_{mn}+A_2[J_m,J_n]+A_3(\{J_m,J_n\}-\tfrac{5}{2}\delta_{mn})+O(\omega^2)). \tag{5}$$

Note that the third element is an irreducible second-order tensor. In this way the generalized form of the Cabibbo-Radicati theorem will involve only the spin-independent amplitude A_1.

Substituting Eq. (5) in Eq. (4) and comparing coefficients, we obtain

$$A_1=\left(\frac{2\mu^V-1}{4M^2}-2F_1^{V\prime}-\frac{Q^V}{3M^2}\right)\omega, \tag{6a}$$

$$A_2(0)=-\frac{\mu^V}{3M}, \tag{6b}$$

$$A_3=\frac{Q^V}{6M^2}\omega, \tag{6c}$$

where $F_1^{V\prime}=[dF_1^V(t)/dt]_0$.

By the usual method we compute the isovector charge mean-square radius[5]

$$\langle r^2\rangle^V I^\alpha=\langle\vec{0}|\int J_0^\alpha(\vec{r})r^2d\vec{r}|\vec{0}\rangle = i^2V\lim_{\vec{p}=0}\lim_{\vec{p}'=\vec{p}}\nabla_p{}^2\langle\vec{p}'|J_0^\alpha|\vec{p}\rangle. \tag{7}$$

Using Eq. (3) we obtain[6]

$$\frac{\langle r^2\rangle^V}{3}=\frac{2\mu^V-1}{4M^2}-2F_1^{V\prime}-\frac{Q^V}{3M^2}, \tag{8}$$

which is exactly the factor present in Eq. (6a). Therefore, the spin-independent part A_1 is given to first order by the isovector charge radius and the spin-dependent part of the amplitude is given by the isovector magnetic and quadrupole moment of the system.

Theorem (6a) corresponds to the Cabibbo-Radicati theorem for the nucleon case[1] which reads

$$A_1=\left(\frac{2\mu^V-1}{4M^2}-2F_1^{V\prime}\right)\omega \quad (J=\tfrac{1}{2}),$$

where F_1^V is the isovector Dirac form factor.

We also quote the result for the case[7] $J=1$ (divided by $2M$):

$$A_1=\left(\frac{\mu^V-1}{3M^2}-2F_1^{V\prime}\right)\omega \quad (J=1),$$

where here[4] $F_1^V(0)=\mu^V$.

For the case $J=0$ one immediately obtains

$$A_1=-(2F^{V\prime})\omega \quad (J=0),$$

where $F^V(q^2)$ is the isovector charge form factor.

Using Eq. (7) with the corresponding expressions for the current matrix element it is easy to check that all factors present in these amplitudes for $J\leq 1$ are equal to $\frac{1}{3}\langle r^2\rangle^V$, as in the case $J=\frac{3}{2}$. We therefore conjecture the following generalization of the Cabibbo-Radicati theorem:

$$A_1=\frac{\langle r^2\rangle^V}{3}\omega, \tag{9}$$

for arbitrary spin.

Theorem (6b) satisfies the generalized theorem

$$A_2(0)=-\frac{\mu^V}{2JM}, \tag{10}$$

that has been conjectured before.[7]

Theorem (6c) is a generalization to spin $\frac{3}{2}$ of a similar theorem[7] for spin-1 targets, which (divided by $2M$) reads

$$A_3=\frac{Q^V}{2M^2}\omega \quad (J=1).$$

A generalization of this theorem is suggested if we introduce the factor $Q^V/2J(2J-1)$. We therefore conjecture

$$A_3 = \frac{Q^V}{2J(2J-1)M^2}, \qquad (11)$$

for arbitrary spin.

The generalized Cabibbo-Radicati theorem, Eq. (9), can also be written as

$$A_1 = -2\omega\left(\frac{dG_0^V}{dt}\right)_{t=0}, \qquad (12)$$

where G_0^V is the isovector part of the physical form factor G_0 introduced by Gourdin,[2] extending for higher spins the definition of the charge form factor first introduced by Yennie, Lévy, and Ravenhall[8] for the nucleon case.

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$$F_3 q_\rho q_\sigma \to F_3(q_\rho q_\sigma - \tfrac{1}{3}q^2\delta_{\rho\sigma})$$

similar to the case (Ref. 4) $J=1$, the quadrupole moment would be absent in the expressions of A_1 and of the charge radius $\langle r^2\rangle^V$, but the relation between them would be maintained,

$$A_1 = \omega(6\mu^V - 7 - 24M^2F_1^{V\prime})/12M^2$$
$$= \tfrac{1}{3}\omega\,\langle r^2\rangle^V.$$

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Bethe-Salpeter Equation for Nucleon-Nucleon Scattering Matrix Padé Approximants*

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We first show that the so-called "kernel subtraction" technique which we used in a previous paper on the Bethe-Salpeter equation is unnecessary. The simplified equations which result make it possible to introduce matrix Padé approximants in a straightforward way. These are found to converge to the same solution that we found in our previous work and more rapidly than the ordinary Padé approximants.

I. ELIMINATION OF THE KERNEL-SUBTRACTION TECHNIQUE

If we do not use the kernel-subtraction technique,[1] introduced in Sec. II E in Ref. 2, everything goes as in Ref. 2 with the following minor and simplifying exceptions. The last term in Eq. (29) is unnecessary. Equations (32) and (34) remain valid, but they are equations for $\phi(p, ip_4, \alpha)$ and $\phi(p, E-E(p), \alpha)$; that is, the primed ϕ's are not introduced as they are in Eq. (24) of Ref. 2. Also, in place of Eqs. (37)–(41), there is a single, simpler equation,

$$\tan\delta = \frac{E}{2p}\,\phi(\hat{p}, 0, 1; \hat{p}, 0, 1), \qquad (1)$$

provided that, in the integrals over q in Eqs. (32) and (34), the principal part is taken.

The principal part is taken in this way. The integrals over q have two contributions, one from the double integral and one from the single integral on the right-hand side of Eqs. (32) and (34). The single integral with $\hat{p}$ as upper limit contributes to one side of the q integration (that is, the side q