# Gauge Theory of Weak and Electromagnetic Interactions with Han-Nambu Quarks*

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The use of Han-Nambu quarks enables us to construct an SO(3) gauge theory of weak and electromagnetic interactions which is compatible with the naive quark model. The model is constrained by the requirements that $K_L \to \mu\bar{\mu}$ is suppressed and the $K_L$-$K_S$ mass splitting is small. This leads to a relation between the $W$ mass and the Cabibbo angle, which determines the $W$ mass to be near 35 GeV.

Recent work indicates the possibility of constructing a renormalizable model of weak and electromagnetic interactions based on a spontaneously broken gauge theory. We present such a model in which the fundamental hadrons are the Han-Nambu[1] quarks. We show that this model has the interesting property that the Cabibbo angle is related to the intermediate-vector-boson mass. First, we briefly review the models already proposed. Although several plausible models of leptonic interactions have been suggested,[2-4] there are difficulties in sensibly incorporating semileptonic and nonleptonic processes. One difficulty is the appearance of neutral currents in lowest-order weak interactions which gives rise to such phenomena as neutrino-proton scattering.[5] The introduction of strangeness violation can lead to another difficulty: It may induce unacceptably large values of the $K_L$-$K_S$ mass difference and the $K_L \to \mu\bar{\mu}$ decay rate.

In a previous paper,[3] we suggested a model based on the gauge group SO(3) in which the only neutral vector boson is the photon. A version of this model with five fundamental hadrons (quarks) has been shown to be unacceptable because it leads to too great a rate for $K_L \to \mu\bar{\mu}$.[6] An alternative model involves two SU(4) quartets of quarks, where the higher-order difficulties are eliminated by the procedure of Glashow, Iliopoulos, and Maiani.[7] However, in such a model, the construction of baryons is unorthodox and awkward: Octets are made up of two quarks and an antiquark while decuplets require three quarks and two antiquarks. All the successful predictions of the naive quark model are lost.[8]

In this paper, we construct an SO(3) model involving the nine Han-Nambu quarks,[1] comprising three SU(3) triplets. The difficulties with higher-order weak interactions can be overcome if and only if the Cabibbo angle is in a definite relation to the mass of the intermediate vector boson, $W$.

The Han-Nambu quarks are denoted by

$$\begin{bmatrix} \mathcal{P}_1 & \mathcal{P}_2 & \mathcal{P}_3 \\ \mathfrak{N}_1 & \mathfrak{N}_2 & \mathfrak{N}_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix},$$

with electrical charges

$$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}.$$

Each column is a triplet under the conventional strong-interaction SU(3) group, $\mathcal{K}$. Each row is a triplet under another SU(3) group, $\mathcal{K}'$, which is also assumed to be an approximate symmetry of strong interactions. All observed hadrons are singlets under $\mathcal{K}'$. If mesons are made of quark-antiquark, and baryons of three quarks, their SU(3) properties are the same as in the naive quark model.

Under the weak and electromagnetic gauge group $\mathfrak{S}$, these nine quarks must transform like two triplets and three singlets, so that the neutral generator is the electrical charge. The charged current may be divided into two parts, $J_\mu$ and $J'_\mu$, where $J_\mu$ involves transitions between quarks in the same column and $J'_\mu$ involves transitions from one column to another. Only $J_\mu$ contributes to the semileptonic decays of observed hadrons, under the assumption that the analogs of $T_3$ and $Y$ in $\mathcal{K}'$ are good symmetries of strong interactions. We shall assume that, up to a multiplicative factor $a$, $J_\mu$ is the conventional weak-interaction current, i.e., it is purely $V-A$, and transforms like a member of an $\mathcal{K}$ octet.

The right-handed and left-handed quark fields must each be assigned to two $\mathfrak{S}$ triplets and three $\mathfrak{S}$ singlets. These assignments are determined by the invariant mass terms and the couplings of the scalar mesons, as in Ref. 3. The assignment of the right-handed quarks must be such as to contribute only to $J'_\mu$. One possible assignment of the triplets is

$$[\mathcal{P}_2; \mathcal{N}_3; \mathcal{N}_1]_R \text{ and } [\mathcal{P}_3; \lambda_2; \lambda_1]_R,$$

with the remaining states transforming as singlets. The neutral current contributes the right-handed part of the electromagnetic current, while the charged current plays no role in observed semileptonic phenomena.

The left-handed quark fields must be assigned into triplets as follows:

$$[\mathcal{P}_2; a\mathcal{N}_2(\theta) + b\lambda_3(\chi) + a\cos\phi\,\mathcal{P}_1; \mathcal{N}_1(\theta-\phi)]_L\,,$$
$$[\mathcal{P}_3; a\mathcal{N}_3(\theta) + b'\lambda_2(\chi') + a\sin\phi\,\mathcal{P}_1; \lambda_1(\theta-\phi)]_L\,,$$

where we use the notation $\mathcal{N}(\xi) = \mathcal{N}\cos\xi + \lambda\sin\xi$, $\lambda(\xi) = \lambda\cos\xi - \mathcal{N}\sin\xi$, and $\theta$ is the Cabibbo angle.[9] The requirement that the neutral members of the two triplets be properly normalized yields

$$a^2(1+\cos^2\phi) + b^2 = a^2(1+\sin^2\phi) + b'^2 = 1\,, \tag{1}$$

while the requirement that they be orthogonal yields

$$b\sin(\theta-\chi) + b'\sin(\theta-\chi') + a\sin\phi\cos\phi = 0\,. \tag{2}$$

Even though our model involves no neutral vector boson coupled to a strangeness-changing current, it is possible to obtain $\Delta S = 2$ nonleptonic transitions in lowest order (Fig. 1). These contributions must vanish. One way to satisfy this constraint is for the couplings of $W_\mu$ to $\bar{\lambda}_2\gamma^\mu(1+\gamma_5)\mathcal{N}_1$ and $\bar{\lambda}_3\gamma^\mu(1+\gamma_5)\mathcal{N}_1$ each to vanish. This yields the equations

$$\begin{aligned} &b'\sin(\theta-\phi)\cos\chi' - a\cos(\theta-\phi)\sin\theta = 0\,,\\ &b\cos(\theta-\phi)\cos\chi - a\sin(\theta-\phi)\sin\theta = 0\,. \end{aligned} \tag{3a}$$

Another possibility is to set the couplings of $W_\mu$ to $\bar{\mathcal{N}}_2\gamma^\mu(1+\gamma_5)\lambda_1$ and $\bar{\mathcal{N}}_3\gamma^\mu(1+\gamma_5)\lambda_1$ equal to zero. This yields

$$\begin{aligned} &b'\cos(\theta-\phi)\sin\chi' - a\sin(\theta-\phi)\cos\theta = 0\,,\\ &b\sin(\theta-\phi)\sin\chi - a\cos(\theta-\phi)\cos\theta = 0\,. \end{aligned} \tag{3b}$$

The remaining possibilities [e.g., the vanishing of the couplings of $W_\mu$ to $\bar{\mathcal{N}}_2\gamma^\mu(1+\gamma_5)\lambda_1$ and $\bar{\lambda}_3\gamma^\mu(1+\gamma_5)\mathcal{N}_1$] are not compatible with the constraints we shall deduce next.

Further constraints on the model follow from the requirement that higher-order effects not contribute terms of order $\alpha G$ to the $K_L \to \mu\bar{\mu}$ amplitude or to the $K_L$-$K_S$ mass difference. The relevant diagrams involve the exchange of two $W$'s between a quark and a lepton (or another quark). We demand that the order-$\alpha G$ contributions of such diagrams to unwanted transitions vanish in the limit that $\mathcal{H}'$ is exact. Then if $\mathcal{H}'$ is not badly broken, these contributions will be suppressed. Thus, we require that the $\mathcal{H}'$ singlet part of such diagrams vanish. For $K_L \to \mu\bar{\mu}$ (as well as other $\Delta Y = 1$, $\Delta Q = 0$ semileptonic transitions), this condition is simply

$$2a^2\sin 2\theta = b^2\sin 2\chi + b'^2\sin 2\chi'\,, \tag{4}$$

while for $\Delta Y = 2$ nonleptonic transitions the conditions are (4) and

$$(bb'\cos\chi\cos\chi' - a^2\sin^2\theta) \times (bb'\sin\chi\sin\chi' - a^2\cos^2\theta) = 0\,. \tag{5}$$

Equation (5) is not independent, since (3a) implies that the first factor vanishes [if $\sin(2\theta - 2\phi) \neq 0$], while (3b) leads to the vanishing of the second factor.

Equations (1), (2), (3a) or (3b), and (4) give six constraints on the seven parameters $a$, $\theta$, $b$, $b'$, $\chi$, $\chi'$, $\phi$. They have the trivial solutions $\theta = 0$ or $\theta = \frac{1}{2}\pi$ for $|a| \leq \frac{4}{5}$, but they also have nontrivial ($\theta \neq 0$ or $\frac{1}{2}\pi$) solutions for $|a| < 1/\sqrt{2}$. In this region there is a relation between $\theta$ and $a$. Comparing with the SO(3) gauge theory of leptonic interactions in Ref. 3, we find that the requirements of universality for leptonic and semileptonic weak interactions imply that $a = \sin\beta$, where $\sin\beta$ is directly related to the $W$ mass by $M_W = 53.0\,|\sin\beta|$ GeV/$c^2$. Thus our relation between $\theta$ and $a$ may be translated into a relation between[10] $\theta$ and $M_W$ which is displayed in Fig. 2. For example, $\theta = 13.3°$ (a typical experimental determination) implies $M_W = 35.0$ GeV.

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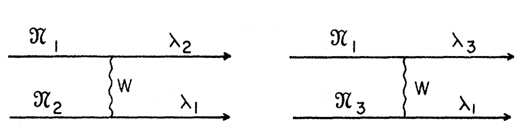


FIG. 1. Lowest-order diagrams contributing to $\Delta Y = 2$ nonleptonic interactions.

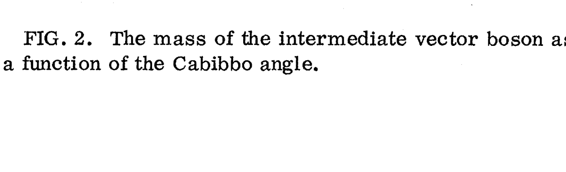


FIG. 2. The mass of the intermediate vector boson as a function of the Cabibbo angle.

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[9]There is no loss of generality in choosing $\mathcal{P}_2$ and $\mathcal{P}_3$ unmixed. However, by choosing all the parameters real, we preclude an explanation of *CP* violation at this level.

[10]We cannot find a closed form for this relation. The curve in Fig. 2 has been constructed from the asymptotic formulas

$$M_W \cong 37.5\ (1-\tfrac{5}{4}\theta^2)\ \text{GeV for } |\theta| \ll 1,$$

$$M_W \cong 150\ |\tfrac{1}{4}\pi-\theta|\ \text{GeV for } |\tfrac{1}{4}\pi-\theta| \ll 1.$$

Note that $M_W(\theta)=M_W(-\theta)=M_W(\frac{1}{2}\pi-\theta)$.

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# First-Order Non-Abelian Compton Effect and Generalized Cabibbo-Radicati Theorem*

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It is shown that the first-order isospin-antisymmetric amplitude for the scattering of isovector photons on a spin $J \le \frac{3}{2}$ target is determined by the isovector charge radius, magnetic moment, and quadrupole moment of the system.

Bég[1] has studied the non-Abelian Compton effect on nucleons to first order in the frequency $\omega$ of the incident photon and showed in particular that the well-known Cabibbo-Radicati sum rule follows from the obtained first-order theorems.

In this paper we extend these theorems for $J \le \frac{3}{2}$ and obtain a generalized expression for the Cabibbo-Radicati theorem which relates the first-order isospin-antisymmetric spin-independent amplitude to the isovector charge mean-square radius of the system. On the other hand, the spin-dependent part is shown to be given by the isovector magnetic moment and quadrupole moment of the system.

Our starting point is the relation

$$k'_m E^{\alpha\beta}_{mn} k_n = \omega\omega' E^{\alpha\beta}_{00} + \omega\omega' U^{\alpha\beta}_{00} - k'_m U^{\alpha\beta}_{mn} k_n + iV(E_{p'}/m)^{1/2}\epsilon^{\alpha\beta\gamma}\langle \vec{p}'\,|\,[\tfrac{1}{2}(\omega+\omega')J^{\gamma}_{0}(0) + \tfrac{1}{2}(k'_m+k_m)J^{\gamma}_{m}(0)]\,|\vec{0}\rangle\,, \qquad (1)$$

where $U^{\alpha\beta}_{\mu\nu}$ and $E^{\alpha\beta}_{\mu\nu}$ are, respectively, the unexcited- and excited-state contributions to the $T$-matrix amplitude $T^{\alpha\beta}_{\mu\nu}$, where

$$(2\pi)^4\delta(p'+k'-p-k)\left(\frac{m}{V^2E_{p'}}\right)^{1/2} T^{\alpha\beta}_{\mu\nu} = i\int d^4x\, d^4y\, e^{-ik'x+iky}\langle \vec{p}'\,|\,[T(J^{\alpha}_{\mu}(x), J^{\beta}_{\nu}(y)) - i\rho^{\alpha\beta}_{\mu\nu}(x)\delta^4(x-y)]\,|\vec{0}\rangle\,. \qquad (2)$$

Equation (1) is a consequence of current conservation and the basic equal-time commutation relations[1] of the current operators $J^{\alpha}_{\mu}$. In these equations $\alpha$ and $\beta$ are isotopic-spin indices, $k'$ and $k$ ($p'$ and $p$) are outgoing and incident "photon" (target) momenta with $\vec{p}=0$.

As is well known, $E^{\alpha\beta}_{00}$ is of order $\omega^2$ and therefore it cannot compete for the determination of $E^{\alpha\beta}_{ij}$ to first order in $\omega$.