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Spectrum-Generating Algebra and No-Ghost Theorem for the Neveu-Schwarz Model*

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The no-ghost theorem of Brower for the conventional dual model (intercept $\alpha_0 = 1$) is extended to the Neveu-Schwarz model, and its spectrum-generating algebra is constructed. These conformally invariant generators (A_n, B_s) are obtained by a systematic use of the gauges G_r. As a consequence it is demonstrated that particular dual models having resonances with positive-definite norms and masses are possible.

I. INTRODUCTION

For several years, dual models with the conformal gauge symmetry of Virasoro[1] have been conjectured to be free of negative-metric or ghost states. Only recently, Brower[2] has presented a proof of this "no-ghost theorem" for the conventional dual model (CDM, intercept $\alpha_0 = 1$).

The critical step was the realization of a set of generators $A_n^{(+)}$ for the longitudinal physical states, whose algebra

$$[A_m^{(+)}, A_n^{(+)}] = (m-n)A_{m+n}^{(+)} + 2m^3\delta_{m+n,0} \tag{1.1}$$

is identical to the conformal Lie algebra[2] of the gauges (L_l) apart from c numbers. They close algebraically with the transverse operators A_n^i of Del Giudice, Di Vecchia, and Fubini (DDF),[3] to form the full spectrum-generating algebra in light-cone variables $[P_\pm = (P_0 \pm P_3)/\sqrt{2},\ \vec{P} = (P_1, P_2)]$.

Here we extend this construction and the no-ghost theorem[4] to the Neveu-Schwarz[5] model (NSM). In this model we have both G-parity-preserving operators $(A_n^i, A_n^{(+)})$ and G-parity-changing operators $(B_r^i, B_r^{(+)})$ to generate the full physical spectrum.[6]

Considerable care has been taken to present the construction as a logical extension of the earlier techniques[2,3,7] employed for the CDM. In the conventional model, the conformal gauge invariance (commutativity with L_l) is manifest for the operators of DDF

$$A_n^i = \langle P^i V_n \rangle \tag{1.2}$$

simply because $P^i V_n$ is a conformal spin-one object.[8] In the NSM we express A_n^i (or B_n^i) in the "G representation,"

$$A_n^i = \langle z^{-r}\{G_r, X\}\rangle \ . \tag{1.3}$$

Gauge invariance or commutativity with G_s can be made manifest by demanding (1) that $z^{-r}\{G_r, X\}$ is independent of r and (2) X is a conformal spin-$\frac{1}{2}$ object.

This G representation combined with a sort of conformally invariant derivative gives the correction terms in $A_n^{(+)}$ and $B_n^{(+)}$ as well.

II. ALGEBRA AND GHOST CANCELLATION

Introducing the complex Koba-Nielsen variable

$$z = \exp(\tau + i\theta), \tag{2.1}$$

the CDM is constructed from the "fields" $Q_\mu(z)$ and $P_\mu(z) = i\partial_\tau Q_\mu(z)$, which satisfy local equal-τ commutation relations

$$[Q_\mu(z_1), P_\nu(z_2)] = 2\pi i g_{\mu\nu}\delta(\theta_1 - \theta_2)\ , \tag{2.2}$$

where $g^{00} = -1$ and $g^{kk} = +1$ $(k = 1, \ldots, D-1)$ in a D-dimensional Lorentz space. The NSM supplements these fields with a conformal spin-$\frac{1}{2}$ field H_μ (see Ref. 8). These new fields satisfy the relations

$$\{H_\mu(z_1), H_\nu(z_2)\} = 2\pi g_{\mu\nu}\,\delta(\theta_1 - \theta_2)\ ,\qquad [H_\mu(z_1), Q_\nu(z_2)] = 0\ , \tag{2.3}$$

and can be expanded in two infinite sets of four-vector annihilation and creation operators defined[8,9] by

$$\alpha_n^\mu = \langle z^n P^\mu(z)\rangle\ ,\quad b_r^\mu = \langle z^r H^\mu(z)\rangle\ , \tag{2.4}$$

where n is any integer and r is a half-integer. Hence they obey

$$[\alpha_m^\mu, \alpha_n^\nu] = m g^{\mu\nu}\delta_{m+n,0}\ ,\qquad \{b_r^\mu, b_s^\nu\} = g^{\mu\nu}\,\delta_{r+s,0},\quad [\alpha_m^\mu, b_r^\nu] = 0\ . \tag{2.5}$$

The amplitude in the CDM is

$$B_N = \int \prod_i \frac{dz_i}{d^3 w}\left\langle 0\left|\prod_i z_i^{-p_i^2} V_0(p_i, z_i)\right|0\right\rangle\ , \tag{2.6}$$

where $p_i^2 = 1$ and

$$V_0 = :\exp[i\sqrt{2}\,p_i\cdot Q(z_i)]:\ .$$

The NSM makes the replacement[10]

$$V_0(p,z) \to \{z^{-r}G_r, V_0(p,z)\} = \sqrt{2}\,p\cdot H(z)V_0(p,z), \tag{2.7}$$

with $p^2=\frac{1}{2}$, where the new gauges G_r are

$$G_r=\langle z^r P_\mu(z)H^\mu(z)\rangle \tag{2.8}$$

and the vertex does not depend on r.

We will construct the on-shell physical states $|\psi,N\rangle$ in the $\mathfrak{F}_2$ formalism[5] where the lowest state $|0_\pi,p\rangle$ is the "pion" ($p^2=\frac{1}{2}$). These states satisfy the on-shell gauge conditions

$$G_r|\psi,N\rangle=0,\quad r=\tfrac{1}{2},\tfrac{3}{2},\ldots \tag{2.9}$$

and

$$(G_{1/2}G_{-1/2}-1)|\psi,N\rangle=2(L_0-\tfrac{1}{2})|\psi,N\rangle=0\ .$$

The Virasoro gauges (L_l) defined by

$$\{G_r,G_s\}=2L_{r+s}+\tfrac{1}{8}D(4r^2-1)\delta_{r+s,0} \tag{2.10}$$

are redundant. Alternatively they can be expressed in terms of the fields as

$$L_n=\tfrac{1}{2}\langle z^n:(P^2-H\dot{H}):\rangle \tag{2.11}$$

and obey

$$\begin{aligned}&[L_m,L_n]=(m-n)L_{m+n}+\tfrac{1}{8}Dm(m^2-1)\delta_{n+m,0}\ ,\\&[L_m,G_r]=(\tfrac{1}{2}m-r)G_{m+r}\ .\end{aligned} \tag{2.12}$$

Two sets of spectrum-generating operators are needed in the NSM: A_n and B_r, with positive and negative G parity, respectively. In Sec. III we will exhibit the construction of these operators which commute with the gauges.

The transverse operators obey the algebraic relations of (nonrelativistic) harmonic oscillators

$$\begin{aligned}&[A^i_m,A^j_n]=m\,\delta_{ij}\delta_{n+m,0}\ ,\\&\{B^i_r,B^j_s\}=\delta_{ij}\delta_{r+s,0}\ ,\quad [A^i_m,B^j_r]=0\ ,\end{aligned} \tag{2.13}$$

where $i,j=1,2,\ldots,D-2$ label transverse components. Thus we have the isomorphism

$$A^i_n\to\alpha^i_n\ ,\quad B^i_s\to b^i_s\ . \tag{2.14}$$

The algebra for the longitudinal operators is similar to that for the gauges

$$\begin{aligned}&[A^{(+)}_m,A^{(+)}_n]=(m-n)A^{(+)}_{m+n}+m^3\delta_{m+n,0}\ ,\\&\{B^{(+)}_r,B^{(+)}_s\}=2A^{(+)}_{r+s}+4r^2\delta_{r+s,0}\ ,\\&[A^{(+)}_m,B^{(+)}_r]=(\tfrac{1}{2}m-r)B^{(+)}_{m+r}\ .\end{aligned} \tag{2.15}$$

These relations are consistent with the isomorphisms

$$A^{(+)}_n\to\mathfrak{L}_n,\quad B^{(+)}_r\to\mathfrak{G}_r,\quad |0_\pi,p\rangle\to|0\rangle\ , \tag{2.16}$$

where $\mathfrak{L}_n$ and $\mathfrak{G}_s$ are the gauge operators constructed from oscillators with eight *spatial* (positive-metric) components. The equation $\mathfrak{L}_0|0\rangle=0$ corresponds to the linear c-number term in $A^{(+)}_0|0_\pi,p\rangle=-\frac{1}{2}|0_\pi,p\rangle$. Hence we can set $p_0=0$, drop the linear term in $\mathfrak{L}_n$, $p_0\cdot\alpha_n$, and use only spatial oscillators.

The algebra of the physical-state operators closes by virtue of the following:

$$\begin{aligned}&[A^{(+)}_m,A^i_n]=nA^i_{n+m},\quad \{B^i_r,B^{(+)}_s\}=A^i_{r+s}\ ,\\&[A^{(+)}_m,B^i_r]=(\tfrac{1}{2}m+r)B^i_{r+m},\quad [A^i_m,B^{(+)}_r]=mB^i_{r+m}\ .\end{aligned} \tag{2.17}$$

Because the metric tensor[7] (norm of a physical state) is calculated solely by use of the commutation relations, the isomorphisms (2.14) and (2.16) allow us to represent the metric tensor in a positive definite space, for $D\leq 10$. Thus all eigenvalues (norms) are either positive or zero.

It is possible to show that we have constructed all physical states by showing that these states are linearly independent and span the space. Any state,

$$|\lambda,N\rangle=\prod_{n,\mu}(\alpha^\mu_{-n})^{\lambda^\mu_n}\prod_{r,\nu}(b^\nu_{-r})^{\epsilon^\nu_r}|0_\pi,p\rangle\ , \tag{2.18}$$

where

$$\sum_{n,\mu}n\lambda^\mu_n+\sum_{r,\nu}r\epsilon^\nu_r=N$$

and $\epsilon=0$ or 1, can be expanded in terms of the physical-state operators A_n,B_r and the auxiliary operators Φ_n, χ_r in exactly the same way as done in the proof of the no-ghost theorem in the CDM.[2] These auxiliary operators are defined as $\Phi_n=\langle V_n\rangle$ and $\chi_r=\langle H_-P_-^{-1/2}V_r\rangle$, where $V_n=\exp(inQ_-)$. Φ_n and χ_r commute with themselves and with the transverse operators A^i_n and B^i_r. They close algebraically with the longitudinal operators

$$\begin{aligned}&[A^{(+)}_m,\Phi_n]=n\Phi_{n+m},\quad \{\chi_r,B^{(+)}_s\}=\Phi_{r+s}\ ,\\&[A^{(+)}_m,\chi_r]=(\tfrac{1}{2}m+r)\chi_{r+m},\quad [\Phi_m,B^{(+)}_s]=m\chi_{m+s}\ .\end{aligned} \tag{2.19}$$

Since this augmented set of operators generates *all* states and is linearly independent, the subsets A_n and B_r which generate *physical* states have no linear dependencies either. Thus an arbitrary physical on-shell state can be constructed as

$$|\psi,N\rangle=\prod_{n,i}(A^i_{-n})^{\lambda^i_n}\prod_{r,i}(B^i_{-r})^{\epsilon^i_r}(A^{(+)}_{-m})^{\lambda^0_m}(B^{(+)}_{-m+1/2})^{\epsilon^0_{m-1/2}}\cdots(A^{(+)}_{-1})^{\lambda^0_1}(B^{(+)}_{-1/2})^{\epsilon^0_{1/2}}|0_\pi,p\rangle\ , \tag{2.20}$$

where

$$\sum_{n,i}n\lambda^i_n+\sum_{r,i}r\epsilon^i_r=N\quad(\epsilon=0\text{ or }1),$$

summing over $i=0,1,2,\ldots,D-2<8$. We can identify the null states as those with λ^0_1 or $\epsilon^0_{1/2}\neq 0$; this follows from the eigenvalue equation

$A_0^{(+)}|0_\pi,p\rangle=-\frac{1}{2}|0_\pi,p\rangle$ and the commutation relations for $A^{(+)}$ and $B^{(+)}$, Eq. (2.15).

At the critical dimension $D=10$, new null states appear. Defining the operators

$$\bar{B}_r^{(+)}=B_r^{(+)}-\sum_{k,i}B^i_{r+k}A^i_{-k}\ ,$$
$$\bar{A}_{2r}^{(+)}=(\bar{B}_r^{(+)})^2 \quad\text{and}\quad \bar{A}_0^{(+)}=\tfrac{1}{2}\{\bar{B}_{1/2}^{(+)},\bar{B}_{-1/2}^{(+)}\}\ , \tag{2.21}$$

we obtain longitudinal generators $(\bar{A}_n^{(+)},\bar{B}_r^{(+)})$ which commute with the transverse generators (A_n^i,B_r^i) and form the subalgebra

$$\{\bar{B}_r^{(+)},\bar{B}_s^{(+)}\}=2\bar{A}^{(+)}_{r+s}+\tfrac{1}{8}(10-D)(4r^2-1)\delta_{r+s,0}\ ,$$
$$[\bar{A}_m^{(+)},\bar{A}_n^{(+)}]=(m-n)\bar{A}^{(+)}_{m+n}+\tfrac{1}{8}(10-D)(m^3-m)\delta_{m+n,0}\ ,$$
$$[\bar{A}_m^{(+)},\bar{B}_r^{(+)}]=(\tfrac{1}{2}m-r)\bar{B}^{(+)}_{m+r}\ . \tag{2.22}$$

At $D=10$ all c numbers are absent, and since $\bar{A}_0|0_\pi,p\rangle=0$ these operators create null states. Since these states decouple,[11] the operators $\bar{B}_r$ $(\bar{A}_n)$ *act* like a new set of G_r (L_n) gauges. Both here and in the CDM[2] the $\bar{A}$ is a true representation of the conformal Lie algebra, *without* c-number anomalies.

III. CONSTRUCTION OF SPECTRUM GENERATORS

A. Transverse Algebra

Following DDF[3] and Brower and Goddard[7] we seek to construct "photon" (i.e., zero-mass vector-meson) transition operators for the NSM.[12] In the CDM the ρ-meson ("photon") vertex operator $A_n^i=\langle P^iV_n\rangle$ is obtained by coupling two ground-state vertices together.[3] In the NSM Rosenzweig[13] coupled two ground-state ("pion") vertices to get the ρ vertex, which we can write as

$$A_n^i=\langle P^iV_n-nH^iH_-V_n\rangle\ . \tag{3.1}$$

It is illuminating to reformulate this operator as

$$A_n^i=\langle z^{-r}\{G_r,H^iV_n\}\rangle\ . \tag{3.2}$$

This "G representation" is r-independent and makes conformal gauge invariance transparent:

$$[G_s,\langle z^{-r}\{G_r,X\}\rangle]=\langle z^{-s}[L_{2s},X]\rangle=0, \tag{3.3}$$

if X has conformal spin $\frac{1}{2}$ and $z^{-r}\{G_r,X\}$ is r-independent.

Comparing (3.1) and (3.2) we note that G_r increases conformal spin by $\frac{1}{2}$ and reverses the G parity of any object. Therefore, to find the other set of transverse physical-state operators B_s^i, we must find a photon-type (i.e., containing V_s) operator with conformal spin $\frac{1}{2}$ and positive G parity that is independent of r when commuted with $z^{-r}G_r$. The simplest choice satisfying these criteria is

$$B_s^i=\langle -z^{-r}[G_r,H^i\dot{H}_-P_-^{-1/2}V_s]\rangle\ . \tag{3.4a}$$

Using the algebra of the gauges and fields we see that

$$B_s^i=\langle H^iP_-^{1/2}V_s-H_-P^iP_-^{-1/2}V_s+\tfrac{1}{2}H^iH_-\dot{H}_-P_-^{-3/2}V_s\rangle\ . \tag{3.4b}$$

By construction this commutes with the gauges and these transverse "photon" operators satisfy the following relations:

$$(A_n^i)^\dagger=A^i_{-n}\ ,\quad A_n^i|0_\pi\rangle=0,\quad n>0$$
$$(B_s^i)^\dagger=B^i_{-s},\quad B_s^i|0_\pi\rangle=0,\quad s>0 \tag{3.5}$$

and the commutation relations (2.13).

Note that A_n^i can be considered as a strongly interacting "photon" transition operator (i.e., it allows transitions between states with the same G parity whose mass and spin differ by integers) and it has the same properties (3.5) and commutation relations (2.13) as A_n^i did in the CDM. Friedman and Rosenzweig[14] have shown that these few properties are all that are needed to derive sum rules and low-energy limits for "Compton" scattering. Therefore we can state that there is a universal "charge" and "magnetic moment" for all states connected by "photon" transitions in the NSM also.

B. Longitudinal Algebra

Following Brower and Goddard[7] we consider the operators

$$A_n^L=\langle z^{-r}:\{G_r,H_+V_n\}:\rangle\ ,$$
$$B_s^L=\langle -z^{-r}:[G_r,H_+H_-P_-^{-1/2}V_s]:\rangle\ , \tag{3.6}$$

which are the longitudinal parts of A_n^μ and B_s^μ $(k_\mu B_s^\mu=0,\ k_\mu A_n^\mu=0$ for $n\neq0)$. Commuting these with the gauges gives nonvanishing anomalies which arise, as in the CDM, because of the normal ordering

$$[G_r,A_n^L]=-n^2r\langle z^rH_-V_n\rangle\ ,$$
$$\{G_r,B_s^L\}=\tfrac{1}{4}(r^2-\tfrac{1}{4})\langle z^r(P_-^{-1/2}+\tfrac{3}{2}H_-\dot{H}_-P_-^{-5/2})V_s\rangle$$
$$-sr\langle z^r(P_-^{1/2}+\tfrac{1}{2}H_-\dot{H}_-P_-^{-3/2})V_s\rangle\ . \tag{3.7}$$

These anomalies can be expressed in the "G representation"

$$[G_r,A_n^L]=-nr\langle[G_r,V_n]\rangle\ ,$$
$$\{G_r,B_s^L\}=\tfrac{1}{4}(r^2-\tfrac{1}{4})\langle\{G_r,H_-P_-^{-3/2}V_s\}\rangle$$
$$-sr\langle\{G_r,H_-P_-^{-1/2}V_s\}\rangle\ . \tag{3.8}$$

It is useful to extend the G representation to objects

$$C_r=\langle z^{-r}[G_r,X_r^{(1/2)}]_\mp\rangle\ ,$$

where $X_r^{(J)}$ depends on r and has conformal spin

J relative to L_{2r}. This ensures that

$$[G_r, C_r]_\pm = \langle z^{-r}[L_{2r}, X_r^{(1/2)}]_-\rangle = 0 \ . \tag{3.9}$$

In order to obtain objects $X^{(1/2)}$ we note that the "covariant derivative" $D_{2rJ} \equiv \partial_\tau + 2rJ$ when acting on a conformal spin-J field[8] $X^{(J)}$ gives $D_{2rJ}X^{(J)} = X_r^{(J+1)}$, which is an r-*dependent* conformal spin-$(J+1)$ object *relative* to L_{2r}.

$$[z^{-r}L_{2r}, D_{2rJ}X^{(J)}] = z^r\left[z\frac{d}{dz} + 2r(J+1)\right]D_{2rJ}X^{(J)} \tag{3.10}$$

for each r.

For example let $X_r^{(1/2)} = (D_r H_-)P_-^{-1}V_r$, then

$$[G_r, \langle z^{-r}\{G_r, (D_r H_-)P_-^{-1}V_n\}\rangle] = 0 \ . \tag{3.11}$$

This gives the identity

$$[G_r, \langle 2rV_n + (\dot{P}_-P_-^{-1} - nH_-\dot{H}_-P_-^{-1})V_n\rangle] = 0 \ . \tag{3.12}$$

We can rewrite Eq. (3.7) as

$$[G_r, A_n^L + n\langle rV_n\rangle] = 0$$

and then use the identity above, Eq. (3.12), to get an r-*independent* expression for the longitudinal-physical-state operator, $A_n^{(+)}$:

$$A_n^{(+)} = \langle z^{-r}\{G_r, H_+V_n\}\rangle - \tfrac{1}{2}n\langle(\dot{P}_-P_-^{-1} - nH_-\dot{H}_-P_-^{-1})V_n\rangle. \tag{3.13}$$

We can find the correction to B_s^L in exactly the same way. The simplest choices for $X^{(1/2)}$ are $P_-^{1/2}V_s$ and $H_-(D_rH_-)(D_{2r}P_-)P_-^{-7/2}V_s$ which give the needed identities. One might have considered $H_-(D_rH_-)P_-^{-3/2}V_s$ or $(D_{2r}P_-)P_-^{-3/2}V_s$ but they give identities redundant with $P_-^{1/2}V_s$. The needed identities are

$$\langle z^{-r}[G_r, P_-^{1/2}V_s]\rangle = \tfrac{1}{2}r\langle H_-P_-^{-1/2}V_s\rangle - \tfrac{1}{2}\langle\partial_\tau(H_-P_-^{-1})P_-^{1/2}V_s\rangle$$

and

$$\begin{aligned}\langle z^{-r}[G_r, H_-(D_rH_-)(D_{2r}P_-)P_-^{-7/2}V_s]\rangle &= -2r^2\langle H_-P_-^{-3/2}V_s\rangle - 2rs\langle H_-P_-^{-1/2}V_s\rangle \\ &\quad + \langle[H_-\dot{H}_-\ddot{H}_-P_-^{-7/2} + \partial_\tau(H_-P_-^{-1})\dot{P}_-P_-^{-3/2}]V_s\rangle,\end{aligned} \tag{3.14}$$

which anticommute with G_r by construction. The terms $rH_-P_-^{-1/2}V_s$ and $r^2H_-P_-^{-3/2}V_s$ in Eq. (3.8) can thus be reexpressed as r-independent terms,

$$\begin{aligned}B_s^{(+)} = \langle z^{-r}&:[G_r, H_+H_-P_-^{-1/2}V_s]:\rangle \\ &-\tfrac{1}{8}\langle[H_-\dot{H}_-\ddot{H}_-P_-^{-7/2} + \partial_\tau(H_-P_-^{-1})\dot{P}_-P_-^{-3/2}]V_s\rangle \\ &+\tfrac{1}{16}\langle[H_-P_-^{-3/2} + 20\partial_\tau(H_-P_-^{-1})P_-^{1/2}]V_s\rangle \ .\end{aligned} \tag{3.15}$$

An unambiguous deductive procedure can be formulated for the correction terms to A_n^L and B_s^L based on their Poincaré properties. Poincaré translations require that any "photon" operator (e.g., V_n itself) that takes p_1 into $p_2 = p_1 + nk$ [$k = \tfrac{1}{2}(1,0,0,1)$] satisfy the condition

$$[p_0^{(+)}, V_n] = [\langle P_+\rangle, V_n] = -nV_n \ . \tag{3.16}$$

In particular the correction terms F_n ($A_n^{(+)} = A_n^L + F_n$) and E_s ($B_s^{(+)} = B_s^L + E_s$) must satisfy

$$\begin{aligned}F_n &= -\frac{1}{n}\langle[P_+, F_n]\rangle \ , \\ E_s &= -\frac{1}{s}\langle[P_+, E_s]\rangle \ .\end{aligned} \tag{3.17}$$

We can derive an explicit expression for $[P_+(\theta), F_n]$ (or $[P_+(\theta), E_s]$) by expressing gauge invariance in terms of the densities $P\cdot H$ and $P^2 + H\cdot\dot{H}$. For example,

$$\begin{aligned}&2[P_+, F_n]H_- + 2[H_+, F_n]P_- = \sum z^{-r}[G_r, A_n^L] \equiv g_n \ , \\ &2[P_+, F_n]P_- + [H_+, F_n]\dot{H}_- + H_-[\dot{H}_+, F_n] = \sum z^{-2r}[L_{2r}, A_n^L] \equiv l_n \ .\end{aligned} \tag{3.18}$$

Note that the known quantities g_n and l_n do *not* depend on r and that $z^{-r}\{G_r, g_n\} = l_n$. We can eliminate $[H_+, F_n]$ from the left-hand side and derive

$$[P_+, F_n] = g_n\dot{H}_-P_-^{-2} - \partial_\tau(g_nP_-^{-1})P_-^{-1}H_- + l_nP_-^{-1}(1 - H_-\dot{H}_-P_-^{-2}) \ , \tag{3.19}$$

and similarly for $[P_+, E_s]$. Thus by (3.17) we get $A_n^{(+)}$ (and $B_s^{(+)}$) simply from the Poincaré properties of the physical-state operators.

IV. CONCLUSION

Several new features are present in this construction:

(i) The introduction of the anticommuting operators B_r (or b_r) removes the tachyon on the leading trajectory ($\alpha_0 = 1$). Hence in the $G = +1$ sector or by displacing the "pion" trajectory ($G = -1$) as suggested by Halpern and Thorn, models with positive norms and no tachyons appear. Indeed giving positive signature to the leading trajectory results in a spectrum with *positive definite* norms and masses.

(ii) The transition operators A_n and B_n correspond to zero-mass "currents" with the quantum numbers of ρ, σ, and π, A_1, respectively. However, since only the zero-mass ρ occurs in the theory, we see that the existence of these generators with lightlike momenta does *not* require the existence of zero-mass particles.

(iii) The construction can be applied to the fermion sector of Ramond[15] by replacing H^μ by Γ^μ

throughout. Positivity for these states leaves the intercept on the leading fermion trajectory unconstrained.

Although we have not stressed the point, clearly the Lorentz covariance and closure properties of the algebra allows many of the generators to be derived from each other. For example given $B_r^{(+)}$ we have $A_{2r}^{(+)} \equiv (B_r^{(+)})^2$ and using appropriate Lorentz generators gives A_n^i and B_r^i. A systematic study of these relationships is underway.

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[5] A. Neveu and J. H. Schwarz, Nucl. Phys. B31, 86 (1971); A. Neveu, J. H. Schwarz, and C. B. Thorn, Phys. Letters 35B, 529 (1971).

[6] We use n, m, l for the integer indices on A_n, a_n, L_n etc., and r, s for the half-integer indices for the anticommuting operators B_r, b_r, G_r, etc.

[7] R. C. Brower and C. B. Thorn, Nucl. Phys. B31, 163 (1971); R. C. Brower and P. Goddard, *ibid.* B40, 437 (1972).

[8] The notation $\langle\cdots\rangle$ denotes the integral $\oint \cdots dz/2\pi i z$, or for $z = \exp(\tau + i\theta)$ the average over θ, $\int \cdots d\theta/2\pi$. A conformal spin-J field $X^{(J)}(z)$ obeys the algebra

$$[L_n, X^{(J)}] = z^n (z d/dz + nJ) X^{(J)} .$$

[9] Consequently,

$$Q^\mu = q_0^\mu - i\alpha_0^\mu \ln z + i \sum_{n\neq 0} \alpha_n^\mu z^{-n}/n ,$$

where $\alpha_0^\mu = p_0^\mu$ and $[q_0^\mu, p_0^\nu] = i g^{\mu\nu}$.

[10] The NSM amplitude is simply a modified beta function [as expressed by D. B. Fairlie, Durham report, 1971 (unpublished)]

$$\int \prod_i \frac{dz_i z_i^{-1/2}}{d^3\omega} \left\| \frac{p_i \cdot p_j}{z_i - z_j} \right\|^{1/2} \prod_{i\neq j} |z_i - z_j|^{p_i \cdot p_j} ,$$

where $\|a_{ij}\|^{1/2}$ is the square root of the determinant of the skew-symmetric matrix a_{ij} $(a_{ii} \equiv 0)$.

[11] E. Del Giudice and P. Di Vecchia, Nuovo Cimento 70A, 579 (1970).

[12] While completing this manuscript we received a report from J. H. Schwarz [Nucl. Phys. B46, 61 (1972)] with similar results.

[13] C. Rosenzweig, Lett. Nuovo Cimento 2, 927 (1971).

[14] K. A. Friedman and C. Rosenzweig, Nuovo Cimento 10A, 53 (1972).

[15] R. Ramond, Phys. Rev. D 3, 2415 (1971). R. C. B. wishes to thank Dr. Ramond for illuminating discussions on this point.