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Mechanism for Octet Enhancement in S-Wave Nonleptonic Decays

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The mechanism of Li and Pagels is used to discuss the S-wave nonleptonic decays of mesons and baryons. It implies that the decays $\Sigma^+ \to n\pi^+$ and $K^+ \to \pi^+\pi^0$ do not occur and thus the $\Delta I = \frac{1}{2}$ rule and the Lee-Sugawara relation hold. Moreover, it also yields a relation between the F/D ratio for nonleptonic hyperon decays and the F/D ratio for the semileptonic decays.

I. INTRODUCTION

We extend the threshold-dominance hypothesis of Li and Pagels to both meson and baryon matrix elements of the parity-conserving weak Hamiltonian $\mathcal{H}_w^{pc}(0)$. In other words, we assume that the techniques of Ref. 1 may also be applied to matrix elements of $\mathcal{H}_w^{pc}(0)$. We shall see that this implies octet enhancement for these matrix elements as well as a relation between the F/D ratio for nonleptonic hyperon decays [denoted by $(F/D)_w$] and the F/D ratio for semileptonic decays [denoted by $(F/D)_A$]. Upon combining the relation so obtained with that derived in Ref. 1 between the F/D ratio for the baryon octet mass splitting [denoted by $(F/D)_B$] and $(F/D)_A$, it then follows that

$$\left(\frac{F}{D}\right)_w = \left(\frac{F}{D}\right)_B = \frac{10}{3}\,\frac{(F/D)_A}{[1-3(F/D)_A{}^2]} = \frac{2}{3}\,\frac{m_N - m_\Xi}{m_\Lambda - m_\Sigma}\,, \tag{1.1}$$

in excellent agreement with experiment.

The technique here employed may be summarized as follows.[1] One assumes that both baryon and meson matrix elements of $\mathcal{H}_w^{pc}(0)$ obey an unsubtracted dispersion relation, so that

$$\langle X_\alpha|\mathcal{H}_w^{pc}(0)|X_\beta\rangle = \frac{1}{\pi}\int_{t_0}^\infty \frac{dt}{t}\,\mathrm{Im}\langle X_\alpha|\mathcal{H}_w^{pc}(0)|X_\beta\rangle\,. \tag{1.2}$$

(In the above, X_α is used to either denote a baryon B_α or a meson P_α.)

It is then stated that the dominant contribution to the dispersion integral in (1.2) is given by the two pseudoscalar-meson states in the threshold region. In this way the matrix element $\langle X_\alpha|\mathcal{H}_w^{pc}(0)|X_\beta\rangle$ is related to the integral up to a certain cutoff M of a bilinear form in the S-wave scattering amplitudes for $X_\alpha\bar{X}_\beta\to P_\gamma P_\delta$ and the matrix elements $\langle P_\gamma|\mathcal{H}_w^{pc}(0)|P_\delta\rangle$. One then calculates the amplitudes for $X_\alpha\bar{X}_\beta\to P_\gamma P_\delta$ in the SU(3) $\otimes$ SU(3) limit by computing the baryon-exchange contribution in pseudovector coupling theory (when $X_\alpha=B_\alpha$ and $\bar{X}_\beta=\bar{B}_\beta$) or by using Weinberg's[2] treatment of meson-meson scattering (when $X_\alpha=P_\alpha$, $\bar{X}_\beta=\bar{P}_\beta$). Furthermore, the threshold is lowered down to zero. In this fashion, the matrix elements $\langle X_\alpha|\mathcal{H}_w^{pc}(0)|X_\beta\rangle$ are thus expressed in terms of an arbitrary parameter M and the matrix elements $\langle P_\gamma|\mathcal{H}_w^{pc}(0)|P_\delta\rangle$. Considering the case $X_\alpha=B_\alpha$ and $X_\beta=B_\beta$, the baryon matrix elements $\langle B_\alpha|\mathcal{H}_w^{pc}(0)|B_\beta\rangle$ are thus expressed in terms of the meson matrix elements $\langle P_\gamma|\mathcal{H}_w^{pc}(0)|P_\delta\rangle$. To determine the latter one sets $X_\alpha=P_\alpha$ and $X_\beta=P_\beta$. One thus obtains a set of bootstrap-type equations for $\langle P_\alpha|\mathcal{H}_w^{pc}(0)|P_\beta\rangle$ which determine completely the ratio between any two of these matrix elements. Returning to the equations relating $\langle B_\alpha|\mathcal{H}_w^{pc}(0)|B_\beta\rangle$ to $\langle P_\alpha|\mathcal{H}_w^{pc}(0)|P_\beta\rangle$, the ratio between any two matrix elements $\langle B_\alpha|\mathcal{H}_w^{pc}(0)|B_\beta\rangle$ is then also completely determined.

II. NOTATION AND CALCULATION

Upon making use of partial conservation of axial-vector current (PCAC) and SU(2) invariance, the matrix elements for the S-wave nonleptonic hyperon decays may be written in the standard fashion[3] as

$$\langle B_\alpha\pi_\beta|\mathcal{H}_w^{pv}(0)|B_\gamma\rangle=-\frac{\sqrt{2}}{f_\pi}\,i\langle B_\alpha|[Q_5^\beta,\mathcal{H}_w^{pv}(0)]|B_\gamma\rangle$$
$$=-\frac{\sqrt{2}}{f_\pi}\,i\langle B_\alpha|[Q^\beta,\mathcal{H}_w^{pc}(0)]|B_\beta\rangle$$
$$\equiv iA(B_\gamma\to B_\alpha\pi_\beta),\quad(2.1)$$

where $\mathcal{H}_w^{pv}(0)$ and $\mathcal{H}_w^{pc}(0)$ denote the parity-violating and parity-conserving parts of the nonleptonic current-current weak Hamiltonian. From Eq. (2.1) one then obtains the well-known[3] sum rules

$$A(\Lambda_-^0)+\sqrt{2}\,A(\Lambda_0^0)=0\,,$$
$$A(\Xi_-^-)-\sqrt{2}\,A(\Xi_0^0)=0\,,\quad(2.2)$$

and

$$A(\Sigma_-^-)-\sqrt{2}\,A(\Sigma_0^+)+A(\Sigma_+^+)=0\,.\quad(2.3a)$$

If one furthermore assumes SU(3) for $\langle B_\alpha|\mathcal{H}_w^{pc}(0)|B_\beta\rangle$, then the relation[3]

$$2A(\Xi_-^-)=A(\Lambda_-^0)+\sqrt{3}\,A(\Sigma_0^+)-(\tfrac{3}{2})^{1/2}A(\Sigma_+^+)\quad(2.3b)$$

also follows.

Equations (2.2) are nothing else but the $\Delta I=\frac{1}{2}$ rule for the Λ and Ξ decays. On the other hand, Eq. (2.3a) would be equivalent to the $\Delta I=\frac{1}{2}$ rule for Σ decays provided that $A(\Sigma_+^+)=0$, in which case Eq. (2.3b) furthermore reduces to the Lee-Sugawara relation. We shall show that under our present assumptions $A(\Sigma_+^+)=0$ in fact follows so that the Lee-Sugawara relation and the $\Delta I=\frac{1}{2}$ rule hold. Moreover, we shall see that Eq. (2.3b) [with $A(\Sigma_+^+)=0$] follows from the Li and Pagels mechanism without needing to make any SU(3) assumptions about the matrix elements $\langle B_\alpha|\mathcal{H}_w^{pc}(0)|B_\beta\rangle$.

We next make use of Eq. (1.2) and the procedure discussed in Sec. I to arrive at

$$\langle\Xi^0|\mathcal{H}_w^{pc}(0)|\Lambda\rangle=(\text{const})[\langle\pi^+|\mathcal{H}_w^{pc}(0)|K^+\rangle(\tfrac{2}{3})^{1/2}(3+8\alpha^2-12\alpha)+\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle(3+8\alpha^2-12\alpha)/\sqrt{3}$$
$$+\langle\eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle(-3+4\alpha)]\,,$$

$$\langle\Lambda|\mathcal{H}_w^{pc}(0)|n\rangle=(\text{const})[\langle\pi^+|\mathcal{H}_w^{pc}(0)|K^+\rangle(\tfrac{2}{3})^{1/2}(3-4\alpha^2)+\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle(3-4\alpha^2)/\sqrt{3}$$
$$+\langle\eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle(-3-4\alpha^2+8\alpha)]\,,$$

$$\langle\Sigma^0|\mathcal{H}_w^{pc}(0)|n\rangle=(\text{const})[\langle\pi^+|\mathcal{H}_w^{pc}(0)|K^+\rangle(3+4\alpha^2-8\alpha)\sqrt{2}+\langle\pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle\tfrac{1}{3}(-3+4\alpha^2)\quad(2.4)$$
$$+\langle\eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle(3+4\alpha^2-8\alpha)/\sqrt{3}\,]\,,$$

and

$$\langle \Sigma^+ |\mathcal{H}_w^{pc}(0)|p\rangle = (\text{const})[\langle \pi^+ |\mathcal{H}_w^{pc}(0)|K^+\rangle \tfrac{1}{3}(6+16\alpha^2-24\alpha)+\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle(3-8\alpha+4\alpha^2)\sqrt{2}$$
$$+\langle \eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle(\tfrac{2}{3})^{1/2}(3+4\alpha^2-8\alpha)],$$

where α is related to the F/D ratio for semileptonic hyperon decays [denoted by $(F/D)_A$] by $(F/D)_A=(1-\alpha)/\alpha$.

In order to compute the meson matrix elements in Eq. (2.4) we once again make use of Eq. (1.2) and the procedure discussed in Sec. I. In this way we obtain

$$\langle \pi^+ |\mathcal{H}_w^{pc}(0)|K^+\rangle = \tfrac{1}{4}a\langle \pi^+ |\mathcal{H}_w^{pc}(0)|K^+\rangle + \frac{3}{4}\frac{a}{\sqrt{2}}\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle + \frac{\sqrt{3}\,a}{4\sqrt{2}}\langle \eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle,$$

$$\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle = \frac{3}{4}\frac{a}{\sqrt{2}}\langle \pi^+ |\mathcal{H}_w^{pc}(0)|K^+\rangle - \tfrac{1}{8}a\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle + \frac{\sqrt{3}}{8}a\langle \eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle, \tag{2.5}$$

and

$$\langle \eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle = \frac{\sqrt{3}}{4\sqrt{2}}a\langle \pi^+ |\mathcal{H}_w^{pc}(0)|K^+\rangle + \frac{\sqrt{3}}{8}a\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle - \tfrac{3}{8}a\langle \eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle,$$

where a (which is related to the cutoff) must be positive in order to reflect the attractive character of meson-meson scattering in the chiral limit.[1] In order for Eq. (2.5) to have a nontrivial solution, the determinant of the coefficients must vanish. This leads to the cubic equation

$$3a^3+8a^2-4a-16=0 \tag{2.6}$$

whose solutions are

$$a_{1,2}=-2 \tag{2.7}$$

and

$$a_3=\tfrac{4}{3}. \tag{2.8}$$

Since $a>0$, it then follows that $a=\frac{4}{3}$ is the only physically acceptable solution of (2.5). Upon substituting the value for a from Eq. (2.8) into Eq. (2.5) one thus arrives at

$$\langle \pi^+ |\mathcal{H}_w^{pc}(0)|K^+\rangle = \sqrt{3}\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle \tag{2.9}$$

and

$$\langle \eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle = \frac{1}{\sqrt{3}}\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle. \tag{2.10}$$

Note that Eqs. (2.9) and (2.10) imply octet dominance of $\langle P_\alpha|\mathcal{H}_w^{pc}(0)|P_\beta\rangle$ [i.e., $\langle P_\alpha|\mathcal{H}_w^{pc}(0)|P_\beta\rangle$ is proportional to $d_{6\alpha\beta}$]. Since the matrix elements in Eqs. (2.9) and (2.10) are octet-dominant, the $\Delta I=\frac{1}{2}$ sum rule for the $K\to\pi\pi$ decays then follows by symmetrically reducing one pion. In fact, carrying out this calculation one obtains the desired sum rule, i.e.,

$$a(K^+\to\pi^+\pi^0)=0$$

and (2.11)

$$a(K^0\to\pi^+\pi^-)+\sqrt{2}\,a(K^0\to\pi^0\pi^0)=0,$$

where a is defined by

$$\langle \pi^\alpha\pi^\beta|\mathcal{H}_w^{pc}(0)|K^\delta\rangle = i\,a(K^\delta\to\pi^\alpha\pi^\beta). \tag{2.12}$$

Incidentally, note that Eq. (2.9) could directly be derived from PCAC in both the quark model [where $\mathcal{H}_w^{pc}(0)$ is proportional to the scalar density u_6] and the current-current models. On the other hand, Eq. (2.10) follows from PCAC in the quark model but not in current-current models. In fact, in the latter models, PCAC for the matrix element $\langle \eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle$ would imply

$$\langle \eta^0|\mathcal{H}_w^{pc}(0)|K^0\rangle = -\sqrt{3}\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle \tag{2.13}$$

rather than Eq. (2.10). However, since in the real world both η and K^0 are considerably heavier than the pion, and since, furthermore, η does not decay into $K\pi$ [so that the matrix element $\langle \eta|\mathcal{H}_w^{pc}(0)|K^0\rangle$ does not have any immediate physical interpretation] one could argue that the result (2.13) does not actually need to hold even if $\mathcal{H}_w^{pc}(0)$ is of the current-current type.

Having derived our results for the mesons, we now substitute Eqs. (2.9) and (2.10) into (2.4). We obtain

$$\langle \Xi^0|\mathcal{H}_w^{pc}(0)|\Lambda\rangle = \frac{2}{\sqrt{3}}(\text{const})\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle \times(3-16\alpha+12\alpha^2),$$

$$\langle \Lambda|\mathcal{H}_w^{pc}(0)|n\rangle = \frac{2}{\sqrt{3}}(\text{const})\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle \times(3+4\alpha-8\alpha^2),$$

$$\langle \Sigma^0|\mathcal{H}_w^{pc}(0)|n\rangle = \tfrac{2}{3}(\text{const})\langle \pi^0|\mathcal{H}_w^{pc}(0)|K^0\rangle \times(9-28\alpha+16\alpha^2), \tag{2.14}$$

and

$$\langle \Sigma^+ | \mathcal{H}_w^{pc}(0) | p \rangle = \frac{4}{3\sqrt{2}} (\text{const}) \langle \pi^0 | \mathcal{H}_w^{pc}(0) | K^0 \rangle \times (9 - 28\alpha + 16\alpha^2).$$

From Eqs. (2.1) and (2.14) it may then be immediately seen that also the baryon matrix elements of $\mathcal{H}_w^{pc}(0)$ are octet-dominant, since

$$A(\Sigma_+^+) = -\frac{1}{f_\pi}[-\langle p | \mathcal{H}_w^{pc}(0) | \Sigma^+ \rangle + \sqrt{2} \langle n | \mathcal{H}_w^{pc}(0) | \Sigma^0 \rangle] = 0. \quad (2.15)$$

Furthermore, from Eqs. (2.2) and (2.3a) [with $A(\Sigma_+^+) = 0$] we see that the $\Delta I = \frac{1}{2}$ rule for the S-wave nonleptonic hyperon decays also follows. Moreover, from Eqs. (2.14) we also immediately obtain

$$2\langle \Xi^0 | \mathcal{H}_w^{pc}(0) | \Lambda \rangle + \langle \Lambda | \mathcal{H}_w^{pc}(0) | n \rangle = \sqrt{3} \langle \Sigma^0 | \mathcal{H}_w^{pc}(0) | n \rangle, \quad (2.16)$$

which upon making use of $A(\Sigma_+^+) = 0$ may also be written as

$$2A(\Xi_-^-) = A(\Lambda_-^0) + \sqrt{3} A(\Sigma_0^+), \quad (2.17)$$

and is nothing else but the Lee-Sugawara relation for the S-wave nonleptonic hyperon decays.[3] Thus this relation follows immediately from the Li and Pagels mechanism without needing to make any SU(3) assumptions about the matrix elements $\langle B_\alpha | \mathcal{H}_w^{pc}(0) | B_\beta \rangle$.

If we furthermore assume SU(3) for $\langle B_\alpha | \mathcal{H}_w^{pc}(0) | B_\beta \rangle$, Eqs. (2.14) then also allow us to compute the F/D ratio for the nonleptonic hyperon decays in terms of the F/D ratio for the semileptonic hyperon decays. For this purpose, we write, for example,

$$\langle \Xi^0 | \mathcal{H}_w^{pc}(0) | \Lambda \rangle = -(\tfrac{2}{3})^{1/2} (D_w - 3F_w)$$

and

$$\langle \Lambda^0 | \mathcal{H}_w^{pc}(0) | n \rangle = (\tfrac{2}{3})^{1/2} (D_w + 3F_w). \quad (2.18)$$

Upon combining Eqs. (2.14) and (2.18), we immediately obtain

$$\left(\frac{F}{D}\right)_w = \frac{10}{3} \frac{(F/D)_A}{1 - 3(F/D)_A{}^2}, \quad (2.19)$$

with

$$\left(\frac{F}{D}\right)_A = \frac{1-\alpha}{\alpha}.$$

Making use of Eq. (2.19) together with the relation obtained in Ref. 1 for the F/D ratio for the baryon octet mass splitting [denoted by $(F/D)_B$] in terms of $(F/D)_A$, we then arrive at

$$\left(\frac{F}{D}\right)_w = \left(\frac{F}{D}\right)_B = \frac{10}{3} \frac{(F/D)_A}{1 - 3(F/D)_A{}^2} = \frac{2}{3} \frac{m_N - m_\Xi}{m_\Lambda - m_\Sigma}. \quad (2.20)$$

Note added in proof. After submission of this paper we received a report by Riazuddin in which similar ideas are discussed.

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