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Mass Differences in a Unified Theory of Weak and Electromagnetic Interactions*

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The soft-pion and soft-kaon mass differences are calculated in the Weinberg model of weak and electromagnetic interactions. Both are found to be finite without using the second spectral-function sum rule. In addition several comments are made about mass differences of massive hadrons.

I. INTRODUCTION

The problem of the divergences which arise when one attempts to calculate the mass difference between particles that belong to the same isotopic multiplet has been with us for some time. Recently, models have been constructed which unify the electromagnetic and weak interactions into a renormalizable theory.[1-3] It might be hoped that such models would remove the divergences in the mass differences. Some work has been done along these lines.[4,5]

From a purely phenomenological point of view it is clear that before one uses these models to calculate mass differences one must first answer the question: How do the masses of the hadrons arise? The masses of the leptons and intermediate mesons are generated through the couplings of these particles to the scalar mesons ϕ when the symmetry is spontaneously broken. If the masses of the hadrons are generated by the same mechanism then there are two possibilities. If the zeroth-order masses of the members of an isotopic multiplet are unequal then the mass difference need not be finite.[4] If, on the other hand, the zeroth-order masses are equal, then we need to know the precise hadron-ϕ coupling in order to calculate the self-energy terms involving the ϕ.

There does seem to exist one case, however, where the precise nature of the hadron-ϕ coupling can be avoided. That case is the calculation of mass differences of massless hadrons. Of course, the fact that the hadrons have no zeroth-order mass does not necessarily mean there are no hadron-ϕ couplings, but at least there is no need, *a priori*, for such couplings. We will return below to the question of which hadron-ϕ couplings we are assuming to be absent.

The calculation of the electromagnetic mass difference of soft pions was done successfully several years ago.[6] The electromagnetic mass difference of soft K mesons was also calculated.[7] Both calculations depended for convergence on the validity of both the first and second spectral-function sum rules.[8] As is well known, the second sum rule is model-dependent,[9] and for strangeness-changing currents is even contradicted experimentally.[10]

In this paper we calculate the soft-pion and soft-kaon mass differences in the $SU_L(2)\times Y_L$ gauge model of Weinberg.[1] In order to incorporate the hadrons, we shall consider the SU(4) quark scheme.[11,12] As is well known, the simpler SU(3) quark model leads to unobserved neutral strangeness-changing currents. Actually, for the (zero-mass) pion-mass-difference problem, we may, if we wish, disregard the strange hadrons altogether and use the much simpler model discussed by Weinberg,[11] which incorporates the nonstrange hadrons only. In fact, for the pion-mass-difference problem it is easy to see that our results in this model are the same as in the SU(4) quark model. For the kaon problem, the consideration of strange hadrons is obviously essential, and as mentioned we choose to work with the SU(4) quark model. Several other models exist in the literature, but we believe that the SU(4) quark model offers a rather simple and instructive example for the study undertaken here.

In order to be more precise about the role of the hadron-ϕ couplings, we would like to present the following consideration. In the SU(4) quark model, we take, to start with, the strong-interaction Lagrangian density to be $SU(4)\times SU(4)$-invariant in the Nambu-Goldstone sense. We then assume that the spontaneous breaking of the $SU_L(2)\times Y_L$ symmetry, which arises when ϕ acquires a nonvanishing vacuum expectation value, also leads to terms, through the hadron-ϕ couplings, which break the chiral SU(4) symmetry of strong interactions. The advantage of this mechanism,[11,13] is that the spontaneous breaking of $SU_L(2)\times Y_L$ symmetry itself generates the explicit terms that break the Nambu-Goldstone chiral symmetry of strong interactions, and leads to nonzero masses for pseudoscalar mesons, consistent with our usual ideas.

Obviously, the nature of these symmetry-breaking terms depends on the hadron-ϕ couplings introduced in the theory. If the pions are considered massless, we may drop any pion-ϕ coupling and require the strong interactions to be invariant under chiral SU(2) symmetry. To be sure, other suitable hadron-ϕ couplings will exist, such that the spontaneous symmetry breaking of $SU_L(2)\times Y_L$ effectively reduces the symmetry of strong interactions from $SU(4)\times SU(4)$ to $SU(2)\times SU(2)$. As usual, the chiral SU(2) symmetry is taken to be realized in the Nambu-Goldstone fashion, with zero-mass pions, and with a vacuum state for strong interactions invariant under SU(2), the usual isospin group. Similarly, in the case of the massless kaon, we require the strong-interaction symmetry to be broken only to $SU(3)\times SU(3)$. This may be achieved by dropping the pseudoscalar-meson–ϕ couplings, but as before, we cannot preclude the existence of other appropriate hadron-ϕ couplings which eventually leave the strong Lagrangian $SU(3)\times SU(3)$ invariant. Consistent with our usual ideas, we shall assume that the SU(3) $\times$SU(3) symmetry is realized in the Nambu-Goldstone manner, with the vacuum state invariant under SU(3) symmetry.

Indeed, $SU_L(2)\times Y_L$-invariant models can be written down where, through the introduction of appropriate hadron-ϕ couplings, one can reduce the strong-interaction symmetry to any arbitrary level. For problems involving massless pions and kaons, the important simplification is the absence of pseudoscalar-meson–ϕ couplings. The weak and electromagnetic interactions of hadrons may then be expressed by [11]

$$\mathcal{L} = \frac{1}{2\sqrt{2}}\, g(W_\mu J_W^{\mu\dagger} + W_\mu^\dagger J_W^\mu) + \tfrac{1}{2}(g^2+g'^2) Z_\mu J_Z^\mu - eA_\mu J_{\text{em}}^\mu. \quad (1)$$

In the $SU(4)\times SU(4)$ model the electromagnetic current is given by

$$J_{\text{em}}^\mu = V_3^\mu + (\tfrac{1}{3})^{1/2} V_8^\mu - (\tfrac{2}{3})^{1/2} V_{15}^\mu. \quad (2)$$

The currents which couple to the Z and W mesons are given by

$$J_Z^\mu = \beta[V_3^\mu + (\tfrac{1}{3})^{1/2} V_8^\mu - (\tfrac{2}{3})^{1/2} V_{15}^\mu] - [A_3^\mu + (\tfrac{1}{3})^{1/2} A_8^\mu - (\tfrac{2}{3})^{1/2} A_{15}^\mu], \quad (3)$$

where β is a function of the Weinberg angle,

$$\beta = 1 - 2\sin^2\theta_W, \quad (4)$$

and

$$J_W^\mu = \cos\theta_C (V_{1+i2}^\mu - A_{1+i2}^\mu) + \sin\theta_C (V_{4+i5}^\mu - A_{4+i5}^\mu) - \sin\theta_C (V_{11+i12}^\mu - A_{11+i12}^\mu) + \cos\theta_C (V_{13+i14}^\mu - A_{13+i14}^\mu), \quad (5)$$

where θ_C is the Cabibbo angle. The SU(4) quarks and λ matrices are given in the Appendix.

The coupling constants g, g', and e satisfy the usual relations

$$e = \frac{gg'}{(g^2+g'^2)^{1/2}}, \quad \frac{g'}{g} = \tan\theta_W, \qquad \frac{G}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{g^2+g'^2}{8m_Z^2}, \quad (6)$$

where G is the Fermi coupling constant.

In Secs. II and III, we use the interaction (1) with the currents given by Eqs. (2), (3), and (5) to study the π^+-π^0 and K^+-K^0 mass differences in the limit of zero pion and kaon masses. The main result of our investigation is that these mass differences are finite in second order, without using the second Weinberg sum rules. The π^+-π^0 mass-difference problem for massive pions is studied in Sec. IV. For this purpose we consider a special model for introducing the pion-ϕ coupling, as was first suggested by Weinberg [1] and recently studied by Palmer.[13] To second order the π^+-π^0 mass difference is found to be finite again. In the end, we make some comments on mass differences of other hadrons.

II. MASS DIFFERENCE OF SOFT PIONS

The electromagnetic mass difference between the charged and neutral pions is given by the familiar expression, correct to second order (hereafter understood),

$$(m_+{}^2 - m_0{}^2)_\gamma = \frac{e^2}{4\pi}\frac{1}{(2\pi)^3}\int \frac{d^4q}{q^2}\left(g^{\mu\nu} - \lambda\frac{q^\mu q^\nu}{q^2}\right) V_{\mu\nu}^{33}, \quad (7)$$

where λ is arbitrary and $V_{\mu\nu}^{ab}$ is the covariant time-ordered product

$$V_{\mu\nu}^{ab} \equiv \int d^4x\, e^{iq\cdot x}\,[\langle\pi^+|(V_\mu^a(x)V_\nu^b(0))_+|\pi^+\rangle - (\pi^+ \to \pi^0)]. \quad (8)$$

The contribution to the mass difference from the Z-meson interaction is

$$(m_+{}^2 - m_0{}^2)_Z = \frac{g^2+g'^2}{4}\frac{1}{4\pi}\frac{1}{(2\pi)^3} \times \int \frac{d^4q}{q^2-m_Z{}^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{m_Z{}^2}\right)(\beta^2 V^{33}_{\mu\nu} + A^{33}_{\mu\nu}), \tag{9}$$

where $A^{ab}_{\mu\nu}$ is the analog of $V^{ab}_{\mu\nu}$ for axial-vector currents, and β is given by (4). It is important to notice that

$$\tfrac{1}{4}(g^2+g'^2)(\beta^2-1) = -e^2. \tag{10}$$

We will use this repeatedly. Finally there is the contribution from the W meson

$$(m_+{}^2 - m_0{}^2)_W = \frac{g^2}{4}\frac{\cos^2\theta_C}{4\pi}\frac{1}{(2\pi)^3} \times \int \frac{d^4q}{q^2-m_W{}^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{m_W{}^2}\right)(V^{+-}_{\mu\nu} + A^{+-}_{\mu\nu}). \tag{11}$$

We have, of course, kept only those parts of the currents (2), (3), and (5) which can be combined to give a change in isospin of two. It is now a trivial exercise in Clebsch-Gordan coefficients to show that for this $\Delta I = 2$ part

$$V^{+-}_{\mu\nu} = -V^{33}_{\mu\nu}, \quad A^{+-}_{\mu\nu} = -A^{33}_{\mu\nu}. \tag{12}$$

Now we reduce in the pions, take them to be soft, use partial conservation of axial-vector current (PCAC) in the form

$$\pi^a(x) = \frac{1}{F_\pi m_\pi{}^2}\partial_\mu A^\mu_a(x), \tag{13}$$

and use the equal-time commutation relations of the local chiral SU(2)×SU(2) current algebra. We find

$$V^{33}_{\mu\nu}(q) = -A^{33}_{\mu\nu}(q) \tag{14a}$$

$$= \frac{2}{F_\pi{}^2}[\Delta^{V(3,3)}_{\mu\nu}(q) - \Delta^{A(3,3)}_{\mu\nu}(q)], \tag{14b}$$

where

$$\Delta^{V(a,b)}_{\mu\nu}(q) \equiv \int d^4x\, e^{iq\cdot x}\langle 0|(V^a_\mu(x)V^b_\nu(0))_+|0\rangle. \tag{15}$$

We see immediately that the contribution to the mass difference from the W meson is zero, while the contribution from the Z meson is proportional to $-e^2$.

Using the spectral representations of the (covariant) propagator functions

$$\Delta^V_{\mu\nu}(q) = -i\int_0^\infty dm^2 \frac{\rho^V(m^2)}{q^2-m^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m^2}\right), \tag{16a}$$

$$\Delta^A_{\mu\nu}(q) = -i\int_0^\infty dm^2 \frac{\rho^A(m^2)}{q^2-m^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m^2}\right) + iF_\pi{}^2\frac{q_\mu q_\nu}{q^2} \tag{16b}$$

and substituting these into (7) and (9), we have

$$(m_+{}^2 - m_0{}^2)_\gamma = -\frac{3ie^2}{(2\pi)^4}\frac{1}{F_\pi{}^2}\int\frac{d^4q}{q^2}\left[\int dm^2\frac{\rho^V(m^2)-\rho^A(m^2)}{m^2} - F_\pi{}^2\right] - \frac{3ie^2}{(2\pi)^4}\frac{1}{F_\pi{}^2}\int\frac{d^4q}{q^2}\int dm^2\frac{\rho^V(m^2)-\rho^A(m^2)}{q^2-m^2} \tag{17}$$

and

$$(m_+{}^2 - m_0{}^2)_Z = \frac{ie^2}{(2\pi)^4}\frac{1}{F_\pi{}^2}\int\frac{d^4q}{m_Z{}^2}\left[\int dm^2\frac{\rho^V(m^2)-\rho^A(m^2)}{m^2} - F_\pi{}^2\right] + \frac{3ie^2}{(2\pi)^4}\frac{1}{F_\pi{}^2}\int\frac{d^4q}{q^2-m_Z{}^2}\int dm^2\frac{\rho^V(m^2)-\rho^A(m^2)}{q^2-m^2}. \tag{18}$$

The first term in both (17) and (18) is zero by the first spectral-function sum rule.[8] The total mass difference is then

$$m_+{}^2 - m_0{}^2 = \frac{3ie^2}{(2\pi)^4}\frac{1}{F_\pi{}^2}\int\frac{d^4q}{q^2}\frac{m_Z{}^2}{q^2-m_Z{}^2} \times \int dm^2\frac{\rho^V(m^2)-\rho^A(m^2)}{q^2-m^2}, \tag{19}$$

which is finite *without the use of the second spectral-function sum rule.* Note in particular the role of the Z contribution. If this contribution were not present, the finiteness of (17) would require both the first and the second spectral-function sum rules.

We may approximate the spectral functions with ρ and A_1 poles

$$\rho^V(m^2) = g_\rho{}^2\delta(m^2 - m_\rho{}^2), \tag{20a}$$

$$\rho^A(m^2) = g_A{}^2\delta(m^2 - m_A{}^2). \tag{20b}$$

The mass difference is then

$$m_+{}^2 - m_0{}^2 = \frac{3e^2}{4\pi}\,\frac{1}{F_\pi{}^2}\left[\frac{g_A{}^2}{4\pi}\,\frac{m_Z{}^2}{m_Z{}^2 - m_A{}^2}\ln\frac{m_A{}^2}{m_Z{}^2} - \frac{g_\rho{}^2}{4\pi}\,\frac{m_Z{}^2}{m_Z{}^2 - m_\rho{}^2}\ln\frac{m_\rho{}^2}{m_Z{}^2}\right]. \tag{21}$$

If we use the content of the second spectral-function sum rule to set $m_A{}^2 = 2m_\rho{}^2$ and $g_A{}^2 = g_\rho{}^2$ and in addition use $g_\rho{}^2 \simeq 2M_\rho{}^2F_\pi{}^2$, we recover the mass difference of Ref. 6 plus a negligible correction,

$$m_+{}^2 - m_0{}^2 = \frac{3e^2}{4\pi}\,\frac{m_\rho{}^2}{2\pi}\left[\ln 2 + O\left(\frac{m_\rho{}^2}{m_Z{}^2}\ln\frac{m_Z{}^2}{m_\rho{}^2}\right)\right]. \tag{22}$$

III. MASS DIFFERENCE OF SOFT K MESONS

The contribution from the electromagnetic interaction to the $\Delta I = 1$ mass difference between the charged and neutral K mesons is

$$(m_{K^+}{}^2 - m_{K^0}{}^2)_\gamma = \frac{e^2}{4\pi}\,\frac{1}{(2\pi)^3}\int \frac{d^4q}{q^2}\left(g^{\mu\nu} - \lambda\,\frac{q^\mu q^\nu}{q^2}\right)\left[(\tfrac{1}{3})^{1/2}V^{3,8}_{\mu\nu}(q) - (\tfrac{2}{3})^{1/2}V^{3,15}_{\mu\nu}(q)\right], \tag{23}$$

where we redefine $V^{ab}_{\mu\nu}$ as

$$V^{ab}_{\mu\nu}(q) = \int d^4x\, e^{iq\cdot x}\left[\langle K^+|(V^a_\mu(x)V^b_\nu(0))_+|K^+\rangle - (K^+ \rightarrow K^0)\right]. \tag{24}$$

The interaction with the Z meson contributes

$$(m_{K^+}{}^2 - m_{K^0}{}^2)_Z = \frac{g^2 + g'^2}{4}\,\frac{1}{4\pi}\,\frac{1}{(2\pi)^3}\int \frac{d^4q}{q^2 - m_Z{}^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_Z{}^2}\right)$$
$$\times\left\{\beta^2\left[(\tfrac{1}{3})^{1/2}V^{3,8}_{\mu\nu}(q) - (\tfrac{2}{3})^{1/2}V^{3,15}_{\mu\nu}(q)\right] + \left[(\tfrac{1}{3})^{1/2}A^{3,8}_{\mu\nu}(q) - (\tfrac{2}{3})^{1/2}A^{3,15}_{\mu\nu}(q)\right]\right\}. \tag{25}$$

Finally, using the W-meson current given by (5), we have

$$(m_{K^+}{}^2 - m_{K^0}{}^2)_W = \frac{g^2}{4}\,\frac{1}{4\pi}\,\frac{1}{(2\pi)^3}\int \frac{d^4q}{q^2 - m_W{}^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W{}^2}\right)$$
$$\times\left\{\sin^2\theta_C\left[V^{4+i5,4-i5}_{\mu\nu}(q) + A^{4+i5,4-i5}_{\mu\nu}(q) + V^{11+i12,11-i12}_{\mu\nu}(q) + A^{11+i12,11-i12}_{\mu\nu}(q)\right]\right\}, \tag{26}$$

where there is no contribution from the term $V^{1+i2,1-i2}_{\mu\nu} - A^{1+i2,1-i2}_{\mu\nu}$ in Eq. (5) because the isovectors $I=1$, $I_3=-1$ and $I=1$, $I_3=+1$ cannot be combined symmetrically to give $I=1$, $I_3=0$.

We now use PCAC for the K mesons, taking them to be soft, and use the local SU(3)×SU(3) commutation relations. The photon contribution comes out proportional to

$$\frac{e^2}{F_K{}^2}\left(\Delta^{V(3,3)}_{\mu\nu} + \Delta^{V(8,8)}_{\mu\nu} - 2\Delta^{A(4,4)}_{\mu\nu} - \sqrt{2}\,\Delta^{V(8,15)}_{\mu\nu}\right), \tag{27}$$

where we neglect the σ terms in the commutators.[7] This is, of course, correct only if chiral SU(3) is exact.

Given the photon result it is easy to see from (2) and (3) that the Z contribution must be proportional to

$$\frac{g^2 + g'^2}{4F_K{}^2}\left[\beta^2\left(\Delta^{V(3,3)}_{\mu\nu} + \Delta^{V(8,8)}_{\mu\nu} - 2\Delta^{A(4,4)}_{\mu\nu} - \sqrt{2}\,\Delta^{V(8,15)}_{\mu\nu}\right) + \left(\Delta^{A(3,3)}_{\mu\nu} + \Delta^{A(8,8)}_{\mu\nu} - 2\Delta^{V(4,4)}_{\mu\nu} - \sqrt{2}\,\Delta^{A(8,15)}_{\mu\nu}\right)\right]. \tag{28}$$

The W-meson contribution is

$$\frac{g^2}{4F_K{}^2}\left[\sin^2\theta_C\left(\Delta^{V(3,3)}_{\mu\nu} - 3\Delta^{V(8,8)}_{\mu\nu} + 2\Delta^{V(4,4)}_{\mu\nu} + \Delta^{A(3,3)}_{\mu\nu} - 3\Delta^{A(8,8)}_{\mu\nu} + 2\Delta^{A(4,4)}_{\mu\nu}\right.\right.$$
$$\left.\left. - 2\Delta^{V(11,11)}_{\mu\nu} - 2\Delta^{A(11,11)}_{\mu\nu} + 2\Delta^{V(13,13)}_{\mu\nu} + 2\Delta^{A(13,13)}_{\mu\nu}\right)\right]. \tag{29}$$

Now, as discussed in the Introduction, for massless kaons we consider the strong interactions to be SU(3)×SU(3)-invariant, with the vacuum state symmetric under SU(3). Then

$$\begin{aligned}\Delta_{\mu\nu}^{V(3,3)} &= \Delta_{\mu\nu}^{V(8,8)} \\ &= \Delta_{\mu\nu}^{V(4,4)}\,, \\ \Delta_{\mu\nu}^{A(3,3)} &= \Delta_{\mu\nu}^{A(8,8)} \\ &= \Delta_{\mu\nu}^{A(4,4)}\,, \\ \Delta_{\mu\nu}^{V(8,15)} &= 0 \\ &= \Delta_{\mu\nu}^{A(8,15)}\,, \\ \Delta_{\mu\nu}^{V(11,11)} &= \Delta_{\mu\nu}^{V(13,13)}\,, \\ \Delta_{\mu\nu}^{A(11,11)} &= \Delta_{\mu\nu}^{A(13,13)}\,. \end{aligned} \tag{30}$$

Thus the W-meson contribution is again zero. The photon contribution is

$$\frac{2e^2}{F_K^{\,2}}(\Delta_{\mu\nu}^{V(3,3)} - \Delta_{\mu\nu}^{A(3,3)})\,, \tag{31}$$

which, using (4), is the negative of the Z contribution. Using the SU(3) relation $F_\pi = F_K$, the soft-K mass difference is precisely equal to the soft-π mass difference. Of course, this must be true because we have assumed exact SU(3).[14] Note in particular that without the Z contribution the photon contribution itself would not lead to a finite result unless we assume the second spectral-function sum rule for the SU(3)×SU(3) group.

IV. MASS DIFFERENCE OF PHYSICAL PIONS

As we discussed in the Introduction, any attempt to calculate the mass difference of physical (massive) particles must specify the precise method of generating the masses themselves and therefore the interactions of the particles with the scalar mesons, ϕ. This general question is beyond the scope of the present work but we would like to make a few comments about the mass difference of physical pions.

If the interactions of the pseudoscalar mesons with ϕ can be neglected, then the convergence of the pion mass difference can be seen from (7), (9), (11), and (12) and the BJL limit to depend on the following commutators having no $\Delta I = 2$ parts:

$$\delta(x_0)\{[\partial_0 V_3^\mu(x), V_3^\nu(0)] + [\partial_0 A_3^\mu(x), A_3^\nu(0)]\}\,, \tag{32a}$$

$$\delta(x_0)[\partial_0 D_3(x), D_3(0)]\,, \quad D_3(x) \equiv \partial_\mu A_3^\mu\,. \tag{32b}$$

Now we could attempt to calculate the pion mass difference from (7), (9), and (11) by using the hard-pion model of Gerstein, Schnitzer, and Weinberg.[15] But such a calculation would be divergent because the hard-pion model is consistent with the algebra-of-fields expression for (32a),[16] and in the field-algebra model this commutator has a $\Delta I = 2$ piece. We could, of course, modify the hard-pion model so that the commutator had no $\Delta I = 2$ part and then use the modified model to calculate the mass difference. The results would then be finite but, since the model could be modified in different ways,[17] the answer would not be unique.

We cannot calculate the 4th-order contribution to the mass difference of soft pions for the same reasons because we need a hard-pion model to evaluate some of the 4th-order terms.[18] For example, the 4th-order off-mass-shell term involves the derivative with respect to the mass of the second-order mass shift. Since in the usual hard-pion model the divergent terms in the mass shift must be proportional to the mass, this 4th-order off-shell term would be divergent.

If we choose to consider an explicit model where the pions do not couple to the scalar mesons we could take the SU(2)×SU(2) σ model discussed by Palmer.[13] Here the term which breaks SU(2)×SU(2) (and generates the pion mass) is not an intrinsic term linear in the σ field but is an induced term which arises from a coupling of scalar fields ϕ to the π and σ fields. However, after a suitable gauge transformation, the pions are decoupled from the ϕ. Thus the pion mass difference depends only on the γ, Z, and W interactions, and the question of a divergence depends only on the commutators (32) which we can calculate from the explicit form of the currents.

In particular, the model has PCAC; so, using the canonical commutation relations, the commutator (32b) has no $I = 2$ piece. Further, we have

$$V_a^\mu = \epsilon_{abc}\pi_b\partial^\mu\pi_c\,, \tag{33}$$

$$A_a^\mu = \pi_a\partial^\mu\sigma - \sigma\partial^\mu\pi_a - \langle\sigma\rangle_0\partial^\mu\pi_a\,, \tag{34}$$

which we can use to calculate the canonical commutators explicitly:

$$\begin{aligned}[\partial_0 V_a^i(x), V_b^j(y)]\delta(x^0 - y^0) = -2i\epsilon_{adc}\epsilon_{bec}\{&\partial_x^i\pi_d(x)\partial_x^j\pi_e(x)\delta^4(x-y) - \pi_d(x)\partial_x^i\partial_x^j\pi_e(x)\delta^4(x-y) \\ &+ \pi_d(x)\partial_x^j\pi_e(x)\partial_y^i\delta^4(x-y) - \tfrac{1}{2}\pi_d(x)\pi_e(x)\partial_y^i\partial_y^j\delta^4(x-y) \\ &+ \tfrac{1}{2}[\pi_d(x)\partial_x^i\pi_e(x) - \pi_e(x)\partial_x^i\pi_d(x)]\partial_y^j\delta^4(x-y)\}\end{aligned} \tag{35}$$

(the index c is summed over), and

$$[\partial_0 A_a^i(x), A_b^j(y)]\delta(x^0-y^0) = -2i\{\partial_x^i \pi_a(x)\partial_x^j \pi_b(x)\delta^4(x-y) - \pi_a(x)\partial_x^i\partial_x^j \pi_b(x)\delta^4(x-y) + \pi_a(x)\partial_x^j \pi_b(x)\partial_y^i \delta^4(x-y) - \tfrac{1}{2}\pi_a(x)\pi_b(x)\partial_y^i\partial_y^j \delta^4(x-y) + \tfrac{1}{2}[\pi_a(x)\partial_x^i \pi_b(x) - \pi_b(x)\partial_x^i \pi_a(x)]\partial_y^j \delta^4(x-y) + \delta_{ab} f(\sigma)\delta^4(x-y)\}, \quad (36)$$

where $f(\sigma)$ is a function which involves only the σ field. The sum of (35) and (36) for $a=b=3$ has an isopsin structure $\vec{\pi}\cdot\vec{\pi}$, i.e., is an isoscalar. The same is true for the commutators of the zeroth components of the currents. Thus, from (32) the pion mass difference is finite. Again notice this would not be true if we did not have the Z meson which contributes the axial-vector term in (32a).

A similar argument does *not* work for the proton-neutron mass difference. In this SU(2) model the currents are

$$J_{\text{em}}^\mu = V_3^\mu + \frac{1}{\sqrt{3}} V_8^\mu,$$

$$J_Z^\mu = \beta V_3^\mu + (\beta-1)\frac{1}{\sqrt{3}} V_8^\mu - A_3^\mu, \quad (37)$$

$$J_W^\mu = V_{1+i2}^\mu - A_{1+i2}^\mu .$$

The photon graph is proportional to

$$e^2 V_{\mu\nu}^{3,8} , \quad (38)$$

where $V_{\mu\nu}^{ab}$ is given by (24) with the proton and neutron replacing the K^+ and K^0, respectively. The Z graph goes as

$$\frac{g^2+g'^2}{4}\beta(\beta-1)V_{\mu\nu}^{3,8} , \quad (39)$$

while the W contribution is zero by the isospin argument which follows (26). The total mass difference therefore has a divergence

$$\int \frac{d^4q}{q^2} g^{\mu\nu}\frac{e^2}{2\cos^2\theta_W} V_{\mu\nu}^{3,8} , \quad (40)$$

which is simply $(2\cos^2\theta_W)^{-1}$ times the divergence which occurs if we consider only the photons.[5] The commutator

$$[\partial_0 V_3^\mu, V_8^\nu] \quad (41)$$

does have a nonzero $I=1$ part in the linear σ model. It would appear that the problem of p-n mass difference requires more care. It shou'd be noted that, in the model under consideration, whereas the finite pion mass arises from the spontaneous breaking of $SU_L(2)\times Y_L$ symmetry, the origin of the nucleon mass is quite different and arises by spontaneously breaking the $SU(2)\times SU(2)$ symmetry of the strong Lagrangian. One might then expect that a proper handling of the p-n mass-difference problem would require a unified theory of strong, electromagnetic, and weak interactions.

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APPENDIX

The currents given in (2), (3), and (5) can be thought of as

$$V_a^\mu \sim \bar{\psi}\gamma^\mu \tfrac{1}{2}\lambda_a \psi ,$$

$$A_a^\mu \sim \bar{\psi}\gamma^\mu\gamma^5 \tfrac{1}{2}\lambda_a \psi ,$$

where the wave functions are given by

$$\psi \sim \begin{pmatrix} \mathcal{P} \\ \mathcal{N} \\ \lambda \\ \mathcal{P}' \end{pmatrix} .$$

$\mathcal{P}$, $\mathcal{N}$, and λ are the usual SU(3) quarks, and $\mathcal{P}'$ is the fourth quark with charm quantum number $C=+1$. The 4×4 λ matrices are

$$\lambda_{1\cdots 8} = \begin{pmatrix} \lambda_{1\cdots 8}^{SU(3)} & 0 \\ 0 & 0 \end{pmatrix} ,$$

$$\lambda_9 = \begin{pmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \end{pmatrix} , \quad \lambda_{10} = \begin{pmatrix} 0&0&0&i \\ 0&0&0&0 \\ 0&0&0&0 \\ -i&0&0&0 \end{pmatrix} ,$$

$$\lambda_{11} = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&1&0&0 \end{pmatrix} , \quad \lambda_{12} = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&i \\ 0&0&0&0 \\ 0&-i&0&0 \end{pmatrix} ,$$

$$\lambda_{13} = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix} , \quad \lambda_{14} = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&i \\ 0&0&-i&0 \end{pmatrix} ,$$

$$\lambda_{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&-3 \end{pmatrix} .$$

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PHYSICAL REVIEW D VOLUME 7, NUMBER 2 15 JANUARY 1973

Mechanism for Octet Enhancement in S-Wave Nonleptonic Decays

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The mechanism of Li and Pagels is used to discuss the S-wave nonleptonic decays of mesons and baryons. It implies that the decays $\Sigma^+ \to n\pi^+$ and $K^+ \to \pi^+\pi^0$ do not occur and thus the $\Delta I = \frac{1}{2}$ rule and the Lee-Sugawara relation hold. Moreover, it also yields a relation between the F/D ratio for nonleptonic hyperon decays and the F/D ratio for the semileptonic decays.

I. INTRODUCTION

We extend the threshold-dominance hypothesis of Li and Pagels to both meson and baryon matrix elements of the parity-conserving weak Hamiltonian $\mathcal{H}_w^{pc}(0)$. In other words, we assume that the techniques of Ref. 1 may also be applied to matrix elements of $\mathcal{H}_w^{pc}(0)$. We shall see that this implies octet enhancement for these matrix elements as well as a relation between the F/D ratio for nonleptonic hyperon decays [denoted by $(F/D)_w$] and the F/D ratio for semileptonic decays [denoted by $(F/D)_A$]. Upon combining the relation so obtained with that derived in Ref. 1 between the F/D ratio for the baryon octet mass splitting [denoted by $(F/D)_B$] and $(F/D)_A$, it then follows that

$$\left(\frac{F}{D}\right)_w = \left(\frac{F}{D}\right)_B = \frac{10}{3}\frac{(F/D)_A}{[1-3(F/D)_A{}^2]} = \frac{2}{3}\frac{m_N - m_\Xi}{m_\Lambda - m_\Sigma}\,, \tag{1.1}$$

in excellent agreement with experiment.

The technique here employed may be summarized as follows.[1] One assumes that both baryon and meson matrix elements of $\mathcal{H}_w^{pc}(0)$ obey an unsubtracted dispersion relation, so that

$$\langle X_\alpha | \mathcal{H}_w^{pc}(0) | X_\beta \rangle = \frac{1}{\pi}\int_{t_0}^{\infty} \frac{dt}{t}\,\mathrm{Im}\langle X_\alpha | \mathcal{H}_w^{pc}(0) | X_\beta \rangle\,. \tag{1.2}$$