fourth-order graphs $A-J$ in Fig. 7 uv-convergent in the Yennie gauge. However, with this choice of λ_2, it is impossible to find a finite value of λ_4 that will render the same class of graphs uv-convergent in sixth order due to the uv-divergent subgraphs $K-S$ illustrated in Fig. 7. Thus, in his scheme, λ still requires an infinite renormalization in fourth and subsequent orders of its expansion in e_0^2.

[27]J. C. Ward, Proc. Phys. Soc. (London) A64, 54 (1951); Phys. Rev. 84, 897 (1951). See also T. T. Wu, *ibid.* 125, 1436 (1962).

[28]Our proof is based on a procedure originally devised by Yang and Mills for treating overlapping divergences in photon self-energy graphs with multiphoton intermediate states. An outline of their prescription is given by T. T. Wu, Phys. Rev. 125, 1436 (1962).

[29]The two-photon-rung ladder graph with finite insertions continues to diverge in the infrared for finite q with p and $p'=0$. However, since $\Gamma_\mu^a(s,0)=s_\mu/C$ and $s^\mu\Gamma_\mu^a(s+q,q)=(s^2+2s\cdot q)/C$ in the gauge in which Z_1 is finite, it is trivial to show that this graph diverges in the uv region as $\ln\Lambda^2$. The two-photon annihilation graphs belonging to

$$K^{a(2n+2)}(p,0;\lambda_2,\dots,\lambda_{2n-2})$$

with finite insertions can be made infrared-convergent by giving the photon a small mass. By setting $p=0$ and differentiating these graphs with respect to the photon mass it is easy to show that these graphs likewise diverge no worse than $\ln\Lambda^2$ in the gauge in which Z_1 is finite to order $\alpha_0^{\,n-1}$.

[30]Had we not assumed $\kappa\geq 1$ and invoked the asymptotic hypothesis, then Eq. (5.6) would have contained an additional term

$$\int d^4k\,\frac{\delta\Sigma(p^2;\,C\tilde{\Delta},e^2\tilde{D})}{\delta(e^2\tilde{D}_{\mu\nu}(k))}\,\frac{\delta(e^2\tilde{D}_{\mu\nu}(k))}{\delta\tilde{m}(p_0^2)}\;.$$

In order to solve (5.6) we would have needed an additional equation for $\partial\tilde{D}_{\mu\nu}/\partial\tilde{m}$ obtained from the functional differentiation of the polarization operator expanded in terms of the full $\tilde{D}$ and Δ functions. The asymptotic solution of this coupled pair of integral equations goes beyond the scope of this prelimin^ry study.

[31]We are now in a position to prove the assertion in Sec. III that all uv divergences in Z_2 are isolated by neglecting h provided it vanishes with power-law behavior. The contribution to $\Gamma_\mu(p,p)$ when any one of the internal photon lines of $\Gamma_\mu^a(p,p)$ is replaced by the nonasymptotic part of $e^2\tilde{D}_{\mu\nu}$ is

$$-i\int\frac{d^4k}{(2\pi)^4}\,\frac{\partial}{\partial p^\mu}\,C^a_{\alpha\beta}(p,k)\left(g^{\alpha\beta}-\frac{k^\alpha k^\beta}{k^2}\right)\left(\frac{h(k^2/m^2,\alpha)}{k^2}\right)\;.$$

An application of Weinberg's theorem to the graphs defining $C^a_{\alpha\beta}(p,k)$ shows that to any finite order of perturbation theory and for $k^2\gg p^2$ and

$$\left\langle\frac{\partial}{\partial p^\mu}\,C^a_{\alpha\alpha}(p,k)\right\rangle_k\sim\frac{p_\mu}{k^2}\times(\text{powers of }\ln k^2).$$

Therefore, the above integral is uv-convergent provided

$$h(k^2/m^2,\alpha)\underset{k^2\gg m^2}{\sim}(m^2/k^2)^\kappa,$$

where $\kappa>0$.

PHYSICAL REVIEW D VOLUME 7, NUMBER 2 15 JANUARY 1973

Action-at-a-Distance Theories and Dual Models*

P. Ramond
Physics Department, Yale University, New Haven, Connecticut 06520
(Received 21 September 1972)

We write the most general classical formulation of Poincaré-invariant action-at-a-distance theories and review their classical applications. We stress their bootstraplike properties. In particular, we try to view dual amplitudes in terms of the radiation reaction of "dual atoms."

I. INTRODUCTION

The study of the strong interactions in the limit of short separations has led to the revival of the conformal group.[1] Yet, the physical applications of this group have been hampered by the noninvariance of the sign of x^2 under its *finite* transformations, thus causing an apparent violation of causality. To circumvent this difficulty, modern "conformists" require only *infinitesimal* conformal invariance and break the full invariance by the specification of physically reasonable boundary conditions (for instance, through an $i\epsilon$ prescription). As it is evident that conformal invariance must be broken in some way, it may prove useful, as well as instructive, to consider alternatives to this procedure. One such alternative is provided by the classical treatment of electrodynamics through action at a distance,[2] as formulated by Feynman and Wheeler.[3] Their formulation, as pointed out by Professor Gürsey,[4] is conformally invariant; in it particles interact by means of a symmetric combination of advanced and retarded signals, instead of the usual retarded interaction. Causality can be restored if one adopts the view that radiation is not just an emission, but a transmission process, that is, there is no such thing as unabsorbed radiation. Under this assumption, Feynman and Wheeler show that the force of radiation reaction on a given particle can be understood in terms of the advanced signals emitted by the particles that have absorbed those sent by the original particle. In short, by assuming perfect absorption of all radiation, they have shown their formulation to be equivalent to the field treatment of electrodynamics.[5] In addition, and perhaps more important, their treatment does not give rise to infinities since in action-at-a-distance theories particles do not act on themselves. Unfortunately, this beautiful and elegant alternative view has resisted all attempts at quantization,[6] thereby explaining its fall from prominence. It is nevertheless instructive to speculate on how a quantized action-at-a-distance theory would compare with its quantized field theory (QFT) analog. First of all, we see that photons (the radiation) are never allowed to be on mass shell so that only the charge carriers (electrons, say) can be on mass shell and consequently have asymptotic states.[7] This presents a clear restriction over QFT where particles are free to exist on or off mass shell. Yet, by an appropriate interpretation of the nature of the absorber, one can produce QFT effects such as vacuum polarization and pair production. Thus, if such an asymmetry between electrons and photons can be made tenable, one would have to say that a quantized action-at-a-distance theory will be equivalent to the renormalized quantum electrodynamics.

In strong interactions, there are in fact many particles which do not have any asymptotic states: the resonances. For example, one usually treats (theoretically) both the ρ and π mesons as particles even though one of them does not exist outside of the strong-interaction region. If the absorber of Feynman and Wheeler can be made to absorb everything except the ground states of the various Regge trajectories, it would then be tempting to take action-at-a-distance theories seriously. In this paper, we take this attitude. In view of the extreme difficulties with quantization, we shall present a general classical treatment at first for arbitrary action-at-a-distance theories. Then, we will consider their possible application to dual resonance models (DRM) which we will use as a guide for quantization. Unfortunately we have not yet been able to determine what type of interaction corresponds to dual models. In Sec. II we describe the general formalism of action-at-a-distance theories. Section III will deal with possible applications. Section IV will stress the similarities of the formalism with dual models when some dynamical restrictions are applied.

II. CLASSICAL FORMALISM

We specifically consider an arbitrary number of scalar particles interacting with one another by means of action-at-a-distance forces.[8] By this we mean that the force acting on any given particle is due *only* to the *other* particles; thus there is no self-interaction as well as no degrees of freedom other than those carried by the particles themselves. This presents a clear alternative to the usual field treatment of relativistic interactions. In the following, we expand the relativistic description of such systems.

Every particle will describe a world line with line element

$$ds_i{}^2 = g_{\mu\nu}\, dx_i^\mu\, dx_i^\nu \;\; (\equiv dx_i \cdot dx_i)\,, \tag{2.1}$$

where i is the particle label, $g_{\mu\nu}$ the Lorentz metric taken to be $(1,-1,-1,-1)$, and x_i^μ is a Lorentz four-vector describing the position of particle i. It is convenient to introduce a scalar parameter λ_i as a label, monotonically increasingly along the world line. In addition, we set

$$u_i^\mu \equiv \frac{dx_i^\mu}{d\lambda_i}\,. \tag{2.2}$$

As in all beautiful physical formulations, the behavior of the system is derivable from an action principle. We postulate the action function[9]

$$S = \sum_i m_i c \int ds_i + \sum_{\substack{i \\ i<j}} \sum_j \int d\lambda_i \int d\lambda_j\, R^{(i,j)}(x_i^\mu, x_j^\mu, u_i^\mu, u_j^\mu)\,. \tag{2.3}$$

Note the absence of a self-interaction term. Also, $R^{(i,j)}$ is required to be symmetric under interchange of i and j in order to preserve the symmetry of the action. The dependence of $R^{(i,j)}$ on the path functions is restricted up to first derivatives. Of course the interaction is nonlocal. The requirements of translation and Lorentz invariance as well as symmetry of the action are ensured by taking $R^{(i,j)}$ to be functions of the variables

$$\begin{aligned}
s_0^{(i,j)} &= (x_i - x_j)\cdot(x_i - x_j)\,,\\
s_1^{(i,j)} &= u_i \cdot u_j\,,\\
s_2^{(i,j)} &= (u_i - u_j)\cdot(x_i - x_j)\,,\\
s_3^{(i,j)} &= [u_i\cdot(x_i - x_j)][u_j\cdot(x_j - x_i)]\,,\\
s_4^{(i,j)} &= (u_i\cdot u_i)(u_j\cdot u_j)\,,\\
s_5^{(i,j)} &= u_i\cdot u_i + u_j\cdot u_j\,.
\end{aligned} \tag{2.4}$$

These are related through the relations

$$s_2^{(i,j)}=\frac{1}{2}\left(\frac{d}{d\lambda_i}+\frac{d}{d\lambda_j}\right)s_0^{(i,j)},$$
$$s_3^{(i,j)}=\frac{1}{4}\left(\frac{ds_0^{(i,j)}}{d\lambda_i}\right)\left(\frac{ds_0^{(i,j)}}{d\lambda_j}\right), \tag{2.5}$$
$$s_1^{(i,j)}=-\frac{1}{2}\frac{d^2}{d\lambda_i d\lambda_j}s_0^{(i,j)}.$$

The (classical) physical motion of the particles is defined to be that for which the action is an extremum. Consequently, a small variation away from the physical motion will not alter the value of S. Assuming that this variation tends to zero at infinity, we obtain the equations of motion

$$m_i c\frac{d}{d\lambda_i}\left[\frac{u_i^\mu}{(u_i\cdot u_i)^{1/2}}\right]=\sum_{j\neq i}\int d\lambda_j\left[\frac{\partial R^{(i,j)}}{\partial x_{i\mu}}-\frac{d}{d\lambda_i}\left(\frac{\partial R^{(i,j)}}{\partial u_{i\mu}}\right)\right],\quad i=1,2,\ldots. \tag{2.6}$$

Unlike their nonrelativistic counterparts, these are integrodifferential equations, since the right-hand side depends on integrals over the whole motion. This novel feature is due to the fact that signals cannot propagate instantaneously. To see this more explicitly, it is convenient to rewrite these equations in terms of the new variables. Straightforward algebra yields

$$\frac{d}{d\lambda_i}(M_i u_i^\mu)=\sum_{j\neq i}\int d\lambda_j\,(x_i-x_j)^\mu F^{(i,j)}(\{s^{(i,j)}\}), \tag{2.7}$$

where

$$M_i=\frac{m_i c}{(u_i\cdot u_i)^{1/2}}+\sum_{j\neq i}\int d\lambda_j\left(\frac{\partial R}{\partial s_5}+2u_j\cdot u_j\frac{\partial R}{\partial s_4}\right) \tag{2.8}$$

and

$$F^{(i,j)}=2\frac{\partial R}{\partial s_0}-\frac{d^2}{d\lambda_i d\lambda_j}\frac{\partial R}{\partial s_1}-\left(\frac{d}{d\lambda_i}+\frac{d}{d\lambda_j}\right)\frac{\partial R}{\partial s_2}-\frac{d}{d\lambda_i}\left[u_j\cdot(x_j-x_i)\frac{\partial R}{\partial s_3}\right]-\frac{d}{d\lambda_j}\left[u_i\cdot(x_i-x_j)\frac{\partial R}{\partial s_3}\right]. \tag{2.9}$$

We have omitted the (i,j) superscripts for convenience. We see that the interaction gives rise to a generalized mass or momentum (distinct from the usual generalized momentum). In addition, $F^{(i,j)}(\{s\})$ is manifestly invariant under the interchange of i and j, rendering the integrand on the right-hand side of Eq. (2.7) antisymmetric under $i\to j$. This corresponds to a generalization of Newton's law of action and reaction which is made possible by the presence of retarded and advanced interactions.

Up to now, we have said nothing about the λ_i's except by requiring them to be Lorentz-invariant labels of the world lines (translation invariance was not required as they appear only as infinitesimals in the action). An obvious (but not unique) way of identifying them with some concrete property of the particle's world line is to set them equal to the path lengths of their respective world lines. This procedure, however, is not always compatible with the equations of motion. We see from Eq. (2.1) that our choice requires that

$$u_i\cdot u_i=1. \tag{2.10}$$

On the other hand, by multiplying the equations of motion (2.6) by $u_{i\mu}$, we obtain, after several manipulations,

$$\tfrac{1}{4}m_i c\frac{d}{d\lambda_i}(u_i\cdot u_i)=\frac{d}{d\lambda_i}\sum_{j\neq i}\int d\lambda_j\left(R-u_i\cdot\frac{\partial R}{\partial u_i}\right). \tag{2.11}$$

Thus, consistency requires that

$$\left(1-u_{i\mu}\frac{\partial}{\partial u_{i\mu}}\right)\sum_{j\neq i}\int d\lambda_j R=\text{constant}. \tag{2.12}$$

Obviously not all interactions will satisfy this criterion. One obvious way to satisfy it is to make $R^{(i,j)}$ homogeneous in u_i of the first order. This is a sufficient but not necessary requirement. Note that if R is homogeneous in $u_{i\mu}$ of any order other than one, there can be no interaction compatible with our identification of λ_i since then the interacting action can be rewritten as a sum of free particle actions by using Eq. (2.12).

Now, it is reasonable to require that the physical behavior of our system be unaltered by an arbitrary change in the parametrization of the world lines. The free part of the action, just being an arc length, is invariant under such a change while the interacting part is (*a priori*) not. It is easy to see that the invariance of S is satisfied if Eq. (2.12) is obeyed, by performing an arbitrary change $\lambda_i\to\lambda_i+\delta\lambda_i$ in the expression for the interacting part of S. Hence, on physical grounds, we must restrict ourselves to interactions that satisfy Eq. (2.12).

The invariance of S under translations and Lorentz transformations has been guaranteed by our construction. Such transformations form a ten-parameter group (four for translations and six for rotations in Minkowksi space), the Poincaré group.

We then expect to have ten conserved quantities. We can obtain them either by looking at the response of the action to infinitesimal translations and rotations[9] or directly by looking at the equations of motion. The four generators of translations are given by

$$P_\mu=\sum_i M_i u_{i\mu}(\lambda_i)+\frac{1}{c}\sum_{i<j}\left(\int_{\lambda_i}^{\infty}\int_{-\infty}^{\lambda_j}-\int_{-\infty}^{\lambda_i}\int_{\lambda_j}^{+\infty}\right)d\lambda_i'\,d\lambda_j'\,(x_i'-x_j')_\mu F^{(i,j)}(\{s'\})\,. \tag{2.13}$$

They are constant in λ_i's as can be seen directly by differentiating the above and using (2.7). It is important to remark that the total four-momentum of the system is not just the sum of the particles' individual four-momenta. (Recall that P_0 is an energy which for a system in interaction contains an extra term, the interaction energy). Similarly, the six generators of Lorentz transformations are given by

$$M_{\mu\nu}=\sum_i(x_{i\mu}\,d^iP_\nu-x_{i\nu}\,d^iP_\mu)\,, \tag{2.14}$$

where d^i refers to the variation induced by a small change in λ_i. It is evident from this that the $M_{\mu\nu}$'s are constant. They can also be written as

$$M_{\mu\nu}=\sum_i M_i(x_{i\mu}u_{i\nu}-x_{i\nu}u_{i\mu})+\frac{1}{c}\sum_{i<j}\left(\int_{\lambda_i}^{\infty}\int_{-\infty}^{\lambda_j}-\int_{-\infty}^{\lambda_i}\int_{\lambda_j}^{\infty}\right)d\lambda_i'd\lambda_j'\,(x_{i\mu}'x_{j\nu}'-x_{i\nu}'x_{j\mu}')F^{(i,j)}(\{s'\})\,. \tag{2.15}$$

Again, notice the presence of an interaction angular momentum. These interaction terms, both in P_μ and $M_{\mu\nu}$, correspond to the fact that the interaction is not instantaneous which means that we have some momentum (or angular momentum) in transit,[10] having left a given particle and not yet arrived to another one. These lie at the origin of the no-go theorems of relativistic particle dynamics.[11] Suppose we postulate some Lie brackets between $x_{i\mu}$ and $p_{i\mu}$ ($\equiv m_i u_{i\mu}$); then it is not clear whether P_μ and $M_{\mu\nu}$ will satisfy the brackets of the Poincaré group. This will depend on the interaction which itself depends on $x_{i\mu}$ and $p_{i\mu}$. Hence the emergence of consistency conditions between the boundary conditions (Lie brackets) and the equations of motion. This is symptomatic of all action-at-a-distance theories where the path of a particle at a given time depends on the path of other particles taken at different times. This is in contradistinction with the usual Hamiltonian formalism[12] where the line evolution of the system is independent of the boundary conditions. This fact seems to suggest that action-at-a-distance theories can be regarded as the subset of field theories which satisfy certain bootstrap requirements.

III. EXAMPLES

We illustrate the formalism of Sec. II with some special examples. All cases will be describing the interaction of scalar particles. When building their interaction we will restrict ourselves to those which admit a consistent definition of proper time (path length). Consequently we omit the variables $s_2^{(i,j)}$ and $s_5^{(i,j)}$ from our models since they are not homogeneous in the velocities.

A. Scalar Interactions

This theory is the analog of the field theory mediated by a massive scalar meson.[13,14] It is derivable from the action S with

$$R^{(i,j)}=-g_i\,g_j(s_4^{(i,j)})^{1/2}\,D_{\text{sym}}(m^2;\,s_0^{(i,j)})\,, \tag{3.1}$$

where g_i and g_j are the coupling constants and D_{sym} is the Green's function of the massive Klein-Gordon operator that is symmetric in time:

$$D_{\text{sym}}(m^2;\,s_0)=\tfrac{1}{2}(D^{\text{adv}}+D^{\text{ret}})=2\left[\delta(s_0)-\theta(s_0)\frac{m}{2\sqrt{s_0}}J_1(m\sqrt{s_0})\right]. \tag{3.2}$$

Introduce the function

$$\phi^{(j)}(x)=g_j\int d\lambda_j\,(u_j\cdot u_j)^{1/2}\,D_{\text{sym}}(m^2;\,(x-x_j)^2)\,, \tag{3.3}$$

which obeys

$$(\Box^2-m^2)\phi^{(j)}(x)=-4\pi\rho^{(j)}(x)\,, \tag{3.4}$$

with

$$\rho^{(j)}(x)=g_j\int d\lambda_j\,\delta^{(4)}(x-x_j)\,. \tag{3.5}$$

We see that $\phi^{(j)}(x)$ plays the role of the field and $\rho^{(j)}(x)$ that of the scalar current. Unlike field theory, however, $\phi^{(j)}(x)$ is just a convenient construct, not a physical entity with degrees of freedom of its own. Then we can write the interaction action as

$$S^{\text{int}} = -\tfrac{1}{2}\sum_i \int d\lambda_i \, (u_i \cdot u_i)^{1/2}\Phi_i(x_i)\,, \tag{3.6}$$

where

$$\Phi_i(x_i) = \sum_{j \neq i} \phi^{(j)}(x_i) \tag{3.7}$$

is the scalar "field" felt by the particle i resulting from the action of all the other particles.

B. Vector Interactions

There are several ways to write such interactions. The most popular one, due to Van Dam and Wigner,[15] lets

$$R^{(i,j)} = g_i g_j s_1^{(i,j)} f(s_0^{(i,j)})\,, \tag{3.8}$$

with $f(s_0)$ arbitrary. If we take f to be the symmetric Green's function of Eq. (3.2), the vector "field"

$$A_\mu^{(i)}(x) = g_i \int d\lambda_i \, u_{i\mu} D_{\text{sym}}(m^2, (x - x_i)^2) \tag{3.9}$$

automatically satisfies

$$(\Box^2 - m^2)A_\mu^{(i)}(x) = -4\pi j_\mu^{(i)}(x)\,, \tag{3.10}$$

with

$$j_\mu^{(i)}(x) = g_i \int d\lambda_i \, u_{i\mu} \delta^{(4)}(x - x_i) \tag{3.11}$$

being the vector current. The special case of physical interest corresponds to letting $m = 0$ in the above. Then we obtain the Tetrode-Fokker action[2] which contains the whole of classical electrodynamics. The field strengths are introduced through

$$F_{\mu\nu}^{(i)} = \partial_\mu A_\nu^{(i)} - \partial_\nu A_\mu^{(i)}\,, \tag{3.12}$$

which explicitly satisfy Maxwell's equations. The equations of motion obtained by varying the action S are

$$m_i \frac{d}{d\lambda_i} u_{i\mu} = e_i \sum_{j \neq i} F_{\mu\nu}^{(j)} u_i^\nu\,, \tag{3.13}$$

where we have used Eq. (2.10). Following Feynman and Wheeler,[3] we observe that

$$\begin{aligned}\sum_{j \neq i} F_{\mu\nu}^{(j)} &= \tfrac{1}{2}\sum_{j \neq i} (F_{\mu\nu}^{(j)\text{ret}} + F_{\mu\nu}^{(j)\text{adv}}) \\ &= \sum_{j \neq i} F_{\mu\nu}^{(j)\text{ret}} + \tfrac{1}{2}(F_{\mu\nu}^{(i)\text{ret}} - F_{\mu\nu}^{(i)\text{adv}}) \\ &\quad - \sum_{\text{all } j} \tfrac{1}{2}(F_{\mu\nu}^{(j)\text{ret}} - F_{\mu\nu}^{(j)\text{adv}})\,. \end{aligned}\tag{3.14}$$

The last term vanishes outside the absorber because complete absorption of the radiation is the hypothesis of their interpretation. If so, being nonsingular on the particle's world lines as well as a solution of Maxwell's equations, it must also vanish inside the absorber; it vanishes everywhere.

The second term has been shown by Dirac[16] to give the force of radiation reaction. Thus, with the Feynman-Wheeler reinterpretation, one obtains the Lorentz-Dirac[16] equations of motion. In conclusion, both the classical field and action-at-a-distance formulations give rise to the same theory in the case of infinite-range vector interaction. It is significant that such agreement disappears in the case of short-range interactions.[13] The difference lies in the force of radiation reaction for which the two formulations give different expressions. In the field theory case, the radiation reaction turns out to depend on the previous motion of the particle, which means that the force felt by the particle at a given point in space depends on how it arrived at that point. This presents a clear conceptual difficulty in the meaning of a local field. In the action-at-a-distance case, similar effects are found, but they do not pose any problems of interpretation. We view this as a hint that an action-at-a-distance formulation of short-range forces might be preferable to that of a field theory.

For a vector interaction, the interacting action is then given by

$$S^{\text{int}} = \tfrac{1}{2}\sum_i \int d\lambda_i \, u_i^\mu A_{i\mu}(x_i)\,, \tag{3.15}$$

where

$$A_{i\mu}(x_i) = \sum_{j \neq i} \int d\lambda_j \, u_{j\mu} D_{\text{sym}}(m^2, s_0^{(i,j)}) \tag{3.16}$$

is the total vector "field" felt by particle i.

Let us note an alternate way of writing vector interaction corresponding to

$$R^{(i,j)} = g_i g_j \, s_3^{(i,j)} f(s_0^{(i,j)})\,, \tag{3.17}$$

which reduces to the previous one.[17]

Finally, we emphasize that the vector "field" (3.9) satisfies

$$\partial^\mu A_\mu^{(i)}(x) = 0 \tag{3.18}$$

automatically, thus eliminating the scalar degree of freedom, and corresponding to a conserved vector current. We see that there is a definite choice of gauge made by specifying the form of the action. This result is important in constructing actions for tensor interactions.

C. Tensor Interactions

One of the nicest features of action-at-a-distance theories lies in their ability to describe higher spin interactions in terms of the existing degrees of freedom, i.e., without introducing new degrees of freedom attributed to a given resonance, as in field theory. We start by looking at the second-rank tensor interactions (which are important in their own right). Such interactions will involve couplings to the energy-momentum tensor of the particles. This tensor is obtained by varying the free particle's action with respect to the metric tensor. The result is (for particle i, say)

$$T_{\mu\nu}^{(i)}(x)=m_i c\int d\lambda_i\,\delta^{(4)}(x-x_i)\frac{u_{i\mu}u_{i\nu}}{(u_i\cdot u_i)^{1/2}}. \qquad (3.19)$$

This form suggests then the following possibilities for the action:

$$R^{(i,j)}=\begin{cases}-g_i g_j\, s_1^{\,2}s_4^{\,-1/2}f_1(s_0)\,, & (3.20\text{a})\\ -g_i g_j\, s_1 s_3 s_4^{\,-1/2}f_2(s_0)\,, & (3.20\text{b})\\ -g_i g_j\, s_3^{\,2}s_4^{\,-1/2}f_3(s_0)\,, & (3.20\text{c})\end{cases}$$

where we have dropped the (i,j) superscripts on the s variables. The actions corresponding to these choices can all be written in the form

$$\sum_{i<j}\int d^4x\, T_{\mu\nu}^{(i)}(x)\psi_a^{(j)\mu\nu}(x)\,, \quad a=1,2,3 \qquad (3.21)$$

with

$$\psi_1^{(j)\mu\nu}(x)=g_j\int d\lambda_j\frac{u_j^\mu u_j^\nu}{(u_j\cdot u_j)^{1/2}}f_1((x-x_j)^2)\,, \quad (3.22\text{a})$$

$$\psi_2^{(j)\mu\nu}(x)=g_j\int d\lambda_j\frac{u_j^\mu(x-x_j)^\nu}{(u_j\cdot u_j)^{1/2}}\times u_j\cdot(x-x_j)f_2((x-x_j)^2)\,, \quad (3.22\text{b})$$

$$\psi_3^{(j)\mu\nu}(x)=g_j\int d\lambda_j\,(x-x_j)_\mu(x-x_j)_\nu\times\frac{[u_j\cdot(x-x_j)]^2}{(u_j\cdot u_j)^{1/2}}f_3((x-x_j)^2)\,. \quad (3.22\text{c})$$

These couplings differ by the presence of derivative couplings, as can be seen by integrating by parts the above expressions. The differences between the various actions can be written as

$$\Delta S=\sum_{i<j}\int d\lambda_i\frac{u_{i\mu}u_{i\nu}}{(u_i\cdot u_i)^{1/2}}\frac{\partial}{\partial x_{i\nu}}B^{(j)\mu}(x_i)\,, \qquad (3.23)$$

where $B^{(j)\mu}(x)$ is some field construct. This expression can then be recast in the form

$$\Delta S=-\sum_{i<j}\int d\lambda_i\, B_\mu^{(j)}(x_i)\frac{d}{d\lambda_i}\left[\frac{u_i^\mu}{(u_i\cdot u_i)^{1/2}}\right]. \qquad (3.24)$$

In this way, higher derivative terms can make their appearance in the action. We will comment on this point in Sec. IV.

Physically, it means that these couplings differ in their vector and scalar contents. It is well known that in the theory of high-spin fields it is needed to introduce subsidiary conditions in order to eliminate the spurious components that always appear as a price for using manifestly covariant constructs. Action-at-a-distance theories have a much more clever and economical way to solve these problems.

Consider the coupling (3.20a). A similar form was introduced by Whitehead[18] as an alternative to the linearized form of Einstein's equations. The important difference is the presence of the factor $s_4^{-1/2}$ in our form. As we have shown, such a factor restores the homogeneity in the action that is needed for the parametrization invariance of the theory. This has important physical consequences. In Whitehead's theory, the "field" corresponding to Eq. (3.22a) has ten independent components which then couple to the matter tensor (3.19). In our case, however, we can use the constraint equation (2.10) in the expression for the "field." We can, for example, eliminate u_{i0} in favor of the three-vector part $\vec{u}_i$, thus reducing the number of independent components to six. We still need an additional constraint for a massive spin-two theory. We require the tracelessness of the "field," which is easily implemented by adding the extra coupling term

$$R_{\text{extra}}^{(i,j)}=\tfrac{1}{4}g_i g_j\, s_4^{(i,j)}f_1(s_0)\,. \qquad (3.25)$$

In field theory, this would correspond to the introduction of a scalar field.[19] The nature of the formalism forces us to add these new couplings which simulate the field part of the energy tensor. We see that the constraint equation (2.10) plays exactly the same role as that of the subsidiary conditions of field theory. This is part of the beauty of these theories. Still-higher-spin interactions can be introduced through coupling terms of the form

$$R^{(i,j)}=g_i g_j\, s_1^{\,n-l}s_3^{\,l}\, s_4^{\,-(n-1)/2}f_{l,n}(s)\,, \quad l=0,1,\ldots,n\,, \qquad (3.26)$$

which correspond to spin n.

In conclusion, it would seem that action-at-a-distance theories provide a much more economical way to describe high spin interactions, since one does not have to add new (stable) particles in order to have tensor forces. This suggests the desirability of casting theories of strong interactions in this language. Furthermore, as indicated earlier, these probably correspond to bootstrap theories. In Sec. IV, we try to pursue this program by investigating the connection with dual models.

IV. DUAL MODELS

Let us start this section with several general remarks concerning the applicability of action-at-a-distance theories to the quantum domain.[20] In this formalism, the interaction is nonlocal in character – thus the requirements of crossing symmetry, spin statistics, . . . , do not follow automatically and must therefore impose strong restrictions on the type of interaction (probably in terms of gauges). Also the requirement of total absorption of the radiation must be included as a constraint in the quantization procedure unless one is prepared to obtain nonunitary answers.[6] At the moment, we do not know how to solve these problems. Conceptually, any alternative (or subset) of field theories where particles can be restricted to exist off mass shell is definitely welcome in the description of the strong interactions. In addition, the commitment of the theory to a non-Hamiltonian form, as shown by the interdependence of the boundary conditions with the equations of motion, raises the possibility of a bootstrap.

As a partial answer to these questions, we shall try, in the following, to obtain the results of dual resonance models,[21] starting from an action-at-a-distance formulation. This should be regarded as an exercise which might throw light on how to quantize this type of theory on the one hand, or on the nature of dual models on the other.

We consider scalar particles of the same mass, m, interacting by means of the action (2.3). At the moment, let us choose (with hindsight!)

$$R^{(i,j)}=\sum_{n=0}^{\infty}\sum_{l=0}^{n} g_i^{(n,l)}g_j^{(n,l)}\,s_1{}^{n-l}s_3{}^{l}s_4{}^{-(n-1)/2}\times D_{\rm sym}(m_0{}^2+n\omega;\,s_0)\,, \tag{4.1}$$

which is very general, but includes linear trajectories, and will not be sufficient to lead to the degeneracy of dual models. We use this form as a prototype for the moment. In order to make contact with dual models, we further assume that these trace out *periodic* paths, i.e.,

$$x_{i\mu}(\lambda_i+T)=x_{i\mu}(\lambda_i)+\frac{T}{m}\Pi_{i\mu}\,. \tag{4.2}$$

This very strong restriction to the type of solutions we want may not be consistent with the interaction (4.1). Since we do not have enough information at this stage to answer this consistency question, we relegate it to the end. Physically, this means that all the scalar particles are performing periodic motions with the *same* period. This universality requirement may be regarded as a phenomenological input. Define the kinematical momenta

$$P_{i\mu}(\lambda_i)=m\,\frac{dx_{i\mu}(\lambda_i)}{d\lambda_i}\,, \tag{4.3}$$

which are therefore periodic functions of λ_i. Further, it follows that

$$\Pi_{i\mu}=\frac{1}{T}\int_0^T d\lambda_i\,P_{i\mu}(\lambda_i)\,. \tag{4.4}$$

The parametrization invariance of the action allows us to choose the λ_i's such that

$$P_i\cdot P_i=m^2\,. \tag{4.5}$$

In view of the periodicity of P_i, this equation is equivalent to the set

$$\frac{1}{T}\int_0^T d\lambda_i\,e^{2\pi i n\lambda_i/T}P_{i\mu}(\lambda_i)P_i^{\mu}(\lambda_i)=m^2\delta_{n,0}\,,\qquad n=0,\pm1,\pm2,\ldots\,. \tag{4.6}$$

These are classical equations. To generalize them to the quantum domain, we should impose them only for n negative, as in the case of the Lorentz condition in the electromagnetic case.

In our theory, the only degrees of freedom come from the particle's motion. Thus quantization will take the form of specifying the commutator between two $x_{i\mu}(\lambda_i)$ taken at different points of the same world line. Since we do not have a Hamiltonian formalism, $x_{i\mu}(\lambda_i)$ and $P_{i\mu}(\lambda_i)$ are *not* conjugate variables; in fact we can show that if they are conjugate, then the requirement that the expressions (2.13) and (2.15) satisfy the commutation relations of the Poincaré group leads to no interaction between the particles. Still, we see from Eq. (4.2) that our particles are translated in space-time. We consequently identify $\Pi_{i\mu}$ with the four-momentum of the particle,[22] which allows us to write[23]

$$[x_{i\mu}(\lambda_i),\Pi_{i\nu}]=-i\,g_{\mu\nu}\,. \tag{4.7}$$

To see what this identification means, let us write the Fourier expansion of the position coordinates as

$$x_{i\mu}(\lambda_i)=\frac{\lambda_i}{m}\Pi_{i\mu}+\sum_{l=-\infty}^{+\infty}x_{i\mu}(l)e^{2\pi i l\lambda_i/T}\,, \tag{4.8}$$

in terms of which one of the constraints (4.5) is written as

$$\Pi_{i\mu}\Pi^{i\mu}=m^2-\frac{4\pi^2m^2}{T^2}\sum_{l=1}^{\infty}l^2[x_{i\mu}^{\dagger}(l)x_i^{\mu}(l)+x_i^{\mu}(l)x_{i\mu}^{\dagger}(l)]\,, \tag{4.9}$$

where we have used Eq. (4.3) as well as the requirement that x be real. This means that the physical momentum of one particle can have different values. In group-theoretical terms, one "particle" spans a *reducible* representation of the Poincaré group. It is more like a collection of states, i.e., more like an atom than a particle. These "particles" we call "dual atoms." We can specify their level structure further by appealing to the existence of linear Regge trajectories, leading to the requirement that the infinite sum in the above equation have successive integer eigenvalues. This is satisfied if we take

$$x_{i\mu}(l)=\left(\frac{T}{2\pi m}\right)^{1/2}\frac{1}{\sqrt{l}}\,a^{\dagger}_{i\mu}(l)\,, \tag{4.10}$$

with

$$[a_{i\nu}(n),\,a^{\dagger}_{i\mu}(l)]=-g_{\mu\nu}\delta_{n,l}\,,\qquad [a_{i\nu}(n),\,a_{i\mu}(l)]=0\,. \tag{4.11}$$

It follows that[24]

$$[x_{i\mu}(\lambda_i),\,x_{i\nu}(\lambda_i')]=-ig_{\mu\nu}\frac{T}{2\pi m}\,\epsilon\left(\frac{2\pi}{T}(\lambda_i-\lambda_i')\right)\qquad \mathrm{mod}\left(\frac{2\pi}{T}\right), \tag{4.12}$$

where

$$\epsilon(x)'=\begin{cases}+1, & x>0\\ -1, & x<0\,.\end{cases} \tag{4.13}$$

These commutation relations, in turn, impose severe constraints on the interaction, as can be seen from Eqs. (2.13) and (2.15). We are assuming all along that consistency is achieved. By differentiating the above, we see that

$$[x_{i\mu}(\lambda_i),\;P_{i\mu}(\lambda_i)]=\infty\,, \tag{4.14}$$

which shows that the dual-model commutation relations cannot be obtained from a Hamiltonian formalism, expressed in terms of space-time quantities.[25]

It is not very instructive to write down the equations of motion explicitly. Their consequences are of course interesting. Our "dual atoms" accelerate since they perform periodic motion. It follows that they will radiate, the nature of the radiation being dictated by the form of the action. As an example, let us look at the vector interaction. The potential due to particle i can be written as

$$A_\mu^{(i)}(x)=g_i\int d\lambda_i\frac{dx_{i\mu}}{d\lambda_i}\,D_{\mathrm{sym}}[m_0{}^2+\omega;\,(x-x_i)^2]=\tfrac{1}{2}[A_\mu^{(i)\mathrm{ret}}+A_\mu^{(i)\mathrm{adv}}]\,, \tag{4.15}$$

where we now take the integration between 0 and T, making use of our periodicity assumption. The radiation reaction, on the other hand, is given by

$$A_\mu^{(i)\mathrm{rad}}(x)=\tfrac{1}{2}[A_\mu^{(i)\mathrm{ret}}-A_\mu^{(i)\mathrm{adv}}]\,. \tag{4.16}$$

Since for the vector interaction,

$$\tfrac{1}{2}(D^{\mathrm{ret}}-D^{\mathrm{adv}})=\frac{1}{2(2\pi)^3 i}\int d^4k\,e^{ik\cdot x}\,\epsilon(k_0)\delta(k^2-m_0{}^2-\omega)\,, \tag{4.17}$$

it follows that

$$A_\mu^{(i)\mathrm{rad}}(x)=\frac{g_i}{2(2\pi)^3 im}\int d^4k\,e^{ik\cdot x}\,\epsilon(k_0)\delta(k^2-m_0{}^2-\omega)\times\int_0^T d\lambda_i\,P_{i\mu}(\lambda_i)e^{-ik\cdot x_i(\lambda_i)}\,. \tag{4.18}$$

This is nothing but the dual vertex for the emission of a spin-one particle,[26] up to the normal ordering, to avoid infinities. The mass-shell constraint enters automatically because we are looking at the radiation. In this light, the dual trees can be thought of describing the radiation from an accelerating "dual atom." Duality comes in by requiring that the emitted radiation be resolved in terms of the states of the "dual atom."[27] Namely, given the hypothesis of equal spacing as expressed by the commutation relations, duality requires that this spacing already be present in the action so that *all* the states implied by the commutation relations and parametrization invariance be explicitly displayed in the action. This is the embodiment of the interdependence between boundary condition and equations of motion. Hence we should set

$$m_0{}^2=m^2,\qquad \omega=\frac{4\pi m}{T}\,. \tag{4.19}$$

Similar expressions for the radiation emitted through tensor interaction may be written, e.g.,

$$A_{\mu_1\cdots\mu_n}^{(i)\mathrm{rad}}(x)=\frac{g_i^{(n)}}{m^n}\,\frac{1}{2(2\pi)i}\int d^4k\,e^{ik\cdot x}\epsilon(k_0)\delta(k^2-m_0{}^2-n\omega)\int_0^T d\lambda_i\,P_{i\mu_1}\cdots P_{i\mu_n}\,e^{-ik\cdot x_i(\lambda_i)}\,, \tag{4.20}$$

which corresponds to the emitted "particle" lying on the mother trajectory.

As noted in Sec. III, the timelike components of these tensors may be eliminated by means of the constraint (4.5) (Ref. 28); in the quantum case, however, (enormous) difficulties arise because of the normal ordering.[29] The daughter vertices, which are known to include higher derivatives of

P_i, arise by means of the "derivative coupling" mechanism outlined in Sec. III. It is unlikely, however, that the degeneracy of the dual models can be obtained from one action, which means that there is a lack of consistency between the action and the commutation relations. This can probably be seen in terms of the expressions for the Poincaré generators. Another constraint, tacitly assumed, is that the position coordinates transform as Lorentz vectors. Finally, there are *nonlinear* constraints on the coupling constants, obtained by demanding that the generators (2.13) and (2.15) form the Lie algebra of the Poincaré group.

So far we have brushed aside the question of crossing symmetry. However, when we compute the amplitude for a "dual atom" to emit certain types of radiation, we find, as is well known,[30] that crossing symmetry of the amplitude exists only for $m^2 = -1$. This means, in our approach, that for a real period, the energies of the "dual atom" in various excited states are complex. There does not seem to be any simple way to overcome this difficulty.[31]

In terms of specifics, we must regard our attempt at describing dual models in the form of action-at-a-distance theories as a first step towards writing the action for the system. We see, at least, that by requiring a strictly space-time description, we are naturally led away from a Hamiltonian formalism. More important it raises the possibility of expressing such models in terms of bootstrap theories. It should also give some incentive for the study of the scattering of two "dual atoms" where one might expect to simulate core effects, thus leading to a description of diffraction.

Note added in proof. After completion of this work, the existence of a Trieste report by P. Cordero and G. C. Ghirardi came to my attention with which this paper shows much overlap.

ACKNOWLEDGMENTS

The author thanks Professor F. Gürsey, Dr. S. Orfanidis, and M. Kalb and C. Marshall for many interesting discussions.

---

*Research supported by the U. S. Atomic Energy Commission under Contract No. AT(11-1) 3075.

[1]G. Mack and A. Salam, Ann. Phys. (N.Y.) 53, 174 (1969); D. J. Gross and J. Wess, Phys. Rev. D 2, 753 (1970); C. G. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (N.Y.) 59, 42 (1970); D. Boulware, L. Brown, and R. Peccei, Phys. Rev. D 2, 293 (1970); 3, 1750 (1971); G. Domokos and S. Kövesi-Domokos, Johns Hopkins Report No. C00-3285-2, 1972 (unpublished); S. Adler, Phys. Rev. D 6, 3445 (1972).

[2]K. Schwarzschild, Göttinger Nachrichten 128, 132 (1903); H. Tetrode, Z. Physik 10, 317 (1922); A. D. Fokker, *ibid.* 58, 386 (1929); Physica 9, 33 (1929); 12, 145 (1932).

[3]R. P. Feynman and J. A. Wheeler, Rev. Mod. Phys. 17, 157 (1945); 21, 425 (1949).

[4]F. Gürsey (unpublished). The Weyl invariance of the Fokker action has been noted by C. M. Andersen and H. C. von Baeyer, Phys. Rev. D 5, 2470 (1972).

[5]This equivalence is strictly true up to a field coming in from infinity, which cannot be experimentally distinguished. See P. Havas, Phys. Rev. 74, 456 (1948).

[6]To see the ferocity of these attempts, see R. P. Feynman's elegant remarks in Science 153, 699 (1966). Modern attempts are by F. Hoyle and J. V. Narlikar, Ann. Phys. (N.Y.) 54, 207 (1969); 62, 44 (1971); and by P. C. W. Davies, Proc. Cambridge Phil. Soc. 68, 751 (1970).

[7]Professor Gürsey has pointed out the existence of the mirror theory where only photons can be on mass shell and therefore have asymptotic states (private communication).

[8]Action at a distance with finite velocity was first discussed by Gauss. Articles in Ref. 3 contain references to earlier works.

[9]Our action is of the same form as that of J. W. Dettman and A. Schild, Phys. Rev. 95, 1057 (1954).

[10]H. Van Dam and E. P. Wigner, Phys. Rev. 142, 838 (1966).

[11]D. G. Currie, T. F. Jordan, and E. C. G. Sudarshan, Rev. Mod. Phys. 35, 350 (1963); D. G. Currie, J. Math. Phys. 4, 1470 (1963); J. T. Cannon and T. F. Jordan, *ibid.* 5, 299 (1964); H. Ekstein, Université d'Aix-Marseille report, 1964 (unpublished); H. Leutwyler, Nuovo Cimento 37, 556 (1965). For a nice review and many more references, see P. Havas, in *Statistical Mechanics of Equilibrium and Non-Equilibrium,* edited by J. Meixner (North-Holland, Amsterdam, 1965).

[12]P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949).

[13]P. Havas, Phys. Rev. 87, 309 (1952).

[14]Amnon Katz, J. Math. Phys. 10, 1929 (1969); in this reference the action contains an arbitrary function of S not the specific Green's function we have written in the text.

[15]H. Van Dam and E. P. Wigner, Phys. Rev. 138, B1576 (1965); 142, 838 (1966).

[16]P. A. M. Dirac, Proc. Roy. Soc. (London) A167, 148 (1938).

[17]This alternative has been noted by Katz in Ref. 14.

[18]A. N. Whitehead, *The Principle of Relativity* (Cam-

bridge Univ. Press, New York, 1922); also, J. L. Synge, Proc. Roy. Soc. (London) A211, 303 (1952).

[19]Such a field was introduced by J. A. Dyer and A. Schild, J. Math. Anal. Appl. 4, 328 (1962). A physical interpretation of this scalar field was given by F. Gürsey, Ann. Phys. (N.Y.) 24, 211 (1963).

[20]See E. C. G. Sudarshan, Fields and Quanta 2, 175 (1972), where the relation with indefinite metric theories is presented. Also E. C. G. Sudarshan, University of Texas Reports No. CPT 81 and No. CPT 122 (unpublished), and A. M. Gleeson, University of Texas Report No. CPT 151, 1972 (unpublished).

[21]V. Allessandrini, D. Amati, M. LeBellac, and D. Olive, Phys. Reports 1C, 269 (1971); P. Ramond, in *Boulder Lectures in Theoretical Physics*, edited by A. Barut and W. E. Brittin (Colorado Univ. Press, Boulder, Colo., 1972), Vol. XIV A.

[22]P. Ramond, Nuovo Cimento 4A, 544 (1971). We can say that if T is sufficiently small, one can only measure the average over the period.

[23]Strictly speaking, one has to include dilatations in order to consider such position operators.

[24]S. Fubini and G. Veneziano, Nuovo Cimento 67, 29 (1970). We are assuming at this stage that the positions of two different particles commute, which we cannot really justify.

[25]One can restore the Hamiltonian formalism by assuming the existence of an internal space in which there are an infinite number of degrees of freedom which represent a string. Then the trajectory function of the "dual atoms" is interpreted as the amplitude of the Nambu string. See Y. Nambu, Copenhagen Lectures, 1971 (unpublished). The definitive treatment of the string formalism has been formulated by L. N. Chang and F. Mansouri, Phys. Rev. D 5, 2535 (1972).

[26]L. Clavelli and P. Ramond, Phys. Rev. D 3, 988 (1971). P. Campagna, S. Fubini, E. Napolitano, and S. Sciuto, Nuovo Cimento 2A, 911 (1971).

[27]One should not attach too much importance to the word atom, otherwise we would have to consider a hydrogen atom which emits other hydrogen atoms to deexcite itself.

[28]In our formalism, we see explicitly how the parametrization invariance of the action eliminates ghosts. See F. Mansouri and Y. Nambu, Phys. Letters 39B, 375 (1972).

[29]Only lately has the no-ghost proof been given by R. C. Brower, Phys. Rev. D 5, 1655 (1972), and P. Goddard and C. B. Thorn, Phys. Letters 40B, 235 (1972).

[30]M. A. Virasoro, Phys. Rev. D 1, 2933 (1970).

[31]This condition, together with that of linear trajectories, requires the presence of a spin-one, massless "field," which is the price one pays in field theory for having local gauge invariance. Only a nonlinear procedure, like spontaneous symmetry breaking, is known to remedy such ills. Thus it seems that one must understand the spontaneous breaking of the parametrization invariance.