ξ-Limiting Process in Spontaneously Broken Gauge Theories*

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We show that the ξ-limiting regularization process of Lee and Yang can be recognized as a nonlinear gauge condition in the general non-Abelian gauge theory. The ξ-limiting process, however, differs from the general gauge-invariant formulation of the spontaneously broken gauge theories for finite ξ. Some of the problems related to the implementation of general gauge conditions are also briefly discussed.

I. INTRODUCTION

Following the pioneering work of Weinberg,[1] 't Hooft,[2] and Lee[3] the spontaneously broken gauge theory (SBGT) has been established as a renormalizable and unitary theory of massive vector particles.[4,5]

The practical applications of SBGT have been tried by various authors.[6-10] Jackiw and Weinberg[6] and also Bars and Yoshimura[7] found ambiguities in the finite quantity calculated in the U gauge; they used the weak correction to the muon magnetic moment as an example of these ambiguities.

These kinds of ambiguities have been resolved by the n-regularization method by 't Hooft and Veltman,[11] and also by the R_ξ-gauge formulation of SBGT.[12]

In the present note we would like to show that the ξ-limiting process[13] by Lee and Yang applied to the U-gauge Lagrangian can be recognized as a nonlinear gauge condition in SBGT. It is interesting to see that the ξ-limiting process, which has been proposed independently of the general non-Abelian gauge theory, can be nicely accommodated in SBGT as a limiting form of a nonlinear gauge condition. This explains why the ξ-limiting process always gave the correct answers[6-12] when combined with the U-gauge Lagrangian of SBGT. This also provides an interesting practical example[14] of the nonlinear gauge condition.

The ξ-limiting process, however, differs from the general gauge-invariant formulation of SBGT for *finite* ξ. For this reason we expect that the renormalization program of the U-gauge Lagrangian with the ξ-limiting process[13] may not be so convenient. The general gauge independence cannot be maintained by the ξ-limiting process at the intermediate stage of the renormalization program, although we expect that the final result based on this prescription is finite and non-Abelian–gauge–independent in the limit $\xi \to 0$.

II. A NONLINEAR-GAUGE CONDITION

A. Review of the Linear-Gauge Condition

We first briefly review the covariant linear-gauge condition (R_ξ gauge) used in the previous study.[12] We consider the Lagrangian by Georgi and Glashow[15] as an example. This Lagrangian is based on the group O(3), and it has the following form:

$$\mathcal{L} = \tfrac{1}{2}|\nabla_\mu \phi|^2 - V(\phi) - \tfrac{1}{4}(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu - g\vec{B}_\mu \times \vec{B}_\nu)^2 + \text{fermion part}. \tag{2.1}$$

The potential $V(\phi)$ induces the spontaneous breaking of the vacuum symmetry. The real triplet scalar ϕ takes the following form after this symmetry breaking:

$$\phi = \begin{pmatrix} S^+ \\ \psi + v \\ S^- \end{pmatrix}, \quad \langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}. \tag{2.2}$$

The covariant derivative in Eq. (2.1) is defined by

$$\nabla_\mu \phi = (\partial_\mu + ig\vec{B}_\mu \cdot \vec{T})\phi, \tag{2.3}$$

where

$$\vec{B}_\mu \cdot \vec{T} = \begin{pmatrix} A_\mu & -W_\mu^+ & 0 \\ -W_\mu^- & 0 & W_\mu^+ \\ 0 & W_\mu^- & -A_\mu \end{pmatrix} \tag{2.4}$$

and

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(B_\mu^1 \mp iB_\mu^2), \qquad A_\mu = B_\mu^3, \quad g \equiv -e. \tag{2.5}$$

The fermion part of Eq. (2.1) is not important for the following discussions; the potential $V(\phi)$ is given in Appendix B.

The linear gauge condition for the R_ξ gauge[12] can be written as

$$\partial_\mu A^\mu = C^3(x)\,,$$
$$\partial_\mu W^{+\mu} + i\frac{ev}{\xi} S^+ = C^+(x)\,, \qquad (2.6)$$

where $C^3(x)$ and $C^+(x)$ are arbitrary functions. Following 't Hooft[2] we can implement the gauge condition (2.6) by adding the following gauge term to the symmetric Lagrangian (2.1):

$$\mathcal{L}_c' = -\frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - \xi\left|\partial_\mu W^{+\mu} + \frac{iev}{\xi} S^+\right|^2. \qquad (2.7)$$

To get a meaningful S matrix by this prescription, we have to impose several restrictions on the form of $C(x)$ in Eq. (2.6). This will be discussed in Appendix A.

B. Nonlinear Gauge Condition

The nonlinear gauge condition which corresponds to the classical ξ-limiting process by Lee and Yang[13] can be written as

$$\partial_\mu A^\mu = C^3(x)\,,$$
$$\partial_\mu W^{+\mu} - ieA_\mu W^{+\mu} + i\frac{ev}{\xi} S^+ = C^+(x)\,. \qquad (2.8)$$

The only difference between Eq. (2.6) and Eq. (2.8) is that the derivative of W_μ^+ is now replaced by a "covariant" derivative $(\partial_\mu - ieA_\mu)W^{+\mu}$ in Eq. (2.8). The gauge term is given by (see also Appendix A)

$$\mathcal{L}_c = -\frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - \xi\left|\partial_\mu W^{+\mu} - ieA_\mu W^{+\mu} + i\frac{ev}{\xi} S^+\right|^2. \qquad (2.9)$$

Thus we finally get the effective Lagrangian from Eqs. (2.1) and (2.9):

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{L} + \mathcal{L}_c \\ &= |\partial_\mu S^+ + ievW_\mu^+ + ieW_\mu^+\psi - ieA_\mu S^+|^2 \\ &\quad + \tfrac{1}{2}|\partial_\mu\psi - ieW_\mu^+ S^- + ieW_\mu^- S^+|^2 \\ &\quad - \tfrac{1}{2}|\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - W_\nu^+ A_\mu)|^2 \\ &\quad - \tfrac{1}{4}|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)|^2 \\ &\quad - \xi\left|\partial_\mu W^{+\mu} - ieA_\mu W^{+\mu} + i\frac{ev}{\xi} S^+\right|^2 \\ &\quad - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - V(\phi) + \text{fermion part}.\end{aligned} \qquad (2.10)$$

The quadratic part of $\mathcal{L}_{\text{eff}}$ is given by

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{quad}} &= |\partial_\mu S^+|^2 - \frac{1}{\xi}M^2|S^+|^2 + \tfrac{1}{2}(\partial_\mu\psi)^2 - \tfrac{1}{2}m_\psi{}^2\psi^2 \\ &\quad - \tfrac{1}{2}|\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+|^2 + M^2|W_\mu^+|^2 - \xi|\partial_\mu W^{+\mu}|^2 \\ &\quad - \tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2\,,\end{aligned} \qquad (2.11)$$

where $M = ev$. The propagators for the various particles are identical to those in the R_ξ gauge[12]:

$$W_\mu^\pm:\ (-i)\frac{g_{\mu\nu} - k_\mu k_\nu(1-\xi)/(M^2 - \xi k^2)}{k^2 - M^2 + i\epsilon}\,, \qquad (2.12)$$

$$A_\mu:\ (-i)\frac{g_{\mu\nu} - (k_\mu k_\nu/k^2)(1-\alpha)}{k^2 + i\epsilon}\,, \qquad (2.13)$$

$$S^\pm:\ i\frac{1}{k^2 - (1/\xi)M^2 + i\epsilon}\,. \qquad (2.14)$$

The W propagator in Eq. (2.12) is the one first derived by Lee and Yang.[13] The difference between the R_ξ gauge and the present nonlinear gauge is given by

$$\begin{aligned}\Delta\mathcal{L}_{\text{eff}} &= eM(A_\mu W^{-\mu}S^+ + A_\mu W^{+\mu}S^-) - \xi e^2|A_\mu W^{+\mu}|^2 \\ &\quad + ie\xi(\partial_\nu W^{-\nu}A_\mu W^{+\mu} - \partial_\nu W^{+\nu}A_\mu W^{-\mu})\,.\end{aligned} \qquad (2.15)$$

The net effect of the nonlinear gauge (2.8) is to replace the trilinear coupling in the R_ξ gauge,

$$-eM(A_\mu W^{-\mu}S^+ + A_\mu W^{+\mu}S^-)\,, \qquad (2.16)$$

by the interaction

$$-e^2\xi|A_\mu W^{+\mu}|^2 + ie\xi(\partial_\nu W^{-\nu}A_\mu W^{+\mu} - \partial_\nu W^{+\nu}A_\mu W^{-\mu})\,. \qquad (2.17)$$

Note that the interaction (2.16) which exists in the R_ξ gauge is absent in the present nonlinear gauge. The gauge-compensating term is also modified by the nonlinear gauge condition, as we will discuss later.

Equation (2.17) is identical to those extra terms introduced by the ξ-limiting regularization into, e.g., the U-gauge Lagrangian of SBGT. Therefore the electromagnetic vertex of the vector boson is identical to that of the ξ-limiting process, and it has the form (see also Sec. III)

$$\begin{aligned}V_{\beta\mu\alpha} &= (ie)[g_{\alpha\beta}(l+l')_\mu - l_\beta g_{\mu\alpha} - l'_\alpha g_{\mu\beta} \\ &\quad + (g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha) + \xi(l'_\beta g_{\mu\alpha} + l_\alpha g_{\mu\beta})]\end{aligned} \qquad (2.18)$$

for

$$W_\alpha^-(l) + \gamma_\mu(q) \to W_\beta^-(l')\,.$$

The major difference between the present formulation and the ξ-limiting process applied to the U-

gauge Lagrangian is the following: We have unphysical scalars $S^{\pm}$ in Eq. (2.11) to cancel the unphysical poles in the vector-boson propagator, whereas the U-gauge Lagrangian with the ξ-limiting regularization contains no unphysical scalars. The S-matrix element evaluated in the present nonlinear gauge is therefore independent of ξ, and it is unitary for arbitrary ξ.[16] On the other hand, the unitarity is restored only in the limit $\xi=0$ by the ξ-limiting regularization of the U-gauge Lagrangian.

Those unphysical scalars $S^{\pm}$ are expected to decouple from the system in the limit $\xi\to 0$; they acquire an infinite mass in this limit. The gauge term in Eq. (2.9) also imposes the condition

$$|S^+|^2=0 \tag{2.19}$$

in the limit $\xi=0$. The perturbation theory, however, could show "anomalous" behavior in this limit. In fact we encountered such an example when we discussed the neutrino static charge. The unphysical scalars with an infinite mass gave a finite nonzero static charge to the neutrino.[12]

We can summarize what we have learned as follows:

(i) The nonlinear gauge condition (2.8) implemented by Eq. (2.11) gives rise to an S-matrix element which is independent of ξ, i.e., gauge independent.[16]

(ii) The nonlinear gauge condition (2.8) and the ξ-limiting regularization of the U-gauge Lagrangian agree with each other in the limit $\xi\to 0$ *if* the perturbation theory behaves as well as the formal manipulation indicates.

We have thus established that the classical ξ-limiting process can be regarded as a limiting form of the nonlinear gauge condition (2.8) in the general framework of SBGT.

We would like to conclude this section with some comments on the formulation of SBGT. The effective action of SBGT is given by

$$S_{\rm eff}=\int d^4x\,\mathcal{L}_{\rm eff}+S'\,, \tag{2.20}$$

where S' is the gauge-compensating term. It can be written as (see also Appendix A)

$$S'=(-i)\,{\rm Tr}\ln(1+\mathcal{G}\cdot\gamma)\,, \tag{2.21}$$

where

$$\begin{bmatrix} -\left(\partial^2+\dfrac{M^2}{\xi}-i\epsilon\right) & 0 & 0\\ 0 & -(\partial^2-i\epsilon) & 0\\ 0 & 0 & -\left(\partial^2+\dfrac{M^2}{\xi}-i\epsilon\right)\end{bmatrix}\cdot\mathcal{G}=\delta^4(x-y) \tag{2.22}$$

and

$$\gamma\equiv\gamma_1+\gamma_2\,, \tag{2.23}$$

with

$$\gamma_1=\begin{bmatrix} ie\partial_\mu A^\mu-\dfrac{eM}{\xi}\psi & -ie\partial_\mu W^{+\mu}+\dfrac{eM}{\xi}S^+ & 0\\ -ie\partial_\mu W^{-\mu} & 0 & ie\partial_\mu W^{+\mu}\\ 0 & ie\partial_\mu W^{-\mu}+\dfrac{eM}{\xi}S^- & -ie\partial_\mu A^\mu-\dfrac{eM}{\xi}\psi\end{bmatrix}\delta^4(x-y) \tag{2.24}$$

and

$$\gamma_2=\begin{bmatrix} e^2(A_\mu)^2-e^2|W_\mu^+|^2+ieA^\mu\partial_\mu & ieW^{+\mu}\partial_\mu-e^2A^\mu W_\mu^+ & e^2W_\mu^+W^{+\mu}\\ 0 & 0 & 0\\ e^2W_\mu^-W^{-\mu} & -ieW^{-\mu}\partial_\mu-e^2A^\mu W_\mu^- & e^2(A_\mu)^2-e^2|W_\mu^+|^2-ieA_\mu\partial^\mu\end{bmatrix}\delta^4(x-y)\,. \tag{2.25}$$

Note that $\partial_\mu A^\mu$, for example, in the matrix γ_1, stands for $\partial_\mu A^\mu\equiv(\partial_\mu A^\mu)+A^\mu\partial_\mu$. γ_1 corresponds to the linear gauge condition (2.6), and γ_2 is the additional contribution from the nonlinear term in Eq. (2.8). For sufficiently small ξ, we have the following form[17] for $\mathcal{G}\cdot\gamma$:

$$\mathcal{G}\cdot\gamma\approx\begin{bmatrix}\frac{1}{-(\partial^2+\xi^{-1}M^2-i\epsilon)}\left(\frac{-eM}{\xi}\right)\psi & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \frac{1}{-(\partial^2+\xi^{-1}M^2-i\epsilon)}\left(\frac{-eM}{\xi}\right)\psi\end{bmatrix}\delta^4(x-y)\,. \tag{2.26}$$

This is the regulated form of the well-known gauge compensating term for the U gauge.[5] The "fictitious" particle has the propagator

$$\frac{i}{k^2-\xi^{-1}M^2+i\epsilon}\ \underset{\xi\to 0}{\sim}\ \xi\frac{i}{(-M^2)}\ . \tag{2.27}$$

It should be noted that the unphysical scalar propagator in Eq. (2.14) and the fictitious particle propagator in Eq. (2.27) both have the same structure as the unphysical part of the W-boson propagator in Eq. (2.12).

III. DISCUSSION

The vector-boson propagator in Eq. (2.12) has the following property

$$(-i)\frac{g_{\mu\nu}-k_\mu k_\nu(1-\xi)/(M^2-\xi k^2)}{k^2-M^2+i\epsilon}\ \underset{k^2\to M^2}{\sim}\ (-i)\frac{g_{\mu\nu}-k_\mu k_\nu/M^2}{k^2-M^2+i\epsilon}\ . \tag{3.1}$$

It takes the canonical form at the physical pole position. This property is important when one performs an actual calculation; we can use the canonical form of the projection operator for the external vector boson. For the internal lines of the vector-boson propagator, the following choice ($\xi=1$) of the propagator simplifies the calculation:

$$(-i)\frac{g_{\mu\nu}}{k^2-M^2+i\epsilon}\ . \tag{3.2}$$

This corresponds to the generalized Feynman gauge discussed by 't Hooft.[2] For this special choice of ξ, the vertex function in Eq. (2.18) takes the interesting form

$$V_{\beta\mu\alpha}=(ie)[g_{\alpha\beta}(l+l')_\mu+2(g_{\alpha\mu}q_\beta-g_{\beta\mu}q_\alpha)]\,. \tag{3.3}$$

The charge-conservation condition is manifestly satisfied by Eq. (3.3); the gyromagnetic $g_w=2$ is also explicit.

For simple lower-order calculations, Eqs. (3.2) and (3.3) provide very convenient Feynman rules. We have, however, an extra photon-S-S coupling compared to the U-gauge Lagrangian to preserve non-Abelian gauge independence. The unphysical scalar S^- has the electromagnetic vertex

$$(-ie)(l+l')_\mu \tag{3.4}$$

for

$$S^-(l)+\gamma_\mu(q)\to S^-(l')\,.$$

For higher-order calculations, the limiting form at $\xi\approx 0$ may be convenient; the gauge compensating term takes the simplest form in this limit.

The renormalization program in the present nonlinear gauge can be best studied if one combines it with the n-regularization method by 't Hooft and Veltman.[11] The ξ-independence of the physical S-matrix element provides a convenient check of the calculation.

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APPENDIX A

In this appendix we shall review the prescription to implement general gauge conditions first discussed by 't Hooft,[2] and discuss several conditions we should impose on the gauge functions $C(x)$ in Eq. (2.8). We discuss the gauge condition on the W boson:

$$\partial_\mu W^{+\mu}-ieA_\mu W^{+\mu}+i\frac{ev}{\xi}S^+=C(x)\,. \tag{A1}$$

We denote the S-matrix in this gauge by $\langle S\rangle_c$. Thanks to the gauge independence of the S-matrix, one may replace $\langle S\rangle_c$ by

$$\langle S\rangle=\frac{\int(dc)\langle S\rangle_c\exp(-i\beta\int d^4x\,|C(x)|^2)}{\int(dc)\exp(-i\beta\int d^4x\,|C(x)|^2)}\,, \qquad \text{(A2)}$$

where β is an arbitrary constant, and $\langle S\rangle_c$ is independent of β. The exponential factor in Eq. (A2) corresponds to the gauge term

$$\mathcal{L}_c=-\beta\left|\partial_\mu W^{+\mu}-ieA_\mu W^{+\mu}+i\frac{ev}{\xi}S^+\right|^2. \qquad \text{(A3)}$$

The limit $\beta\to\infty$ reduces the gauge condition to $C(x)=0$. However, one may lose the gauge condition in the limit $\beta=0$, and the theory may become "underdetermined." The special choice, $\beta=\xi$, adopted in the present study resolves this difficulty; it also simplifies the theory by eliminating undesirable cross terms in the free part of the Lagrangian. By this special choice of β, the gauge condition is retained for arbitrary β ($=\xi$). We can thus smoothly interpolate between the U gauge and the general R gauge.[18] The gauge invariance of the underlying Lagrangian is now characterized by the ξ independence of $\langle S\rangle$.

To get a meaningful $\langle S\rangle$, we should make sure that there is actually a gauge which satisfies Eq. (A1) for $C(x)$ with the constraint $|C(x)|^2\lesssim 1/\beta$. This is important for nonlinear gauge conditions (see, e.g., Ref. 21).

To make the perturbation theory well defined, we should also satisfy the following requirements:

(i) The gauge function $C(x)$ in Eq. (A1) should contain one or more linear terms in gauge fields.[19]

(ii) Propagators for gauge fields should be influenced by the gauge term added to the Lagrangian [our simple form in Eq. (A3) satisfies this condition]. The physical poles in the propagator, however, should not be modified by this gauge term.[20]

In the present framework, therefore, it is difficult to handle a special class of nonlinear gauge conditions such as $A_\mu A^\mu=\lambda=$ constant of Dirac and Nambu[21]; a straightforward exponentiation of $A_\mu A^\mu-\lambda$ following Eq. (A3) shifts the physical pole of the photon propagator.

One way to handle the condition $A_\mu A^\mu=\lambda$ is to go back to the basic formulation of the general gauge theory, and treat it as a Landau-type condition[5]

$$\delta(A_\mu A^\mu-\lambda)=\frac{\delta(A^0-(\lambda+\vec{A}^2)^{1/2})+\delta(A^0+(\lambda+\vec{A}^2)^{1/2})}{2(\lambda+\vec{A}^2)^{1/2}}. \qquad \text{(A4)}$$

If one eliminates A^0 from the Lagrangian and expands every term in the Lagrangian in terms of $\vec{A}/\sqrt{\lambda}$ one recovers Nambu's result.[22] The denominator factor $(\lambda+\vec{A}^2)^{1/2}$ in Eq. (A4) is canceled by a part of the gauge-compensating term.[23]

Finally, a brief comment on the gauge compensating term S' in Eq. (2.21) is in order. Equation (2.9) in our formulation has an appearance of a Fermi-type gauge condition. On the other hand the gauge compensating term S' in Eq. (2.21) corresponds to a Landau-type gauge condition. The unitarity of the S-matrix based on Eqs. (2.9) and (2.21), namely consistency between these two equations, can be formally shown by a straightforward generalization of the method utilized by Fradkin and Tyutin.[19] The existence of an *identity* due to the local gauge invariance of the original Lagrangian is essential in this discussion. 't Hooft[2] also gives another proof of unitarity based on Ward-Takahashi identities.

APPENDIX B

The potential in Eq. (2.1) is given by

$$\begin{aligned}V(\phi)&=\tfrac{1}{2}\mu_0^2|\phi|^2+\lambda|\phi|^4\\&=\tfrac{1}{2}m_\psi^2\psi^2+\lambda[4v\psi(2S^+S^-+\psi^2)\\&\qquad+(2S^+S^-+\psi^2)^2]\\&\quad+(\mu_0^2+4v^2\lambda)(S^+S^-+v\psi)\,,\end{aligned} \qquad \text{(B1)}$$

where

$$m_\psi^2=\mu_0^2+12\lambda v^2.$$

$\mu_0^2<0$ and we have the condition

$$\mu_0^2+4v^2\lambda=0 \qquad \text{(B2)}$$

in the tree approximation.

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[1]S. Weinberg, Phys. Rev. Letters 19, 1264 (1967); 27, 1688 (1971).

[2]G. 't Hooft, Nucl. Phys. B35, 167 (1971); B33, 173 (1971).

[3]B. W. Lee, Phys. Rev. D 5, 823 (1972). This article contains many references to related works.

[4]B. W. Lee and J. Zinn-Justin, Phys. Rev. D 5, 3121 (1972); 5, 3137 (1972); 5, 3155 (1972). See also T. W. Appelquist and H. R. Quinn, Phys. Letters 39B, 229 (1972).

[5]A. Salam and J. Strathdee, Nuovo Cimento 11A, 397 (1972).

[6]R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2396 (1972).

[7]I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972).

[8]S. Y. Lee, Phys. Rev. D 6, 1701 (1972); 6, 1803 (1972); T. W. Appelquist, J. R. Primack, and H. R. Quinn,

ibid. 6, 2998 (1972).

[9]J. R. Primack and H. R. Quinn, Phys. Rev. D 6, 3171 (1972).

[10]W. A. Bardeen *et al.*, Nucl. Phys. B46, 319 (1972).

[11]G. 't Hooft and M. T. Veltman, Nucl. Phys. B44, 189 (1972).

[12]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972). A similar gauge condition in an Abelian theory was also discussed by Y.-P. Yao, Phys. Rev. D (to be published).

[13]T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962). See also T. D. Lee, Phys. Rev. 128, 899 (1962).

[14]After the completion of the present work, I learned that Gervais and Neveu discussed a class of nonlinear gauge conditions similar to ours in connection with the dual resonance model. J. L. Gervais and A. Neveu, Nucl. Phys. B46, 381 (1972). I am grateful to Professor B. Lee for calling their work to my attention.

[15]H. Georgi and S. L. Glashow, Phys. Rev. Letters 28, 1494 (1972).

[16]We have checked this ξ independence for simple S-matrix elements, the anomalous magnetic moment of the muon and the static charge of the neutrino. See also Refs. 4, 5, and 12. These works are based on the following work by Faddeev and Popov: L. D. Faddeev and V. N. Popov, Phys. Letters 25B, 29 (1967), and Kiev Report No. ITF-67-36, 1967 (unpublished).

[17]Here we assume that $S^{\pm}$ decouple from the system in the limit $\xi = 0$.

[18]Because of the ξ dependence of the W-boson vertex, the Feynman rules are not well defined at $\xi = \infty$. The S matrix is, however, well defined in the limit $\xi \rightarrow \infty$. In this respect, the R_ξ gauge in Ref. 12 is better defined than the present gauge condition.

[19]For the more precise meaning of this condition, see E. S. Fradkin and I. V. Tyutin, Phys. Rev. D 3, 2841 (1970). These authors gave essentially the same prescription to implement general gauge conditions in SBGT as that given by 't Hooft.

[20]The physical pole shifted by the gauge term could be restored to the original position by the gauge-compensating term. The perturbation procedure, however, would become rather complicated.

[21]Y. Nambu, Suppl. Progr. Theoret. Phys. (Kyoto), Extra Number, 190 (1968); P. A. M. Dirac, Proc. Roy. Soc. (London) A209, 291 (1951).

[22]Eq. (A4), however, preserves the charge-conjugation symmetry of the original Lagrangian. We can thus avoid, e.g., a three-photon coupling.

[23]This fact explains why Nambu (Ref. 21) obtained Lorentz-invariant S-matrix elements based on a procedure which is not manifestly Lorentz-invariant. Although the gauge-compensating term does not appear in the lower-order processes discussed by Nambu, it becomes important when one considers higher-order processes.