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Explicit and Spontaneous Chiral-Symmetry Breaking in Terms of Asymptotic Fields

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The Hamiltonian density for asymptotic scalar- and pseudoscalar-meson fields is analyzed in the presence of explicit and spontaneous breaking of chiral SU(3)×SU(3) symmetry down to $SU_I(2)\times U_Y(1)$. It is found that the representations $(3,3^*)+(3^*,3)$, $(1,8)+(8,1)$, $(6,6^*)+(6^*,6)$, and $(8,8)$ (and the singlet) are all present. The connection of the results with earlier work is discussed.

It is usually assumed that the part of the hadron Hamiltonian density which explicitly breaks chiral SU(3)×SU(3) symmetry down to the level of isospin and hypercharge transforms as[1] a $(3,3^*)+(3^*,3)$ representation of the group. A number of authors have considered as alternatives the representations $(1,8)+(8,1)$, $(6,6^*)+(6^*,6)$, and $(8,8)$. On the assumption that, to lowest order in symmetry breaking, the asymptotic (in- or out-) fields should transform linearly (in the limit of no explicit breaking, we know that they must[2]),

$$[T_A, S_j] = i f_{Ajk} S_k, \quad [T_A, P_j] = i f_{Ajk} P_k,$$
$$[X_A, S_j] = -i d_{Ajk} P_k, \quad [X_A, P_j] = i d_{Ajk} S_k, \tag{1}$$

where T_A and X_A (A runs from 1 to 8) are SU(3) and chiral generators, and where S_j and P_j (j runs from 0 to 8) are scalar and pseudoscalar asymptotic fields, the hadron Hamiltonian density

$$H = (\text{kinetic terms}) + \tfrac{1}{2}(m_\pi^2\pi^2 + m_K^2K^2 + m_\eta^2\eta^2 + m_{\eta'}^2\eta'^2 + M_\delta^2\delta^2 + M_\kappa^2\kappa^2 + M_\sigma^2\sigma^2 + M_{\sigma'}^2\sigma'^2) \tag{2}$$

has been shown[3] to contain, in addition to the singlet, all of the four above-mentioned representations. Specifically, the analysis of Ref. 3 yielded

$$\begin{aligned} H = H_0 &+ \{A_{33}[(d,\bar{d}) + (u,\bar{u})] + B_{33}(s,\bar{s}) + \text{c.c.}\} + \{A_{18}[(2M_3{}^3 - M_1{}^1 - M_2{}^2, M_\alpha{}^\alpha) + \text{c.c.}]\} \\ &+ \{A_{66}[(M_{11}, M^{11}) + (M_{22}, M^{22}) + (M_{12}, M^{12})] + B_{66}[(M_{13}, M^{13}) + (M_{23}, M^{23})] + C_{66}(M_{33}, M^{33}) + \text{c.c.}\} \\ &+ \{A_{88}[(M_1{}^1, M_1{}^1) + (M_2{}^2, M_2{}^2)] + B_{88}[(M_1{}^1, M_2{}^2) + (M_2{}^2, M_1{}^1)] + (-B_{88} + A_{88})[(M_2{}^1, M_1{}^2) + (M_1{}^2, M_2{}^1)] \\ &\quad + (A_{88} + B_{88})[2(M_3{}^3, M_3{}^3) - (M_3{}^3, M_1{}^1) - (M_1{}^1, M_3{}^3) - (M_3{}^3, M_2{}^2) - (M_2{}^2, M_3{}^3)] \\ &\quad + C_{88}[(M_1{}^3, M_3{}^1) + (M_2{}^3, M_3{}^2) + (M_3{}^1, M_1{}^3) + (M_3{}^2, M_2{}^3)]\}\,. \end{aligned} \tag{3}$$

The u, d, s stand for the members of the quark triplet; $M_{\alpha\beta}$ ($M^{\alpha\beta}$) stands for the members of the $\underline{6}$ ($\underline{6}^*$) representation of SU(3). $M_\beta{}^\alpha$ stands for the members of the SU(3) nonet, and $M_\alpha{}^\alpha$ for the corresponding trace. The first (second) term in the parentheses (A, B) refers to the transformation property under the left-hand (right-hand) SU(3) of chiral SU(3)×SU(3). Finally, c.c. refers to interchange of A and B in the parentheses (A, B). H_0, the invariant part of the Hamiltonian density, has the form

$$H_0 = (\text{kinetic terms}) + A_{00}(M_\alpha{}^\alpha, M_\beta{}^\beta). \tag{4}$$

The coefficients in Eqs. (3) and (4) are expressed in terms of the scalar and pseudoscalar masses and mixing angles[4] as follows:

$$2A_{00} = \tfrac{1}{9}(8M_\kappa{}^2 + 6M_\delta{}^2 + 2M_\sigma{}^2 + 2M_{\sigma'}{}^2) + (S \to P),$$

$$2A_{33} = M_\kappa{}^2 + \tfrac{1}{3}(M_\sigma{}^2 - M_{\sigma'}{}^2)[\cos 2\theta - (1/2\sqrt{2})\sin 2\theta] - (S \to P),$$

$$2B_{33} = \tfrac{3}{2}M_\delta{}^2 - \tfrac{1}{4}(M_\sigma{}^2 + M_{\sigma'}{}^2) + \tfrac{1}{12}(M_\sigma{}^2 - M_{\sigma'}{}^2)(\cos 2\theta + 2\sqrt{2}\sin 2\theta) - (S \to P),$$

$$2A_{18}=\tfrac{1}{18}[4M_\kappa{}^2-6M_\delta{}^2+(M_\sigma{}^2+M_{\sigma'}{}^2)+(M_\sigma{}^2-M_{\sigma'}{}^2)(\cos 2\theta+2\sqrt{2}\sin 2\theta)]+(S\to P),$$

$$2A_{66}=\tfrac{1}{2}M_\delta{}^2+\tfrac{1}{4}(M_\sigma{}^2+M_{\sigma'}{}^2)-\tfrac{1}{12}(M_\sigma{}^2-M_{\sigma'}{}^2)(\cos 2\theta+2\sqrt{2}\sin 2\theta)-(S\to P),$$

$$2B_{66}=M_\kappa{}^2-\tfrac{1}{3}(M_\sigma{}^2-M_{\sigma'}{}^2)[\cos 2\theta-(1/2\sqrt{2})\sin 2\theta]-(S\to P),$$

$$2C_{66}=\tfrac{1}{2}(M_\sigma{}^2+M_{\sigma'}{}^2)+\tfrac{1}{6}(M_\sigma{}^2-M_{\sigma'}{}^2)(\cos 2\theta+2\sqrt{2}\sin 2\theta)-(S\to P),$$

$$2A_{88}=\tfrac{1}{18}[-8M_\kappa{}^2-6M_\delta{}^2+7(M_\sigma{}^2+M_{\sigma'}{}^2)-(M_\sigma{}^2-M_{\sigma'}{}^2)(\cos 2\theta+2\sqrt{2}\sin 2\theta)]+(S\to P),$$

$$2B_{88}=\tfrac{1}{18}[-8M_\kappa{}^2+12M_\delta{}^2-2(M_\sigma{}^2+M_{\sigma'}{}^2)+2(M_\sigma{}^2-M_{\sigma'}{}^2)(\cos 2\theta+2\sqrt{2}\sin 2\theta)]+(S\to P),$$

$$2C_{88}=-\tfrac{2}{3}(M_\sigma{}^2-M_{\sigma'}{}^2)[\cos 2\theta-(1/2\sqrt{2})\sin 2\theta]+(S\to P),$$

where $(S\to P)$ stands for all the terms preceding it with scalar masses and mixing angle replaced by their pseudoscalar counterparts. Of the terms in Eq. (3), those in the first curly bracket transform according to the $(3,3^*)+(3^*,3)$ representation, those in the second according to $(1,8)+(8,1)$, those in the third according to $(6,6^*)+(6^*,6)$, and those in the fourth according to $(8,8)$.

The authors of Ref. 3 went on to determine, for example, under what conditions on the $0^\pm$ masses certain representations in Eq. (3) might be absent or might have a simple SU(3) content. The purpose of this note is to cast their result, Eqs. (3) and (4), into a more accessible form, to include the effect of spontaneous symmetry breaking, and to comment on the relevance of this to a recent investigation[5] of the $(3,3^*)+(3^*,3)$ model.

Now that the original form of the Hamiltonian density in terms of asymptotic fields has been decomposed into the various representations, the terms for each representation can be separately expressed in terms of the asymptotic fields:

$$H_{(1,1)}=(\delta^2+\kappa^2+S_8{}^2+S_0{}^2)(\tfrac{1}{4}A_{00})+[S\to P], \tag{5a}$$

$$H_{(3,3^*)+(3^*,3)}=\delta^2(\tfrac{1}{4}B_{33})+\kappa^2(\tfrac{1}{4}A_{33})+S_8{}^2(\tfrac{1}{3}A_{33}-\tfrac{1}{12}B_{33})-S_0{}^2(\tfrac{1}{3}A_{33}+\tfrac{1}{6}B_{33})+\tfrac{1}{6}\sqrt{2}\;S_8S_0(A_{33}-B_{33})-[S\to P], \tag{5b}$$

$$H_{(1,8)+(8,1)}=-\delta^2(\tfrac{1}{2}A_{18})+\kappa^2(\tfrac{1}{4}A_{18})+S_8{}^2(\tfrac{1}{2}A_{18})-\sqrt{2}\;S_8S_0(A_{18})+[S\to P], \tag{5c}$$

$$\begin{aligned}H_{(6,6^*)+(6^*,6)}=\;&\delta^2(\tfrac{1}{4}A_{66})+\kappa^2(\tfrac{1}{4}B_{66})+S_8{}^2(\tfrac{1}{4}A_{66}-\tfrac{1}{3}B_{66}+\tfrac{1}{3}C_{66})\\&+S_0{}^2(\tfrac{1}{2}A_{66}+\tfrac{1}{3}B_{66}+\tfrac{1}{6}C_{66})+\sqrt{2}\;S_8S_0(\tfrac{1}{2}A_{66}-\tfrac{1}{6}B_{66}-\tfrac{1}{3}C_{66})-[S\to P],\end{aligned} \tag{5d}$$

$$\begin{aligned}H_{(8,8)}=\;&\delta^2(\tfrac{1}{4}B_{88})-\tfrac{1}{4}\kappa^2(A_{88}+B_{88})+S_8{}^2(\tfrac{1}{2}A_{88}+\tfrac{1}{4}B_{88}-\tfrac{1}{3}C_{88})\\&+S_0{}^2(\tfrac{1}{2}A_{88}+\tfrac{1}{3}C_{88})-\sqrt{2}\;S_8S_0(\tfrac{1}{2}B_{88}+\tfrac{1}{6}C_{88})+[S\to P],\end{aligned} \tag{5e}$$

where $H_{(1,1)}$ is the singlet part of H_0 coming from the mass terms in H, and where $[S\to P]$ stands for all the terms preceding it with scalar asymptotic fields replaced by their pseudoscalar counterparts.

If the pion and kaon are to decay into leptons, and to do so with unequal decay constants [experimentally,[6] $F_K/F_\pi f_+(0)=1.27\pm0.03$], then it can be shown[2] that certain of the commutators in Eqs. (1) cannot have a vanishing vacuum expectation value. But the physical asymptotic fields, which only create and destroy physical one-particle states, cannot have a vacuum expectation value. So, in Eqs. (1) we must replace S_i by S_i+C_i, with c numbers $C_i=\delta_{i0}C_0+\delta_{i8}C_8$. This implies that the symmetry of the vacuum is spontaneously broken down to isospin and hypercharge. (In the limit of no explicit breaking, then, the nine pseudoscalar mesons and the κ meson would be massless, Goldstone bosons; since in that limit the linear transformations are exact, the $0^\pm$ bosons should then all be degenerate in mass, and, due to the spontaneous breakdown, all become massless.[7]) The resulting commutation relations were proposed in Ref. 2.

In the presence of spontaneous breaking, the Hamiltonian in terms of the physical asymptotic fields still must be in the form of Eq. (2). But the decomposition of H into the various representations must be done with fields which transform linearly *with no inhomogeneous term*, i.e., with fields $S_i'\equiv S_i+C_i$ and $P_i'\equiv P_i$. Therefore, expressing H in terms of S' and P' yields the same decomposition as before [Eqs. (5) with S,P replaced by S',P'], but produces a term linear in S', or, up to a c number, linear in S. It must be realized that because $\langle 0|S'|0\rangle\neq0$, in each of the various representations there is a part linear in S, the sum of all which can exactly cancel the aforementioned extra term if one chooses to add up Eqs.

(5) and this term to reconstruct H as given in Eq. (2). In summary, the net effect of the spontaneous breaking is to require the replacement of $S_0\,(S_8)$ by $S_0+C_0\;(S_8+C_8)$ in Eqs. (5) and to add on to Eq. (5b) the term

$$H^{\text{extra}}_{(3,3^*)+(3^*,3)} = -S_8(C_8M_8^2+C_0\,\Delta M^2) - S_0(C_0M_0^2+C_8\,\Delta M^2), \tag{6}$$

with

$$\Delta M^2 \equiv \tfrac{1}{2}[\sin 2\theta(M_\sigma^2 - M_{\sigma'}^2)].$$

The values of C_0 and C_8 can be obtained[2,5] from the experimental values of F_K and F_π.

Reference 5, which utilized the chiral SU(3)× SU(3) transformation properties of the $0^\pm$ asymptotic fields, studied the consequences of assuming that, at least for evaluating matrix elements $\langle 0|[G,H]|1\rangle$ and $\langle 0|[G,[G,H]]|0\rangle$ [G being any chiral SU(3)×SU(3) generator], one could effectively equate H as given by Eq. (2) with H as given in the $(3,3^*)+(3^*,3)$ model, $\epsilon_0 u_0+\epsilon_8 u_8$ [$u_i(x)$ being 0^+ densities which along with 0^- densities $v_i(x)$ belong to a $(3,3^*)+(3^*,3)$ representation]. It was thought that perhaps the spontaneous breakdown mechanism, which permits these matrix elements to differ from zero, might conspire so as to make the contribution from representations other than $(3,3^*)+(3^*,3)$ negligible. This assumption yielded just the equations of the Glashow-Weinberg[1] model; the analysis by use of the transformation properties of the asymptotic fields also yields relations[2,5] among the various leptonic decay constants, which then makes solution[5] of the model possible. Nevertheless, it is appropriate to check the consistency of this assumption.

If one takes, instead of $\epsilon_0 u_0+\epsilon_8 u_8$, the $(3,3^*)+(3^*,3)$ contribution given by the sum of Eq. (5b) (with S replaced by S') and Eq. (6), and tentatively equates it with the entire Hamiltonian of Eq. (2) for the purpose of computing, say, $\langle 0|[G,H]|1\rangle$, the resulting conditions on the $0^\pm$ masses cannot be satisfied. Therefore, the treatment of broken chiral symmetry given in Ref. 5 was not internally consistent.

Nonetheless, the detailed demonstration of the nature of two complementary solutions [Hamiltonian nearly chiral SU(2)×SU(2) symmetric[8] vs Hamiltonian nearly SU(3) symmetric[9]] of the $(3,3^*)+(3^*,3)$ model did reveal a genuine, intrinsic feature of the model, which should persist in any alternative, but internally consistent, solution, if no explicit or implicit assumptions (not necessarily directly about the approximate symmetry of H) are made which bias the theory so as to exclude the possibility of obtaining one or the other of the solutions. For example, certain indirect assumptions might amount to $\langle 0|u_8|0\rangle \simeq 0$ or[7] $\langle 0|v_K|K\rangle \simeq \langle 0|v_\pi|\pi\rangle$, either of which will favor $\epsilon_8/\epsilon_0 \simeq -\sqrt{2}$, the popular value.

At any rate, it is our expectation that the simple, and hence attractive, assumption that H transforms purely as a $(3,3^*)+(3^*,3)$ representation will ultimately prove untenable, even in theories which are not concerned with the transformation properties of asymptotic fields.

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[1]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[2]L. Bessler, T. Muta, H. Umezawa, and D. Welling, Phys. Rev. D 2, 349 (1970). For a proof that, in the limit of no explicit and no spontaneous breaking, the asymptotic fields transform linearly, see J. T. Lopuszanski, J. Math. Phys. 12, 2401 (1971).

[3]N. Papastamatiou, H. Umezawa, and D. J. Welling, Phys. Rev. D 3, 2267 (1971).

[4]In Ref. 3, the mixing is defined by: $\eta=P_8\cos\varphi+P_0\sin\varphi$, $\eta'=-P_8\sin\varphi+P_0\cos\varphi$, $\sigma=S_8\cos\theta+S_0\sin\theta$, and $\sigma'=-S_8\sin\theta+S_0\cos\theta$.

[5]L. Bessler and D. Welling, Nuovo Cimento Lett. 4, 466 (1972); Phys. Rev. D 6, 1092 (1972).

[6]L.-M. Chounet, J.-M. Gaillard, and M. K. Gaillard, Phys. Rep. 4C, 199 (1972).

[7]For a discussion of this, see Ref. 5.

[8]This solution was obtained by Gell-Mann, Oakes, and Renner (Ref. 1).

[9]R. Brandt and G. Preparata, Phys. Rev. D 7, 218 (1973).