Gauge Model of Weak and Electromagnetic Interactions with No Neutral Currents or Heavy Leptons

Ngee-Pong Chang* and Ernest Ma†

Department of Physics, City College of the City University of New York, New York, New York 10031

(Received 5 February 1973)

A gauge-invariant model in which the only gauge particles are the photon and the W boson is constructed without the introduction of heavy leptons. The resulting Lagrangian is nonpolynomial; the question of its renormalizability is discussed.

To obtain a unified description of weak and electromagnetic interactions, several models[1-3] have been proposed, each based on the local gauge invariance of some non-Abelian group.[4] A common feature of these models is that the leptons (and hadron quarks) are assigned to linear representations of the gauge group and the resulting minimally gauge-invariant Lagrangian is of simple polynomial form and, as has been proven by 't Hooft[5] and by Lee and Zinn-Justin,[6] is renormalizable. In all these cases, however, there are present (1) neutral currents and/or (2) heavy leptons, neither of which have yet been observed. We propose here to assign the leptons (and hadron quarks) to nonlinear representations of the gauge group, resulting in a nonpolynomial Lagrangian, and find that we do not have to introduce either neutral currents or heavy leptons as before. The usual renormalization procedure[5,6] is, of course, no longer applicable, but there are indications, based on the work by Efimov,[7] Fradkin,[8] Salam and co-workers,[9] Shafi,[10] and others,[11] that this theory may nevertheless be finite.

Following Georgi and Glashow,[2] we base our model on an SO(3) gauge group. (In fact, we differ only in the treatment of the leptonic part.) Taking the Lagrangian

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}_{\mu\nu} - \tfrac{1}{2}|(\partial_\mu - ig\vec{A}_\mu\cdot\vec{t})\Phi|^2 - F(\Phi^2)\\ &-\bar{\nu}\gamma_\mu\partial_\mu\nu - \bar{e}\gamma_\mu(\partial_\mu - igB_\mu^0)e - m\bar{e}e\\ &+ig'[\bar{\nu}\gamma_\mu(1+\gamma_5)eB_\mu^- + \bar{e}\gamma_\mu(1+\gamma_5)\nu B_\mu^+],\end{aligned}$$

where

$$\vec{G}_{\mu\nu} = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu + g\vec{A}_\mu\times\vec{A}_\nu,$$

$$\Phi = \begin{pmatrix}\phi^+\\ \phi^0\\ \phi^-\end{pmatrix} = e^{ig(\omega_1 t_1 + \omega_2 t_2)}\begin{pmatrix}0\\ \eta\\ 0\end{pmatrix},$$

$$t_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}, \quad t_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&-i&0\\ i&0&-i\\ 0&i&0\end{pmatrix},$$

$$t_3 = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix},$$

$$\begin{aligned}B_\mu^a = &[e^{-ig(\omega_1 t_1+\omega_2 t_2)}]_{ab}A_\mu^b\\ &+\frac{1}{2g}\epsilon_{abc}[\partial_\mu e^{-ig(\omega_1 t_1+\omega_2 t_2)}]_{bd}\\ &\times[e^{ig(\omega_1 t_1+\omega_2 t_2)}]_{dc},\end{aligned}$$

and F is a suitably chosen polynomial; we note that, under a local gauge transformation $\vec{\alpha}$, if we let

$$\Phi \to e^{ig\vec{\alpha}\cdot\vec{t}}\,\Phi,$$

and

$$A_\mu^a \to [e^{ig\vec{\alpha}\cdot\vec{t}}]_{ab}A_\mu^b + \frac{1}{2g}\epsilon_{abc}[\partial_\mu e^{ig\vec{\alpha}\cdot\vec{t}}]_{bd}[e^{-ig\vec{\alpha}\cdot\vec{t}}]_{dc},$$

so that

$$e^{ig(\omega_1 t_1+\omega_2 t_2)} \to e^{ig\vec{\alpha}\cdot\vec{t}}e^{ig(\omega_1 t_1+\omega_2 t_2)} = e^{ig(\omega_1' t_1+\omega_2' t_2)}e^{ig\zeta' t_3},$$

where ω_1', ω_2', and ζ' are nonlinear functions of ω_1, ω_2, and $\vec{\alpha}$, we will have

$$B_\mu^0 \to B_\mu^0 + \partial_\mu\zeta'$$

and

$$B_\mu^\pm \to e^{\mp ig\zeta'}B_\mu^\pm.$$

Therefore, $\mathcal{L}$ is unchanged if we also let $\nu\to\nu$, and

$$e \to e^{-ig\zeta'(\omega_1,\omega_2,\vec{\alpha})}e,$$

so that the electron field transforms in a nonlinear fashion, dependent upon the angular fields ω_1 and ω_2, which, of course, are themselves functions of $\phi^\pm$ and ϕ^0.

Several features of the Lagrangian $\mathcal{L}$ are worth noting:

(1) The fundamental fields are $\vec{A}_\mu$, Φ, ν, and e. $\vec{B}_\mu$ is merely a construction in terms of $\vec{A}_\mu$ and Φ. The interaction part of $\mathcal{L}$ is, therefore,

basically nonpolynomial.

(2) $\mathcal{L}$ is gauge-invariant, so that by a suitable choice of gauge (namely, the U gauge of Weinberg[1]) $\phi^\pm$ can be transformed away. In any other gauge, however, these redundant fields do appear as Goldstone bosons[12] (but they do not contribute to the unitarity relations[5,6]), and, in particular, it is in the R gauge of Lee and Zinn-Justin[6] that one can offer a proof of renormalizability. In our case, due to the nonpolynomial nature of $\mathcal{L}$, complications do arise, and we will discuss this very important point in a later paragraph.

(3) In $\mathcal{L}$, g is the electromagnetic coupling constant (e), g' is the semiweak coupling constant (g_W), m is the mass of the electron, A_μ^0 is the photon (A_μ), $A_\mu^\pm$ is the charged intermediate vector boson ($W_\mu^\pm$), and ϕ^0 is the Higgs scalar, which develops a nonzero vacuum expectation value, λ, by a suitable choice of the polynomial F.[13] The mass of $W_\mu^\pm$ is then $m_W = g\lambda = e\lambda$. We note, therefore, that g_W and e are not necessarily related, so the value of m_W remains arbitrary in this model. In particular, it does not have to be equal to or greater than 37.3 GeV.[1,14]

If we furthermore set $g' = \frac{1}{2}g$, then the model Lagrangian is, in fact, easily derived from a usual minimal Lagrangian involving a triplet ψ which transforms linearly. ψ is then related to ν and e by

$$\psi = e^{ig(\omega_1 t_1 + \omega_2 t_2)} \begin{pmatrix} 0 \\ \nu \\ e \end{pmatrix} ,$$

and it is clear that, in this case, our nonpolynomial Lagrangian is obtainable from a usual polynomial one, with a constraint on ψ. (Of course, with this identification, the mass of the W boson has to be exactly 53.0 GeV.[2])

(4) The mass of the electron in $\mathcal{L}$ is considered given. If we were to ascribe the mass wholly to spontaneous breakdown, the corresponding mass term could be obtained from an interaction term of the form $g_1 \bar{e} e \eta$, which is itself gauge-invariant. The mass m_e is then λg_1. From our nonlinear point of view there appears to be no advantage gained by this choice, and from a calculational point of view, in the R gauge (see below), it is not very attractive.

(5) Finally we remark that this model can be generalized to include muons and quarks in a straightforward manner. (A wide class of previously proposed models, including some with maximal CP violation,[15] can be cast into this form.) There need not be new unobserved particles anywhere, the reason being that the scalar triplet is used here to full advantage, serving as the carrier of the transformation property of the gauge group.

We now come to the question of renormalizability. Let us first review the procedure of 't Hooft[5] and Lee.[6] In the U gauge, one finds that, by power counting, the usual perturbation expansion is nonrenormalizable. The trouble here lies in the high-energy behavior of the vector-boson propagator in the canonical form

$$\frac{\delta_{\mu\nu} + k_\mu k_\nu / M^2}{k^2 + M^2} .$$

By a judicious choice of gauge, however, one can replace it, for instance, with

$$\frac{\delta_{\mu\nu} - k_\mu k_\nu / k^2}{k^2 + M^2} ,$$

which is now well behaved for large k. By power counting, the theory is then renormalizable. Since the S matrix is gauge-independent, one can also show, by a formal limiting procedure,[16] that meaningful calculations of S matrix elements are indeed possible in the U gauge. In particular, one finds that in one-loop graphs divergences do cancel in the U gauge.[17]

The situation, however, is not so simple in our case. In the U gauge, $\vec{B}_\mu = \vec{A}_\mu$, so the weak-interaction part of $\mathcal{L}$ is in the usual form of the $V-A$ theory with a charged intermediate vector boson, which, as it stands, is well known to be nonrenormalizable in perturbation theory. Furthermore, without a heavy lepton or a neutral current, there is no mechanism for the cancellation of divergences in higher-order graphs.[17] The theory, therefore, does not appear to be finite. This may, however, be a deceptive argument. The important point is whether one can find a *particular* form of $\mathcal{L}$ in which the theory can be shown to be finite. Such an example has been given by Shafi.[10] The idea there is to treat a part of $\mathcal{L}$ nonperturbatively, using nonlinear techniques,[7-11] and the rest by the usual perturbation expansion. In our case, let us follow 't Hooft[5] and Lee,[6] take advantage of gauge invariance, and choose a gauge in which the vector-boson propagator is well behaved at high energies; we shall discuss the problem of renormalizability there.

In the R gauge, $\vec{B}_\mu$ is no longer equal to $\vec{A}_\mu$, and $\phi^\pm \neq 0$. We have, instead,

$$B_\mu^0 = \frac{\phi^0}{\eta} A_\mu^0 + \frac{\phi^+}{\eta}\left(A_\mu^- + \frac{i}{g}\,\frac{\partial_\mu \phi^-}{\eta + \phi^0}\right)$$

$$- \frac{\phi^-}{\eta}\left(A_\mu^+ + \frac{i}{g}\,\frac{\partial_\mu \phi^+}{\eta + \phi^0}\right) ,$$

and

$$B_\mu^\pm = \frac{1}{2}\left(1+\frac{\phi^0}{\eta}\right)\left(A_\mu^\pm + \frac{i}{g}\frac{\partial_\mu \phi^\pm}{\eta}\right)$$
$$\mp \frac{\phi^\pm}{\eta}\left(A_\mu^0 \pm \frac{i}{g}\frac{\partial_\mu \phi^0}{\eta}\right)$$
$$-\left(\frac{\phi^\pm}{\eta}\right)^2 \frac{1}{1+\phi^0/\eta}\left(A_\mu^\mp - \frac{i}{g}\frac{\partial_\mu \phi^\mp}{\eta}\right),$$

where

$$\eta^2 = (\phi^0)^2 - 2\phi^+\phi^-$$

and

$$(\phi^+)^* = -\phi^- \;.$$

A naive perturbation-series expansion of $\mathcal{L}$ in powers of ϕ is clearly term-by-term nonrenormalizable. We must, therefore, use the nonperturbative method of Efimov[7] and Fradkin,[8] and perform a sum over the nonpolynomial portion of $\mathcal{L}$ which, based on the generalized power-counting argument of Salam and co-workers,[9] may in fact be renormalizable. (The Dyson index is 4 here, instead of 5, because of the form of the vector-boson propagator in the R gauge.)

In the usual linear gauge models, there is a simple one-to-one correspondence between the terms in the loop expansion in the R gauge and those in the U gauge. If the sum of one-loop graphs in the R gauge is finite, the corresponding sum in the U gauge will also be finite. In fact, they will be equal to each other, and this is the source of confidence for previous calculations[17] in the U gauge. In our model, however, the nonperturbative summation of the Efimov-Fradkin technique no longer respects the loop classification of the perturbation series, and viewed in the U gauge this becomes a particular infinite sum of loop graphs, whose finiteness is otherwise not obvious. (This point was demonstrated explicitly in the model of Shafi,[10] quoted earlier.)

To summarize, the point we are making is that power-counting arguments (in the U gauge) cannot be used to "prove" that the theory is not finite; and since there is gauge invariance, we can take advantage of it and use the R gauge, in which nonperturbative techniques can be relied upon to indicate renormalizability. We have, therefore, constructed a possibly renormalizable model of weak and electromagnetic interactions with a minimum number of new particles (namely, just the W boson and the Higgs scalar). The drawback is that actual calculations with nonpolynomial Lagrangians are very difficult and further progress in this direction will have to be made before numerical estimates of cross sections can be given in this model. But if the optimism expressed by Salam[9] is any indication, there is hope that this problem will eventually be solved.

After the completion of this work, it was pointed out to us that a similar model has been proposed by Faddeev in a recent paper,[18] which unfortunately has not yet been translated. In addition, it should be mentioned that Shafi's model[10] has been enlarged upon by Delbourgo,[19] and that the idea of gauge symmetries with nonlinear realizations has been discussed by Gottlieb.[20]

---

*Work supported in part by the Research Foundation of the City University of New York.

†Work supported by the National Science Foundation.

[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971).

[2]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972).

[3]B. W. Lee, Phys. Rev. D 6, 1188 (1972); J. Prentki and B. Zumino, Nucl. Phys. B47, 99 (1972); A. Pais, Phys. Rev. Lett. 29, 1712 (1972). For other references, see B. W. Lee, report to the Sixteenth International Conference on High Energy Physics, Batavia, Ill., 1972 (unpublished).

[4]C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).

[5]G. 't Hooft, Nucl. Phys. B33, 173 (1971); B35, 167 (1971).

[6]B. W. Lee, Phys. Rev. D 5, 823 (1972); B. W. Lee and J. Zinn-Justin, *ibid.* 5, 3121 (1972); 5, 3137 (1972); 5, 3155 (1972).

[7]G. V. Efimov, Zh. Eksp. Teor. Fiz. 44, 2107 (1963) [Sov. Phys.-JETP 17, 1417 (1963)]; Nuovo Cimento 32, 1046 (1964); Nucl. Phys. 74, 657 (1965).

[8]E. S. Fradkin, Nucl. Phys. 49, 624 (1963).

[9]R. Delbourgo, A. Salam, and J. Strathdee, Phys. Rev. 187, 1999 (1969); A. Salam and J. Strathdee, Phys. Rev. D 1, 3296 (1970); C. J. Isham, A. Salam, and J. Strathdee, *ibid.* 3, 1805 (1971); 5, 2548 (1972); A. Salam, in *Coral Gables Conference on Fundamental Interactions at High Energy II,* edited by A. Perlmutter, G. J. Iverson, and R. M. Williams (Gordon and Breach, New York, 1970); in *Nonpolynomial Lagrangians, Renormalization and Gravity,* 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. I; in *Particles and Fields—1971,* proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Melissinos and P. F. Slattery (A.I.P., New York, 1971).

[10]Q. Shafi, Phys. Rev. D 3, 2363 (1971).

[11]S. Okubo, Progr. Theor. Phys. 11, 80 (1954); H. M. Fried, Nuovo Cimento 52A, 1333 (1967); Phys. Rev. 174, 1725 (1968); M. K. Volkov, Ann. Phys. (N.Y.) 49,

202 (1968).

[12]J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[13]P. Higgs, Phys. Lett. 12, 132 (1964); T. W. B. Kibble, Phys. Rev. 155, 1554 (1967).

[14]T. D. Lee, Phys. Rev. Lett. 26, 801 (1971); J. Schechter and Y. Ueda, Phys. Rev. D 2, 736 (1970).

[15]S. Okubo, Nuovo Cimento 54A, 491 (1968); Ann. Phys. (N.Y.) 49, 219 (1968); R. E. Marshak, R. N. Mohapatra, S. Okubo, and J. S. Rao, Nucl. Phys. B11, 253 (1969); R. E. Marshak, Y. W. Yang, and J. S. Rao, Phys. Rev. D 3, 1640 (1971).

[16]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972).

[17]T. W. Appelquist and H. R. Quinn, Phys. Lett. 39B, 229 (1972); T. W. Appelquist, J. R. Primack, and H. R. Quinn, Phys. Rev. D 6, 2998 (1972); J. R. Primack and H. R. Quinn, *ibid.* 6, 3171 (1972); S. Y. Lee, *ibid.* 6, 1701 (1972); 6, 1803 (1972).

[18]L. D. Faddeev, "Gauge Invariant Model of Electromagnetic and Weak Interactions of Leptons," Dokl. Akad. Nauk SSSR (to be published).

[19]R. Delbourgo, Nuovo Cimento 11A, 943 (1972).

[20]H. P. W. Gottlieb, Nucl. Phys. (to be published).