Therefore, $\Delta m|_{n-p}$ is always negative for any values of the parameters.

V. DISCUSSION

We have shown that the sign of $\Delta m|_{n-p}$ is invariably negative in a class of models based on SU(2). Several other papers have shown the same situation. Although we have not shown that the SU(2) models always give the wrong sign, we feel that the one-loop contributions to the mass difference in SU(2) models are not sufficient to give the positive result.[7]

ACKNOWLEDGMENT

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Lightlike Asymptotic Behavior of Local Operators and the Vacuum Annihilation Property of "Lightlike Charges"*

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It is shown that under lightlike translations the matrix elements of local operators between suitable states decrease asymptotically faster than any inverse power of the amount of translations. This result is used to establish the vacuum annihilation property of "lightlike charges." The physical basis of these asymptotic behaviors is discussed, and the circumstances under which these can be associated with the propagation of the virtual hadrons in a projectile are analyzed.

I. INTRODUCTION

As is well known, there are certain asymptotic properties of local operators at large spacelike or timelike distances that are valid on general grounds. In theories with a mass gap, the truncated vacuum expectation values of local fields, or of quasilocal operators, decrease faster than any inverse power of the amount of spacelike translation of a given cluster.[1] At a large time t [with $x=(\vec{v}t,t)$ and $|\vec{v}|<c$], the matrix element of a quasilocal operator between suitable states may have terms which decrease as $t^{-3/2}$ and as t^{-3}, giving rise to the LSZ (Lehmann-Symanzik-Zimmermann) asymptotic condition and its generalizations.[2]

In this note we wish to study the asymptotic properties of local operators under translations by a four-vector b, with $b^2=0$, and $|b_4|\to\infty$ in a given Lorentz frame. This lightlike asymptotic limit is of interest for several reasons. First, this asymptotic behavior determines whether an integration over a nullplane in defining the so-called lightlike charges[3,4] is meaningful. Second, inasmuch as certain high-energy processes such as deep-inelastic electron-nucleon scattering are probably controlled by the behavior of the product of local currents at lightlike separations,[5-8] it is important to know more about the lightlike asymptotic behaviors of operators and their physical significance.

In Sec. II of this paper we discuss the asymptotic behavior of the matrix element of a local field between states chosen from suitable dense sets, and give sufficient conditions under which there is a strong falloff with increasing $|b_4|$. In Sec. III we comment on the physical basis of the asymptotic behavior, and make a conjecture about the asymptotic behavior when products of local operators are involved. It is pointed out that the lightlike asymptotic behavior of a product of smeared local fields, if one keeps only the most singular piece on the light cone, is very different from the case when the nonsingular part is kept. In Sec. IV we establish the vacuum annihilation property of the "lightlike charges." In Sec. V we examine in what sense long-range lightlike correlations can be associated with the propagation of virtual hadrons, as suggested by Gribov, Ioffe, and Pomeranchuk.[9]

II. LIGHTLIKE ASYMPTOTIC BEHAVIOR

We summarize first, in the following two statements, what can be readily deduced on general grounds about the lightlike asymptotic behavior of a local field. We assume the usual framework of general quantum field theory, and also assume the presence of a mass gap. The asymptotic behavior will be with respect to translation by a vector $b=(0,0,-\lambda,\lambda)$ for large $|\lambda|$.

(i) Let $A(x)$ be a tempered local field with $\langle 0|A(x)|0\rangle=0$. Let D_1 be the domain of finite linear combinations of vectors obtained by applying to the vacuum $B(f)=\int f(x)B(x)d^4x$, where $B(0)$ is a polynominal in local fields smeared with test functions in $\mathfrak{D}(R^4)$, $B(x)=U(x,1)B(0)U^{-1}(x,1)$, and $f\in\mathcal{S}(R^4)$ with its Fourier transform $\tilde{f}(p)$ vanishing for $p_4<m$ $(m>0)$.

Then, for any $\psi\in D_1$, and given x and N,

$$\lim_{|\lambda|\to\infty}\lambda^N\langle\psi|A(b+x)|0\rangle=0.$$

(ii) Let $Q(x)$ be a local field or quasilocal operator of infinite order satisfying $\langle 0|Q(x)|0\rangle=0$. Let D_+ be the domain of finite linear combinations of vectors with a finite number of out-particles, whose wave functions $\in\mathcal{S}(R^3)$ in momentum space with nonoverlapping support in velocities; similarly D_- is defined with in-particles replacing out-particles. Then, for $\psi_\pm$ and $\phi_\pm\in D_\pm$, and any given x and N,

$$\lim_{\lambda\to\pm\infty}\lambda^N\langle\psi_\pm|Q(b+x)|\phi_\pm\rangle=0,$$

where the upper and lower subscripts refer to $\lambda\to+\infty$ and $-\infty$, respectively.

Note that although b is a lightlike vector, $(b+x)$ may not be. As far as the argument $y=b+x$ is concerned, we are considering the limit $y_-\to\infty$ with y_+ fixed $(y_\pm\equiv y_4\pm y_3)$.

The first statement follows from the JLD (Jost-Lehmann-Dyson) representation[10]

$$\langle 0|[B^*(y),A(b+x)]|0\rangle=\int_{m^2}^\infty ds\int d^3\eta\,[\rho_1(\vec{\eta},s)\Delta(y-b-x-\eta,s)+\rho_2(\vec{\eta},s)\partial_4\Delta(y-b-x-\eta,s)]\quad[\eta=(\vec{\eta},0)],\tag{1}$$

where the ρ's have compact support in $\vec{\eta}$. Smearing with f eliminates the $\langle 0|AB^*|0\rangle$ term on the left-hand side due to spectral properties. Hence

$$\langle\psi|A(b+x)|0\rangle=\int_{m^2}^\infty ds\int d^4p\,e^{i(b+x)p}\tilde{f}(p)\delta(p^2-s)\theta(p_4)[\tilde{\rho}_1(\vec{p},s)+ip_4\tilde{\rho}_2(\vec{p},s)]\tag{2}$$

or

$$\langle\psi|A(b+x)|0\rangle=\int_{m^2}^\infty ds\int d^2p\,dp_-\,e^{i\lambda p_-+ixp}\,\tilde{f}\left(\underline{p},p_+=\frac{\underline{p}^2+s}{p_-},p_-\right)\theta(p_-)p_-^{-1}\left[\tilde{\rho}_1(\vec{p},s)+i\left(p_-+\frac{\underline{p}^2+s}{p_-}\right)\tilde{\rho}_2(\vec{p},s)\right],\tag{3}$$

where we have denoted by $\underline{p}$ the two-dimensional vector (p_1,p_2), and by $p_\pm$ the combination $\frac{1}{2}(p_4\pm p_3)$. $\tilde{f}$ is a rapidly decreasing function in $\underline{p}$, p_-, and s; and the $\tilde{\rho}$'s are C_∞ functions in $\vec{p}$ because of the support properties of ρ in coordinate space, with at most polynominal growths in $\vec{p}$ and s. Because there is a lower bound on the range of values of s, $\tilde{f}$ vanishes faster than any power of p_- as $p_-\to 0$. Statement (i) then follows from the Riemann-Lebesgue lemma.

To prove statement (ii), one notes that if both ψ_-, $\phi_-\in D_-$, then from the Haag-Ruelle collision theory one can approximate $\langle\psi_-|Q(b+x)|\phi_-\rangle$ by expressions of the form

$$\int\prod_{k,l}d^3y_k\,d^3z_l\,\bar{g}_k(\vec{y}_k,\ t=\lambda+x_4)g_l(\vec{z}_l,\ t=\lambda+x_4)\times\langle 0|\cdots q^*(\vec{y}_k,\ t=\lambda+x_4)\cdots Q(\underline{x},\ x_3-\lambda,\ t=x_4+\lambda)\cdots q(\vec{z}_l,\ t=\lambda+x_4)\cdots|0\rangle\tag{4}$$

up to terms of order λ^{-M} for any given M. Here q^*, q are one-particle excitation operators, and the g's are solutions to Klein-Gordon equations. By a linked-cluster expansion, and using spacelike asymptotic properties, as well as the fact that the conditions on $D_\pm$ imply that $g(\underline{z},\ z_3=-\lambda,\ t=\lambda)$

vanishes faster than any inverse power of λ for large $|\lambda|$, one concludes that

$$\lim_{\lambda \to -\infty} \lambda^N \langle \psi_- | Q(b+x) | \phi_- \rangle = 0. \quad (5)$$

By a similar argument, for ψ_+, $\phi_+ \in D_+$, one has

$$\lim_{\lambda \to +\infty} \lambda^N \langle \psi_+ | Q(b+x) | \phi_+ \rangle = 0. \quad (6)$$

III. PRODUCTS OF OPERATORS

We now comment on the physical basis of the asymptotic behaviors. As was emphasized by Araki and Haag,[2,11] the asymptotic behavior of a local operator for large time has simple physical interpretations, which are purely *kinematic and geometric*. The $t^{-3/2}$ behavior of the leading matrix element of the operator, i.e., $\langle 0 | Q(x) | 1 \rangle$ [where $x = (\vec{v}t, t)$ for fixed $\vec{v}$ and $|1\rangle$ is a state of one particle prepared in some given region], corresponds to a t^{-3} dependence of the square of the matrix element, representing the decrease in the probability of finding the particle in a region around $(\vec{v}t, t)$, owing to the spreading of the wave packet. Analogously, if a self-adjoint quasilocal operator C satisfies $C|0\rangle = 0$, it has essentially the properties of a counter.[2,11] Again, the spreading of wave packets implies that the effectiveness of its detecting a single particle and making a measurement of its relevant properties decreases as t^{-3} as it is moved along the path $(\vec{v}t, t)$. So its leading matrix elements (which are 1-particle to 1-particle matrix elements) already decrease as t^{-3}.

Similarly, the asymptotic behavior in a lightlike direction obtained above also has a kinematic origin. A long-range correlation along a lightlike direction requires the propagation of particles at the speed of light. The choice of wave functions for the dense set of states in D_1 or $D_\pm$ is such that the momentum wave functions decrease faster than any inverse power asymptotically. Consequently the rapid decrease along the null plane found above is merely a statement of the paucity of Fourier components with $|\vec{v}| = c$ in the states. One can easily see, from Eq. (3) for example, that if the infinite-momentum contribution were not suppressed by the strong decrease of $\tilde{f}$ for large p_+, there would have been long-range lightlike correlations. Also, if there were no mass gap, and if the lower limit of integration in s is zero, $p_- \to 0$ no longer would necessarily imply $p_+ \to \infty$, and there could be again long-range correlations.

These simple considerations are nevertheless worth mentioning because they are based on general physical principles, and not on the details of dynamics. The physical considerations may be useful in suggesting heuristically the asymptotic behavior in cases where rigorous results are still lacking, particularly when products of local fields are involved. In the matrix element of a products of local fields A and B like

$$\langle \psi | A(x) B(y) | \phi \rangle = \sum_n \langle \psi | A(x) | n \rangle \langle n | B(y) | \phi \rangle,$$

one is summing over the intermediate states, and therefore the statements (i) and (ii) in Sec. II, which hold for fixed states, do not apply. Nevertheless, the physical picture mentioned above suggests the following.

Since the intermediate states can contain high-energy Fourier components, in general there can be long-range correlations along the light cone; and to the extent that it is possible to speak of the dependence of $\langle \psi | A(x) B(y) | \phi \rangle$ on $(x-y)_-$ for some fixed $(x-y)_+ \neq 0$, there will *not* always be a rapid decrease with increasing $(x-y)_-$. On the other hand, if $|\psi\rangle$ and $|\phi\rangle$ have rapidly decreasing high-energy components and the fields are smeared with testing functions having the same property in momentum space, then by energy-momentum conservation the intermediate-state wave functions in momentum space must also decrease rapidly with increasing energies. Therefore, if a smeared field is translated along a lightlike direction, the matrix element should decrease rapidly with the amount of translation. Hence we have the following conjecture: If $|\psi\rangle$, $|\phi\rangle \in D_1$, $A(x)$ and $B(x)$ are local fields; $A_f(x) \equiv \int A(y) \times f(y+x) dy$ and similarly for $B_g(y)$ with g, $f \in \mathcal{S}(R^4)$, then for any fixed N and y

$$\lim_{|\lambda| \to \infty} \lambda^N \langle \psi | A_f(b) B_g(y) | \phi \rangle = 0. \quad (7)$$

We have checked that Eq. (7) is valid for $|\psi\rangle$ and $|\phi\rangle$ being 1-particle states in some perturbation calculations to low but nontrivial orders. For $|\psi\rangle$ being a 1-particle state and $|\phi\rangle = |0\rangle$, the validity of Eq. (7) follows from the DGS (Deser-Gilbert-Sudarshan) representation[12] for 3-point functions. We omit the details but remark that the essential ingredient for the validity of Eq. (7) in this case is the fact that the lower limit of integration in the "effective mass variable" does not reach zero. That is, for the representation[12] in the form

$$\langle 1, p | T\, A(x) B(0) | 0 \rangle = \int_{a_0(b)}^{\infty} da \int_0^1 db\, e^{ibp\cdot x} \rho(a,b) \Delta(x,a)$$

it can be verified that in theories with a mass gap one has

$$\min_b \, [a_0(b)] > 0.$$

One also notes that smearing the momentum p of the state does not by itself remove long-range correlations, as long as $b=0$ is an end point of integration which is reached by the support of $\rho(a,b)$.

Finally, with respect to some recent applications which involve extracting the most singular part of operators on the light cone, we must emphasize that the analog of Eq. (7) will not be true if one first extracts the most singular part of $A(x)B(y)$ on the light cone, and then smears in x with f and translates the support of f. This is because the most singular part is sometimes mass-independent and is the same as that in a theory with zero-mass particles.

These points are very simply illustrated by considering the two-point function for a free scalar field of mass m:

$$\langle 0|\phi(x)\phi(0)|0\rangle = \Delta^+(x,m).$$

Without smearing in x, for fixed x^2 there is no decrease at all with increasing x_- for simple reasons of covariance. Obviously by considering derivations of $\phi(x)$ one can get a growth as any given power of x_- along the light cone. If instead of fixed x^2 one considers $x_-\to\infty$ with fixed $x_+\neq 0$, there is a slow decrease as $(x_-)^{-3/4}$. In short, without smearing the fields, long-range correlations exist in a lightlike direction. On the other hand, if we smear in x_+ and $\underline{x}$ with testing functions $f(x_+)$ and $g(\underline{x})$ from $\mathcal{S}$, then

$$W(x_-)\equiv\int f(x_+)g(\underline{x})\Delta^+(x,m)dx_+d^2x$$

vanishes faster than any inverse power of $|x_-|$ as $|x_-|\to\infty$. Finally, if one smears the singular piece of $\Delta^+(x,m)$, viz., $1/(x^2-i\,x_0\epsilon)$, one finds that

$$W_s(x_-)\equiv\int f(x_+)g(\underline{x})(x^2-i\,\epsilon x_0)^{-1}dx_+d^2x$$

decreases only as $(x_-)^{-1}$ for large $|x_-|$. We conclude, therefore, that as far as the long-range correlations along the light cone is concerned, it can be misleading to consider only the most singular piece on the light cone. When smeared, the contribution from the nonsingular part may exactly cancel the long-range correlations from the singular part.

IV. PROPERTIES OF LIGHTLIKE CHARGES

The lightlike charge Q_L associated with a local current j_μ has been formally defined as the integral

$$Q_L=\int d^4x\;\delta(x\cdot n)\;n\cdot j(x),$$

with $n=(0,0,-1,1)$, and arose originally[3,4] in the study of the use of infinite-momentum limit in current-algebra calculations. For the ordinary charge Q, defined formally by the integral

$$Q=\int d^4x\;\delta(x\cdot n)\;n\cdot j(x)$$

with $n^2=1$ it is well known[13] that the following tight connections exist in the presence of a mass gap:

$$\begin{aligned}\partial_\mu j_\mu = 0 &\longleftrightarrow \langle\psi|Q|0\rangle = 0\\ &\longleftrightarrow Q \text{ defines an operator in the Hilbert space,}\end{aligned}\tag{8}$$

where $\psi\in D_l$, the domain of strictly localized states, and the space integration and restriction to sharp time are defined by suitable limits. In case of Q_L the tight link in (8) is broken, since it is known that in a free-field theory $j_\mu(x)=\phi_1^*(x)\times\overleftrightarrow{\partial}_\mu\phi_2(x)$ is not conserved when $m_1\neq m_2$, but the corresponding Q_L defines an operator.[4] Aside from such trivial cases, what happens in general is not known. In Ref. 4 an argument is made that $\langle\psi|Q_L|0\rangle=0$ holds in general, since Q_L commutes with the generator of translations P_-, so Q_L is an eigenstate of P_- with zero eigenvalue, and is hence a multiple of $|0\rangle$. However, in order for this argument to be meaningful one must first show the existence of the integral

$$\int_{-\infty}^{\infty}d^2x\;dx_-\;j_+(x),$$

at least between suitable states. In the case of the ordinary charge Q, the space integral exists between strictly localized states due to spacelike asymptotic properties. For Q_L, on the other hand, we need to know the lightlike asymptotic properties. The result of Sec. II allows one to show the convergence of the relevant integrals, establishing the vacuum annihilation property.[14]

We denote by $g_R(x_-)$ and $g_L(\underline{x})$ infinitely differentiable functions with the properties

$$\begin{aligned}g_R(x_-)&=1,\quad |x_-|<R-1\\ &=0,\quad |x_-|>R\\ g_L(\underline{x})&=1,\quad |\underline{x}|<L-1\\ &=0,\quad |\underline{x}|>L\end{aligned}$$

and consider, for $\psi\in D_1$, the matrix element

$$\left\langle\psi\left|\int d^2x\;dx_-g_L(\underline{x})g_R(x_-)j_\mu(x)\right|0\right\rangle.$$

It follows from statement (i) of Sec. II that the limit $R\to\infty$ of the above matrix element exists, and is in fact equal to zero since $\tilde{\bar{f}}$ in Eq. (3)

vanishes faster than any power of p_- as $p_- \to 0$. The integration over $\underline{x}$ is unnecessary for this result, but it is possible to let $L \to \infty$, as one readily sees from Eq. (3). It is also clear that the considerations so far are independent of which component of the current (i.e., which index μ) is being considered.[15] Thus, for all ψ in the dense set D_1,

$$\langle \psi | Q_L(x_+) | 0 \rangle = 0. \tag{9}$$

This establishes the vacuum annihilation property.

Instead of examining the explicit form of Eq. (3), it is also instructive to return to the line of argument used in Ref. 4. Once one establishes that $\langle \psi | Q_L(x_+) | 0 \rangle$ is well defined, from $\langle \psi | [P_-, Q_L(x_+)] | 0 \rangle = 0$ one finds $\langle P_- \psi | Q_L(x_+) | 0 \rangle = 0$. The crucial difference from the case of ordinary charges is that every state $\psi' \in D_1$ can be written as $P_- \psi$, with $\psi \in D_1$ (which would not be true for $\vec{P}\psi$). Hence one again arrives at the above conclusion.

In contrast to the second link in Eq. (8) for the ordinary charge Q, it does not follow from Eq. (9), at least with the present techniques, that Q_L exists as a bounded bilinear form defining an operator, or even just a bilinear form. [The physical arguments in the latter part of Sec. III suggests that for $\psi, \phi \in D_1$, $\langle \psi | J_\mu(x) | \phi \rangle$ vanishes strongly with x_- for large $|x_-|$, but this is not yet rigorously proved.] Thus the question of the existence of lightlike charges in general remains unsolved.

V. THE ASSOCIATION OF LONG-RANGE CORRELATIONS WITH THE SCATTERING OF VIRTUAL PARTICLES

Some time ago Gribov, Ioffe, and Pomeranchuk gave an interpretation to almost lightlike correlations in high-energy scattering.[9] Consider, for example, a forward elastic scattering amplitude for spinless particles,

$$M(p\cdot q, q^2) = \int dx\, e^{iq\cdot x} \langle p | Tj(x/2) j(-x/2) | p \rangle, \tag{10}$$

where the exponent $(q\cdot x) \simeq q_0(x_0 - x_3) + q^2 x_3 / 2q_0$ if the projectile with a given mass q^2 has a large momentum along the 3 axis. They note that since for large q_0 the contribution comes mainly from $x_0 - x_3 \simeq 0$, a relevant question is whether a finite interval of the remaining variable x_3 gives most of the contribution to the integral, or whether a large interval of x_3, growing with increasing q_0, gives important contributions. This interval they call the "longitudinal range of interaction", arguing that $x/2$ corresponds to the point of absorption of the incoming projectile, and $-x/2$ corresponds to the point of emission of the outgoing projectile. They state that it will be extremely interesting if the second possiblity is realized, leading to "a curious physical picture in which, for example, at extremely high energies the dimensions of the region of interacting will exceed those of the atom." We will show below that under reasonable circumstances the second possibility is generally realized, and that with a suitable interpretation this situation is entirely natural and unsurprising.

Part of the reason that people may have considered the second possibility strange is probably the use of the terminology "range of interaction" in this context, which could lead to confusion with the conventional notion of an interaction range. The two notions are actually quite different. The conventional interaction range refers to a spatial extension at a given time, whereas different times are involved in the "range" being considered here. A clearer exposition of what the authors of Ref. 9 mean by their range is given in a later article by Gribov.[16] Essentially, the projectile is said to turn into virtual hadrons, which virtual state lasts for a relatively long time as a result of time dilation, and the longitudinal range is the distance traversed by these hadrons.

Although this picture is attractive in its simplicity, one may nevertheless be puzzled by several ambiguities in this interpretation. For instance, it may be confusing to simultaneously speak of a particle with definite momentum q and sharp space-time localization at the point $x/2$. Furthermore, while to a resonance that actually decays eventually one can associate a meaningful lifetime, a virtual state never lives; what does one mean then by its lifetime? To put it differently, a virtual state, being not physically realized, can presumably be associated with a particle at any time during its existence. If the particle is stable, is there any basis for calling $(\text{mass})^{-1}$ the lifetime of its virtual states?[16]

We will try to give a formulation of this suggested picture by means of a simple diagrammatic analysis of physical singularities, which, in our opinion, renders the notion more precise. Since part of the confusion is due to the fact that one seems to be talking about particles with definite momenta and sharp space-time localization at the same time, one way to avoid the confusion is to build wave packets. This, however, complicates the formulas and discussions. To keep as close as possible to the simplicity of the original analysis of Ref. 9, while at the same time avoiding possible confusions, we divide the question into two parts

(i) One first asks: Does $F(p\cdot x, x^2) \equiv \langle p | Tj(x/2) j(-x/2) | p \rangle$ have a long range in the

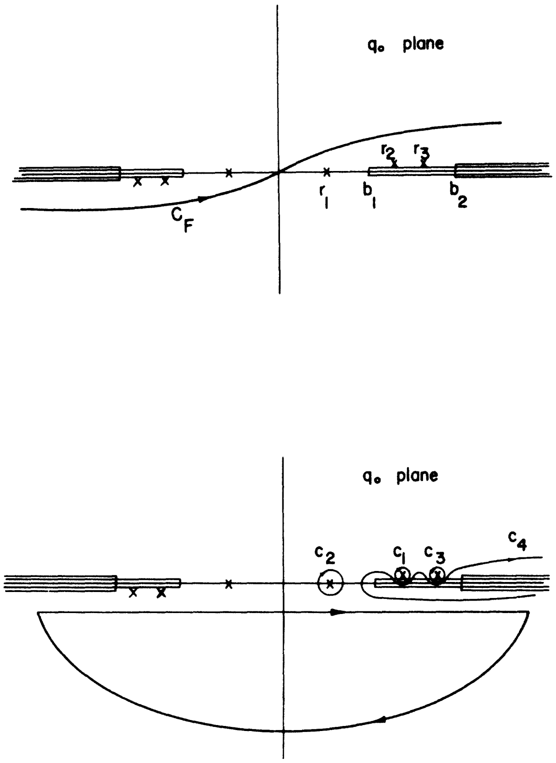

FIG. 1. Contours of integration in the q_0 plane. $r_2 = (k^2+\mu^2)^{1/2}$, $r_1 = -1+(k^2+1)^{1/2}$, $r_3 = 1+(k^2+1)^{1/2}$; s-channel threshold at $-1+(k^2+\mu^2+2\mu+1)^{1/2}$, and q^2-channel threshold at $(k^2+4\mu^2)^{1/2}$.

variable $(p\cdot x)$ for fixed x^2? If it does, what kind of processes contribute to it?

(ii) If the answer to the first part of (i) is yes, one then asks: Which of the long-range parts of F (i.e., the long-range part arising from which processes) contributes to M in Eq. (10), as q_3 becomes large for a given q^2?

To study the first question, we write

$$F(x\cdot p, x^2) = \int dq\, e^{-ix\cdot q} M(p\cdot q, q^2), \tag{11}$$

where the contour in the q_0 plane, for given $k = |\vec{q}| > 0$, is shown in Fig. 1(a), with p taken to be (1, 0, 0, 0,). For positive $x_0 > |\vec{x}|$, we close the contour as shown in Fig. 1(b), and drop the contribution from the infinite semicircle.[17] We can then evaluate the contributions from the contours C_i separately, identify the physical process giving rise to each of the singularities, and see which of these give dominant contributions to the long-range part of F. Obviously, one can make a similar analysis for large negative x_0. We will use the notation F_i to denote the contribution of C_i to F.

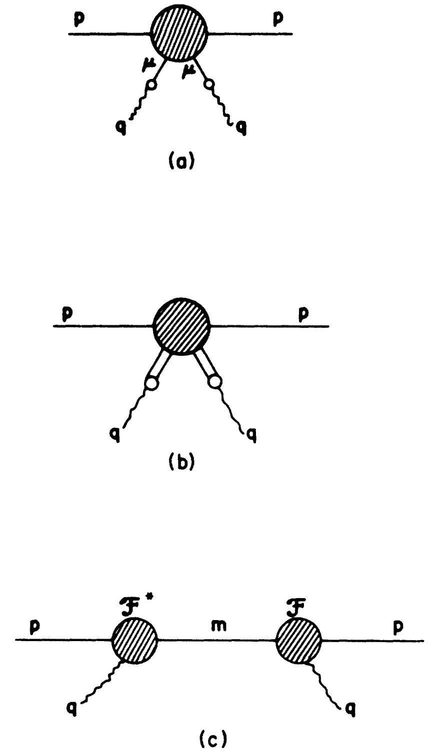

FIG. 2. (a) One-particle intermediate states in the q^2 channel; (b) two-particle intermediate states in the q^2 channel; (c) one-particle intermediate states in the s channel.

F_1 receives contributions from double poles arising from the process of Fig. 2(a), if the source $j(0)$ connects the vacuum to 1-particle states. Near a double pole at $q^2 = \mu^2$ we write

$$M(q_0, q^2) = R(q_0, q^2)(q^2 - \mu^2 + i\epsilon)^{-2},$$

where $R(q_0, \mu^2)$ = the forward scattering amplitude $A(q_0)$ for the scattering of the mass-μ particle against the target. As $x_0 \to \infty$ with $x_- = x_0 - |\vec{x}|$ fixed, one finds that the dominant contribution is from a stationary phase point,[18] giving

$$F_1(x\cdot p, x^2) \underset{\substack{p\cdot x\to\infty \\ x^2 < Cp\cdot x}}{\sim} \frac{\text{constant}}{(x^2)^{1/4}}\, e^{-i(\mu^2 x^2)^{1/2}} \times A\left(q_0 = \frac{\mu p\cdot x}{(x^2)^{1/2}}\right). \tag{12}$$

So the long-range part in F_1 is directly related to the high-energy behavior of the elastic scattering amplitude $A(q_0)$. If, for instance, $A(q_0)$ has the Regge behavior $A(q_0) \to (q_0)^{\alpha(0)}$ for large q_0, then

$$F_1(x\cdot p, x^2) \underset{\substack{p\cdot x\to\infty \\ x^2<Cp\cdot x}}{\sim} \frac{\text{constant}}{(x^2)^{1/4}}\, e^{-i(\mu^2x^2)^{1/2}} (p\cdot x)^{\alpha(0)} . \tag{13}$$

The contribution from C_2, corresponding to the process in Fig. 2(c), gives

$$F_2(x\cdot p, x^2) \underset{\substack{p\cdot x\to\infty \\ x^2<Cp\cdot x}}{\sim} \frac{\text{constant}}{(x^2)^{3/4}}\, e^{i\,p\cdot x - i(\mu^2x^2)^{1/2}} \left| \mathfrak{F}\left(-\frac{2\mu p\cdot x}{(x^2)^{1/2}}\right)\right| , \tag{14}$$

where $\mathfrak{F}(q^2)$ is the form factor of the target particle.[19] If the form factor vanishes as $q^2 \to -\infty$, while $\alpha(0)$ is of the order of unity, clearly F_1 dominates over F_2 in magnitude as $p\cdot x \to \infty$ for given x^2. But more important is the phase factor $e^{ip\cdot x}$ in F_2, not present in F_1, to which we will return later.

The contribution from C_3 corresponding to a u-channel pole is similar to that from C_2, and need not be discussed in detail. The contribution from C_4 is of the form

$$F_4 = \frac{\text{constant}}{|\vec{x}|}\int k\, dk \int_{b_0}^{\infty} dq_0\, e^{-iq_0x_0} \sin(k|\vec{x}|)\, \text{disc} M, \tag{15}$$

where b_0 depends on k; disc$M$ has singularities from the q^2-channel processes as well as from s- and u-channel processes. We first examine the q^2-channel singularities, the strongest one being a possible $(q^2 - b^2)^{-1/2}G(q_0, q^2)$ contribution arising from a 2-particle threshold at $q^2 = b^2$, corresponding to the process of Fig. 2(b). The asymptotic contribution to F_4 is proportional to the large-q_0 behavior of $G(q_0, b^2)$. If $G(q_0, b^2)$ is proportional to $(q_0)^{\alpha(0)}$ also, as suggested from Regge-pole considerations, then this particular contribution to F_4, as $(p\cdot x) \to \infty$ with $(x^2) < C(p\cdot x)$, is

$$\frac{\text{constant}}{x^2}\left(\frac{p\cdot x}{(x^2)^{1/2}}\right)^{\alpha(0)} e^{-i(b^2x^2)^{1/2}} . \tag{16}$$

Upon comparing F_1 and the above partial contribution to F_4, one notes that the different nature of the singularities gives rise to different x^2 dependences. But for fixed x^2 the dependence on $(p\cdot x)$ for both is proportional to $(p\cdot x)^{\alpha(0)}$ within a Regge-pole approximation,[20] and is in any case proportional to the high-energy scattering amplitude for one particle, or for two particles traveling with the same velocity (corresponding to thresholds in the q^2 channel), against the target particle, independent of any Regge-pole approximation for the description of such amplitudes.[21]

The same qualitative features persist for contributions from higher intermediate states in the q^2 channel. They give contributions decreasing more rapidly with increasing x^2, but for fixed x^2 the dependence on $(p\cdot x)$ is again like $(p\cdot x)^{\alpha(0)}$ within the Regge-pole approximation. Thus one sees that $F(p\cdot x, x^2)$ does have long-range parts in $(p\cdot x)$ arising from the opening of each intermediate state in the q^2 channel. If the coefficients of the $(p\cdot x)^{\alpha(0)}$ factor from different intermediate states are such that the sum does not diverge,[22] then the net contribution of the q^2-channel intermediate states to F_4 is of order $(p\cdot x)^{\alpha(0)}$. It can be shown that the s-channel contributions to F_4 are smaller in magnitude than the above, if q_0 disc$M$ is bounded as $|q^2| \to \infty$, both for fixed $(p+q)^2$ and for fixed q^2/q_0 (as would be the case with canonical Bjorken scaling). The exact dependence on $(p\cdot x)$ requires knowledge of detailed dynamics, but, independent of the dynamical assumptions, the contributions to F_4 from s-channel processes with bounded $(p+q)^2$ carry the phase $e^{ip\cdot x}$ as in F_2. Similar statements apply to u-channel contributions with $p \to -p$.

We summarize the results of the above considerations as follows:

(a) $F(p\cdot x, x^2)$ does not have a long range in $(p\cdot x)$ due to singularities in $M(p\cdot q, q^2)$ arising from the presence of q^2-channel processes. If the forward elastic scattering amplitudes for the scattering of particles in the q^2-channel intermediate states against the target behave like $(q_0)^{\alpha(0)}$ for large q_0, then the q^2-channel contributions to $F(p\cdot x, x^2)$ behave like $(p\cdot x)^{\alpha(0)}$ for large $(p\cdot x)$ with fixed x^2.

(b) The s- and u-channel processes can also give long-range contributions to $F(p\cdot x, x^2)$, if, e.g., the form factor in Eq. (14) decreases slowly with $-q^2$. But if one makes additional assumptions on the behavior of the matrix element $M(p\cdot q, q^2)$, extracted from the behavior of electromagnetic current matrix elements, then these contributions are dominated in magnitude by the q^2-channel contributions.

Therefore, one can say that, in the sense of (a) and (b) above, the longest range in $F(p\cdot x, x^2)$ is due to $j(0)$ turning into hadrons, which scatter against the target.

These conclusions are entirely consistent with the physical criterion for the presence of long-range correlations suggested in Sec. III. We see that the q^2-channel contributions are proportional to the hadron-target scattering amplitudes at high energies; therefore the propagation of high-energy particles is responsible for the long-range correlations. These are *real* particles, in the sense that the discontinuities in $M(p\cdot q, q^2)$ are nonzero only for those values of q for which the intermediate states in the q^2 channel can actually occur. At this stage no "virtual" states need to be invoked. If the matrix element $\langle p|j(x)j(y)|p\rangle$ is considered as the limit of $\langle p|j(f)j(g)|p\rangle$ when the supports of the smearing functions f and g shrink to the points x and y, corresponding to measurements localized in increasingly small space-time regions, large amounts of energy and momentum are involved, and real particles can be created in the measurement process. Since one is interested in the connected matrix elements, it is the propagation of these real particles in the presence of the target that is responsible for the long-range correlations in $F(p\cdot x, x^2)$. It is therefore quite natural that the long-range correlations are proportional to the scattering amplitudes.

As mentioned in footnote 18, there can be also some long-range contributions, vanishing only as some inverse power of $p\cdot x$, arising from a coincidence of an s- or u-channel threshold with a q^2-channel threshold, in apparent contradiction to our criterion because no large momentum states seem to be involved. But this is owing to the fact that a plane-wave state $|p\rangle$ extending over all space has been used. When one smears in p to obtain a localized target, these particular long-range parts will be removed, as seen from the $e^{ip\cdot x}$ factor multiplying these contributions.

Having established the presence of long-range correlations in $F(p\cdot x, x^2)$, and having discussed their physical origin, we can now turn to the second question: In the spirit of the Gribov-Ioffe-Pomeranchuk analysis, which long-range part in F contributes to $M(p\cdot q, q^2)$ for some fixed q^2 as $p\cdot q\to\infty$? As mentioned earlier, the question they raised is: For $x_3\simeq x_0+O(1/q_0)$, what is the behavior of the remaining integral

$$\int dx_0\, e^{ix_0(q^2/q_0)}\, F(x_0, x^2),$$

for large q_0? Since for fixed q^2 the ratio (q^2/q_0) goes to zero, the integral gets contributions over a large interval, of the order (q_0/q^2), provided $F(x_0, x^2)$ has a long-range part not multiplied by a phase factor that varies rapidly with x_0. For $x_0 \lesssim (q_0/q^2)$ and $x_3\simeq x_0+O(1/q_0)$, x^2 is bounded and the earlier results on $F(x_0, x^2)$ apply. We know that there *is* such a long-range part in $F(x_0, x^2)$ not multiplied a rapidly oscillating phase, coming from q^2-channel contributions. Because of the time ordering involved in the definition of $F(x_0, x^2)$, this long-range piece can contribute to the above integral even for fixed values of q^2 not equal to those at which the physical q^2-channel processes occur. It is perhaps in this sense that one can speak of the projectile turning into *virtual* hadrons, which propagate for a period of the order (q_0/q^2).

The above analysis provides a basis for the intuitive picture of high-energy scattering taking place through the projectile turning into virtual hadrons. At the same time we see that caution is required so as not to extend this interpretation too far, since the s- and u-channel processes can also give long-range parts that may be important under different circumstances. Further dynamical inputs, such as the behavior of form factors or scaling functions, are needed in each case to ascertain when the q^2-channel contributions to long-range correlations indeed dominate.

The picture of a projectile turning into virtual hadrons has also been applied to virtual Compton scattering in the deep-inelastic region,[23] and long-range correlations corresponding to $\omega=0$ ($\omega=-q^2/q_0$) have been discussed. In principle, both q^2-channel discontinuities and s-channel discontinuities in the region $\omega=0$ can contribute long-range parts to $F(x_0, x^2)$.[24] However, the s-channel contributions will be present also for finite ω, whereas the smoothness of the experimental scaling function $f_2(\omega)$ together with its vanishing behavior near $\omega=\pm 1$ imply that the long-range correlation corresponding to $\omega=0$ dominates, provided $f_2(0)$ is not zero.[25] So if experimentally $f_2(0)$ is nonzero, the q^2-channel contributions to long-range correlations are also dominant in the deep-inelastic virtual Compton process.

One sees, therefore, that in practice the process of the source first turning into hadrons, with the scattering between these hadrons and the target taking place afterwards, seems to be the most important according to present evidences; but in principle the propagation of hadrons created after the target and the source have already interacted, corresponding to s-channel intermediate states, can also give rise to long-range correlations. In either case, however, the propagation of particles with $v\simeq c$ in the target rest frame is responsible for the long range, if the target is localized.

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[14]When this paper was under preparation, we learned that R. Brandt and P. Otterson have also established the vacuum annihilation property of "lightlike charges" [J. Math. Phys. 13, 1714 (1972)]. For a study of "lightlike charges" in perturbation theory, see F. Jegerlehner, thesis, Univ. of Bern, 1972 (unpublished).

[15]In this connection, it does not matter even whether the field is a component of a current or whether it is some other local field.

[16]V. N. Gribov, Zh. Eksp. Teor. Fiz. 57, 1306 (1969) [Sov. Phys.-JETP 30, 709 (1970)].

[17]As is well known, the contribution from the semicircle is actually not negligible if there are strong singularities at the origin in configuration space, and subtractions of derivatives of δ functions have to be made. However, since one is interested here in the large-x_0 behavior, the subtraction will not contribute to it, and one can imagine that one is actually closing the contour for the subtracted amplitude.

[18]This is true if the forward elastic amplitude $A(q_0) > \text{const}(q_0)^{-3/2}$ when q_0 is large, since the contributions from the s-channel singularities of $R(q_0,q^2)$ coinciding with the double pole at $q^2=\mu^2$ are of order $(x_0)^{-3/2}$.

[19]If the particle in the s-channel intermediate state differs from the target particle, transition form factors enter instead of elastic form factors.

[20]This was pointed out by C. A. Orzalesi [Phys. Rev. D 7, 488 (1973)] and independently by M. Robillota, A. Silva, and C. H. Woo [in Proceedings of the Fourth Brazilian Symposium on Theoretical Physics, 1972 (unpublished)].

[21]If the total cross sections grow logarithmically, for instance, F_i will have $\ln(p\cdot x)$ dependences also, as seen from Eq. (12).

[22]The coefficients have different x^2 dependences, and it is unlikely that they sum to zero identically for all x^2.

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[24]In this case the separation into q^2-channel and s-channel contributins is not gauge-invariant, although for the contribution to the longest range the separation may be still meaningful; see below.

[25]We use $f_2(\omega)$ instead of $F_2(\omega)$ to denote the scaling function to avoid confusion with our earlier notations. The connection between the behavior of $f_2(\omega)$ and long-range correlations has been discussed by Pestieau *et al*. (Ref. 23); however, an error in their erratum should be corrected. What is relevant as $\omega \to 0$ is whether the even derivatives of $\omega^{-1}f_2(\omega)$ vanish, not how fast $f_2(\omega)$ approaches zero.