$$W=\exp(i3\pi Y_-). \quad (12)$$

This restriction, combining with the assumption of octet dominance, limits the symmetry breaking to the (1, 8), (8, 1), $(3, \bar{3})$, $(\bar{3}, 3)$, $(6, \bar{6})$, $(\bar{6}, 6)$, and (8, 8) representations.[21]

Unfortunately, a W-invariant irreducible Hamiltonian gives unpleasant results in the $(3, \bar{3})+(\bar{3}, 3)$ system ($m_\pi^2=0$ and $\sigma_{\pi N}=0$).[19] However, a reducible, W-invariant Hamiltonian certainly contains sufficient free parameters to fit all the presently known data on meson-meson and meson-baryon scattering.

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Goldstone Bosons as Bound States in the Quark-Gluon Model*

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We examine the implications of a Nambu-Goldstone realization of chiral symmetry in the quark-gluon model. The context of this examination is that of a renormalizable, finite theory, so eigenvalue conditions are assumed to be satisfied. We discuss the solutions to the Schwinger-Dyson gap equation for the fermion self-energy $\Sigma(p^2)$ that exhibit spontaneous breaking of the vacuum symmetry. In the leading-order Bethe-Salpeter approximation the boundary conditions to the homogeneous, linear integral equations stipulate the vacuum symmetry. It is shown how the Goldstone bosons emerge as bound states, as suggested by Nambu and Jona-Lasinio. We also examine the Goldstone alternative in the Bethe-Salpeter equation for fermion-fermion scattering. Explicit symmetry breaking is introduced by additional Abelian vector gluons coupling to hypercharge and isospin besides baryon number. The eigenvalue condition for the fine-structure constant is consequently model-dependent but takes a simple form. We also consider the influence of explicit symmetry breaking on the ground-state mesons and indicate how the solutions to the eigenvalue problem regulate the structure of symmetry breaking.

I. INTRODUCTION

The standard model for implementing the Nambu-Goldstone realization of chiral symmetry has been the renormalized Σ model.[1,2] This model has the property that to lowest order in the coupling constants and in the tree approximation one may examine the spontaneous symmetry breaking of the ground state. In particular the mechanism of the Goldstone theorem is explicit in this model, and most of the consequences of chiral symmetry like soft-pion theorems can be examined in the tree approximation.

In this article we will examine how a Nambu-

Goldstone realization of a chiral symmetry can occur in the quark-gluon model. If such a realization is possible, then the ground-state mesons must be bound states of quark-antiquark pairs. Consequently such a realization can never be obtained to any finite order in perturbation theory in the quark-gluon coupling.

Our undertaking is very much in the spirit of the original Nambu–Jona-Lasinio model[3] in which the ground-state pion emerged as a massless bound state. However, their model required a cutoff due to the divergences associated with the four-fermion interaction, while the quark-gluon model, after renormalization, will be cutoff-independent.

We are motivated by the hope of incorporating the feature of chiral symmetry, which has experimental success in its own right, with the features of the quark model into a single model of the strong interactions. Both the Σ model and the quark model have undesired features. In the Σ model one has elementary scalar and pseudoscalar fields contributing to the currents so that the Callan-Gross sum rule,[4] which seems to be supported experimentally, would be violated. Further, one does not have the same light-cone algebra as abstracted from the quark model. On the other hand, in the quark-gluon model, as it is usually implemented in perturbation theory, one does not have Goldberger-Treiman relations or soft-pion theorems. We would like to construct a model that retains the desirable features of both the Σ model and the quark model and dispenses with the undesired features.

In the model we will describe the ground-state mesons would be bound states, like other hadrons, and could lie on a Regge trajectory. In this model the energy scale is set by the quark mass or, equivalently, the meson decay constant f_π.[5] Ultimately one might amend a gauge model of weak interactions to such a hadron model.

The quark-gluon model we will examine is essentially quantum electrodynamics (QED) with a massive photon and zero-bare-mass fermion. We will appeal to some of the known features of this theory which lie outside the direct context of perturbation theory. For reviews on the topic of finite QED, the reader should consult the articles of Johnson and Baker,[6] and Adler.[7]

There is, however, an important difference between the solutions to QED that are experimentally relevant and those of the gluon model as a model of strong interactions. While we want Goldstone bosons in the symmetry limit for the gluon model, we do not want this for QED. The way in which this difference can come about is described here. The main point is that the formal γ_5 invariance of the Lagrangian implies that the matrix elements of the divergence of the axial-vector current obey *homogeneous* Dyson-Schwinger equations. If we pick the trivial solution to such homogeneous equations, then the Goldstone theorem applies and we must have Goldstone bosons. This is the case desired for the gluon model. Alternatively, if we assume the Goldstone bosons do not couple then we cannot have the trivial solutions to the homogeneous equations, and the Goldstone theorem does not apply. This is the solution for QED.

The plan of this article is as follows. First we will review the conditions for a finite theory. We discuss the conditions that the Schwinger-Dyson gap equation for the fermion propagator possess a nontrivial solution. The fermion mass is generated purely from the interactions (as in the Σ model). From the axial-vector Ward identities, we examine the conditions that Goldstone bosons exist or do not exist. We also examine the Bethe-Salpeter equation for zero-mass poles in the fermion-fermion scattering amplitude and show how the Goldstone alternative emerges.

In Sec. III we consider explicit symmetry breaking by introducing additional Abelian vector mesons which couple to Y (=hypercharge) and I_3 (=component of isospin) besides B (=baryon number). The eigenvalue conditions for a finite theory turn out to have the simple form $f(g_i^2)=0$, where the g_i ($i=1,2,3$) are linear combinations of the coupling constants to B, Y, and I_3. A consequence is that the eigenvalue condition for the electric charge e is model-dependent. It depends on the couplings of the additional vector-meson species. One may still in principle compute the fine-structure constant; however, it is no longer the eigenvalue of the simplified eigenvalue condition for pure QED. Further, we indicate how the solutions to the eigenvalue problem can regulate the structure of symmetry breaking.

The model is considered in the leading-order Bethe-Salpeter approximation. In this approximation the integral equations for the fermion self-energy are homogeneous linear integral equations. It is the specification of the vacuum symmetry that establishes the boundary conditions required to solve such equations. Some symmetry-breaking effects in the ground-state meson spectrum are considered. All symmetry-breaking effects are completely finite in this model. In particular, radiative corrections are all finite.

II. THE QUARK-GLUON MODEL

A. Finite Quantum Electrodynamics

We will briefly review the conditions for a finite theory[6,7] and specify our assumptions. The Lagrangian is $\mathcal{L}=\mathcal{L}_0+\mathcal{L}'$, with

$$\mathcal{L}_0 = -i\bar{q}(x)\gamma\cdot\partial q(x) - \tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} - \tfrac{1}{2}\mu_0{}^2 W_\mu W_\mu - g_0 W_\mu \bar{q}(x)\gamma_\mu q(x) - [\partial_\mu W_\mu(x)]^2/\lambda^2\,, \quad G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu\,. \tag{1}$$

λ is the gauge parameter and $\mathcal{L}'$ is a term which breaks the formal $U(3)\times U(3)$ symmetry of $\mathcal{L}_0$. This theory is renormalizable to every finite order in perturbation theory.

As is well known, the divergences associated with the unrenormalized theory can be lumped into the vertex, wave-function, and coupling-constant renormalizations, $Z_1 = Z_2$ and Z_3, respectively. By a suitable choice of the gauge parameter λ, corresponding to the generalized Landau gauge, the gauge-dependent constants $Z_1 = Z_2$ are rendered finite. We will choose this gauge to work in.

The condition that Z_3, a gauge-independent constant, be finite, or equivalently that the renormalized gluon propagator have asymptotic behavior no worse than free-field behavior, implies an eigenvalue condition on the coupling constant.[8] If one sums the vacuum polarization graphs by a "vacuum polarization insertionwise" summation, then this condition is on the bare coupling g_0, with no implied restriction on the physical coupling other than $g_0{}^2 \geq g^2$. If one sums in renormalized perturbation theory by "loopwise summation" as described by Adler,[7] then the condition is on the physical coupling constant. It could turn out that both of these summation techniques are consistent, in which case they simply define different theories.[6] For definiteness we will assume that the condition is on the physical coupling constant.

If one sums all the graphs of the "cracked-eggshell" variety shown in Fig. 1 to the vacuum polarization in renormalized perturbation theory, then, as Baker and Johnson showed,[9] they sum to at most a single logarithm

$$\pi_c(q^2) \underset{q^2\to\infty}{\sim} C(g^2) + f(g^2)\ln(-q^2/q_0{}^2)\,. \tag{2}$$

Here $f(g^2)$ is the Baker-Johnson function. A necessary condition for a finite Z_3 is the eigenvalue condition

$$f(g^2) = 0\,. \tag{3}$$

If the zero of this function were a simple zero,

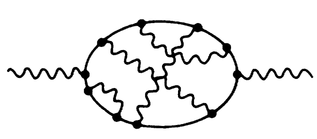

FIG. 1. "Cracked-eggshell" contribution to vacuum polarization.

then the renormalized gluon propagator would approach its asymptotic value rapidly. In this instance one may replace the gluon propagator with its free-field value as far as examining asymptotic behavior of other Green's functions is concerned. However, a zero of $f(g^2)$ implies that it must be an essential zero, in which case the asymptotic behavior of the gluon propagator is approached very slowly.[7] Then it is not clear that one can ignore the nonasymptotic pieces in the gluon propagator in establishing the asymptotic behavior of other Green's functions. We will assume that we may treat the gluon as if it were a free field if the eigenvalue condition is valid. In particular we will consider the simplified model which completely ignores gluon self-energy insertions.[10]

B. The Gap Equation for the Fermion Propagator

In this model we want a formal chiral symmetry of $\mathcal{L}$ so that the bare fermion mass vanishes.[11] The physical fermion mass M need not vanish. The mechanism by which this comes about was described first by Nambu and Jona-Lasinio,[3] and in the present context by Johnson, Willey, and Baker,[9] and Maris, Herscovitz, and Jacob.[12]

Ignoring gluon self-energies so that $\mu^2 = \mu_0{}^2$, $g_0 = g$, we see that the Schwinger-Dyson gap equation for the fermion propagator is

$$S^{-1}(p) = \not{p} + \frac{ig^2}{(2\pi)^4}\int \frac{d^4l}{l^2-\mu^2} \times \left(g_{\mu\nu} - \xi\frac{l_\mu l_\nu}{l^2}\right)\gamma_\mu S(p-l)\Gamma_\nu(p-l,p)\,, \tag{4}$$

where $\xi(g^2)$, a gauge parameter, is chosen so that the integral is finite (this defines the Landau gauge), and $S^{-1}(p) = C(p^2)[\not{p} - \Sigma(p^2)]$ is the fermion propagator, and Γ_ν is the vertex function. These are unrenormalized quantities which differ from the renormalized amplitudes only by the multiplication of finite constants $Z_1 = Z_2$ in the Landau gauge.

Little is known about the solutions to the nonlinear equation (4), even in the approximation $\Gamma_\mu = \gamma_\mu$. Suppose that μ is the only mass scale in the problem. Then if $S^{-1}(p^2, \mu^2, g^2)$ is a solution of (4) so is $\beta^{-1}S^{-1}(\beta^2 p^2, \beta^2\mu^2, g^2)$, which, in this instance, is identical to $S^{-1}(p^2, \mu^2, g^2)$. For the self-energy one has

$$\Sigma(p^2) = \mu F(p^2/\mu^2, g^2)\,, \tag{5}$$

with F a dimensionless function of its arguments. Then the fermion mass M is determined from the gluon mass by

$$M = \mu F(M^2/\mu^2, g^2) \tag{6}$$

and is not arbitrary. QED could be a solution to (6) corresponding to $\mu = 0$, and M an arbitrary mass. Equation (6) could also possibly have solutions $\mu/M \neq 0$ corresponding to the gluon model.

Alternatively it may not be possible to solve the integral equations without stipulating the value of $S^{-1}(p)$ at some point p_0^2. This introduces a new parameter with the dimensions of mass so that $\beta^{-1}S^{-1}(\beta^2 p^2, \beta^2\mu^2, g^2)$ is not identical to $S^{-1}(p^2, \mu^2, g^2)$. Then the boundary condition $\Sigma(M^2) = M$ fixes β, and the masses M and μ can be arbitrary.

To leading order in the Bethe-Salpeter expansion, the fermion mass M is arbitrary. To this approximation the equation (4) for $\Sigma(p)$ is

$$\Sigma(p) = \frac{-3ig^2}{(2\pi)^4}\int \frac{d^4l\, \Sigma(p-l)}{(l^2-\mu^2)[(p-l)^2 - M^2]}, \tag{7}$$

i.e., a linear homogeneous equation. So if $\Sigma(p)$ is a solution, so is any constant multiple of $\Sigma(p)$ with the constant specified by the boundary condition $\Sigma(M^2) = M$.

It is known from a study of the Callan-Symanzik equations[10] that the asymptotic behavior of the fermion propagator is given by

$$\begin{aligned} C(p^2) &\xrightarrow[p^2\to\infty]{} C_1(\mu^2/M^2, g^2), \\ \Sigma(p^2) &\underset{p^2\to\infty}{\sim} MC_2(\mu^2/M^2, g^2)(M^2/p^2)^{\epsilon(g^2)}, \end{aligned} \tag{8}$$

with $C_2(\mu^2/M^2, g^2)$ finite as $\mu/M \to 0$. For the integral (4) to exist we require

$$\epsilon(g^2) > 0\,. \tag{9}$$

This is valid to the first few orders of perturbation theory $\epsilon(g^2) = 3g^2/(4\pi)^2 + \frac{3}{2}g^4/(4\pi)^4 + \cdots$. We will assume (9) is satisfied.

C. The Goldstone Alternative in the Gluon Model

That the gluon model can exhibit the Goldstone mode has been shown elsewhere.[13, 14] Here we will briefly review the argument. Let ${}^a\bar{\Gamma}^5_\mu(p', p)$ be the renormalized axial-vector vertex function corresponding to $\gamma_\mu\gamma_5\lambda_a$, and $2M\,{}^a\bar{\Gamma}^5_D(p', p)$ the vertex corresponding to the divergence of the axial-vector current. The Ward identity for the axial-vector vertex then is

$$(p'-p)_\mu\, {}^a\bar{\Gamma}^5_\mu(p', p) = 2M\,{}^a\bar{\Gamma}^5_D(p', p) + (Z_A/Z_2)[\bar{S}_F^{-1}(p')\gamma_5\lambda_a + \lambda_a\gamma_5\bar{S}_F^{-1}(p)], \tag{10}$$

where $\bar{S}_F^{-1}(p) = \gamma\cdot p - \Sigma(p)$ is the renormalized fermion propagator, and Z_A/Z_2 a ratio of renormalization constants which is cutoff-independent. Further, it can be shown that the Schwinger-Dyson equation for ${}^a\bar{\Gamma}^5_D(p', p)$ is homogeneous, a consequence of the formal chiral symmetry of $\mathcal{L}_0$. The integral equation is

$${}^a\bar{\Gamma}^5_D(p', p) = \int \frac{d^4l}{(2\pi)^4}\bar{S}_F(p'+l)\,{}^a\bar{\Gamma}^5_D(p'+l, p+l)\bar{S}_F(p+l)\bar{K}(p', p, l), \tag{11}$$

where $\bar{K}$ is the renormalized fermion-antifermion scattering kernel.

If there exist Goldstone bosons, then

$${}^a\bar{\Gamma}^5_\mu(p', p) \sim \lambda^a\gamma_5 G(p)(p'-p)_\mu/(p'-p)^2$$

as $p' \to p$, where $G(p)$ is the coupling of the Goldstone meson to the quarks. From (10) one has

$$\gamma_5\lambda_a G(p) = 2M\,{}^a\bar{\Gamma}^5_D(p, p) - (Z_A/Z_2)\gamma_5\lambda_a 2\Sigma(p)\,. \tag{12}$$

If the Goldstone mode is absent, as is desired for QED, then $G(p) = 0$ and $2M\,{}^a\bar{\Gamma}^5_D(p, p) = (Z_A/Z_2) \times\gamma_5 2\Sigma(p)$. This fixes a boundary condition on the homogeneous integral equation (11), so we cannot have a trivial solution in spite of the formal chiral symmetry of $\mathcal{L}_0$. This is not a violation of the Goldstone theorem because the formal chiral symmetry of $\mathcal{L}_0$ is not a true symmetry and the matrix elements of the divergence of the axial-vector currents do not vanish.

Alternatively, if we chose the trivial solution ${}^a\bar{\Gamma}^5_D(p', p) = 0$ to the homogeneous integral equation, then we must have, from Eq. (12), $G(p) = 2(Z_A/Z_2)\Sigma(p)$, a Goldberger-Treiman relation. This is the solution desired for our strong-interaction model in the absence of explicit symmetry breaking (which gives the mass of the Goldstone bosons). In this instance the Goldstone theorem applies since the matrix elements of the divergence of the axial-vector current vanish.

The alternatives are the following: *Either* (I) the Goldstone mode is absent and the formal symmetry of the Lagrangian is broken explicitly for matrix elements, *or* (II) the formal symmetry of the Lagrangian is valid for matrix elements of the divergence of the axial-vector current and the vacuum symmetry is thus spontaneously broken, requiring massless Goldstone bosons.

There is no evident inconsistency for either (I) or (II), but it could turn out that they cannot both be solutions for all values of the masses and coupling constants. The reason is that the integral equations to which we assume solutions exist may not have the required solution for all values of the input parameters.

An interesting by-product of this discussion is that a formal symmetry of $\mathcal{L}$, corresponding to a formally conserved current, need not be a true symmetry of the world. This is in fact what happens for QED. Perhaps this is a way of introducing explicit symmetry breaking without explicit symmetry breaking on the formal level, while preserving the renormalizable structure of the theory.

The Ward identities for axial-vector currents have anomalous terms coming from fermion triangles. In the present case there is an anomaly in the Ward identity for the current corresponding to $\gamma_5\gamma_\mu\lambda_0$. However, these anomalies do not affect our considerations here since the Goldstone theorem is a statement at zero momentum transfer and all anomalies of the triangle type vanish at this point.[15]

D. Bethe-Salpeter Equation

It is instructive to examine the Goldstone alternative in the context of the Bethe-Salpeter equation (BSE) for fermion-antifermion scattering. We will suppress the internal symmetry factors since they do not affect the discussion.

The BSE for the fermion-antifermion amplitude is

$$ {}_{ab}T_{cd}(p,p',k) = {}_{ab}K_{cd}(p,p',k) + \int \frac{d^4 l}{(2\pi)^4}\, {}_{ae}T_{fd}(p,l,k) S_{eg}(l_+)\, {}_{gb}K_{ch}(l,p',k) S_{hf}(l_-)\,, \qquad l_\pm = l \pm \tfrac{1}{2}k \tag{13} $$

and the kinematics is illustrated in Fig. 2. Here K is the kernel, a, b, c, d, etc. refer to spin indices, and $S^{-1}(l) = \not{l} - M$. If there is a massless 0^- Goldstone boson of the type discussed in the last section, then it appears as a pole in T,

$$ {}_{ab}T_{cd}(p,p',k) = i\gamma^5_{ad}\gamma^5_{cb}\frac{g(p)g(p')}{k^2} + {}_{ab}T^R_{cd}(p,p',k)\,, \tag{14} $$

with T^R regular as $k \to 0$, and $g(p)$ the coupling constant of the Goldstone boson to the fermions. It is sufficient to consider the projection

$$ 12igF(p,p',k) = \gamma^5_{ad}\, {}_{ab}T_{cd}\gamma^5_{cb}\,. \tag{15} $$

We will also consider the kernel only to leading order and in the Landau gauge,

$$ {}_{ab}K_{cd}(p,p',k) = -ig^2\left[g_{\alpha\beta} - \frac{(p-p')_\alpha(p-p')_\beta}{(p-p')^2}\right]\frac{\gamma^\alpha_{ab}\gamma^\beta_{cd}}{(p'-p)^2-\mu^2}\,, \tag{16} $$

which has the projection

$$ \frac{12ig^2}{(p'-p)^2-\mu^2} = \gamma^5_{ad}\, {}_{ab}K_{cd}\gamma^5_{cb}\,. $$

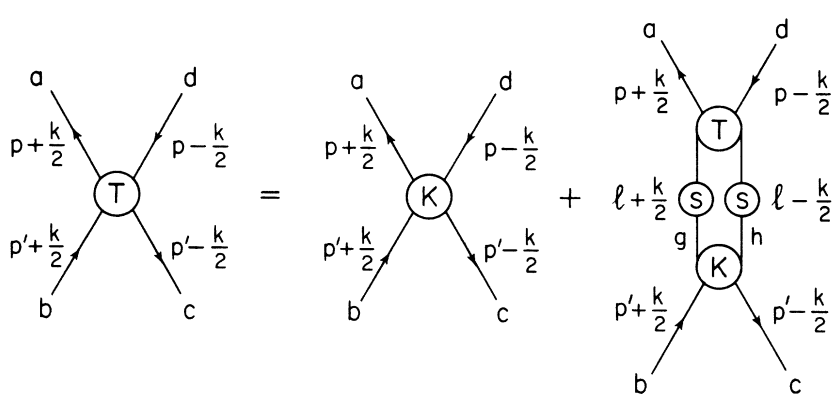

FIG. 2. Bethe-Salpeter-equation kinematics.

Next we write

$$F(p,p',k)=\frac{G(p)G(p')}{k^2}+F^R(p,p',k)\,, \tag{17}$$

where $G(p)=2g(p)/3g$ and $F^R(p,p',k)$ is regular as $k\to 0$. With the definitions (13)–(17), substituting in (13), one obtains

$$\frac{G(p)}{k^2}\left[G(p')+3ig^2\int\frac{d^4l}{(2\pi)^4}\frac{G(l)}{[(p'-l)^2-\mu^2](l^2-M^2)}\right]+F^R(p,p',k)$$

$$=\frac{1}{(p'-p)^2-\mu^2}-3ig^2G(p)\int\frac{d^4l}{(2\pi)^4}\frac{G(l)}{(p'-l)^2-\mu^2}\left[\frac{R(l^2,l\cdot k,k)}{k^2}\right]$$

$$-3ig^2\int\frac{d^4l}{(2\pi)^4}\frac{F^R(p,l,k)(l^2-m^2-k^2/4)+M^R(p,l,k)}{[(p'-l)^2-\mu^2](l_+{}^2-M^2)(l^2-M^2)}\,, \tag{18}$$

where

$$R(l^2,l\cdot k,k^2)=\frac{-\frac{3}{4}k^2(l^2-M^2)-\frac{1}{16}k^4+(l\cdot k)^2}{(l_+{}^2-M^2)(l_-{}^2-M^2)(l^2-M^2)}$$

so that

$$\frac{R(l^2,l\cdot k,k^2)}{k^2}\to\frac{-\frac{1}{2}l^2+\frac{3}{4}M^2}{(l^2-M^2)^3}\quad\text{as } k\to 0$$

and is nonsingular, and

$$12ig^2M^R(p,l,k)=\gamma^5_{ad}{}_{ae}T^R_{fd}(p,l,k)\gamma^5_{eh}[\tfrac{1}{2}(\not{l}\not{k}-\not{k}\not{l})-M\not{k}\,]_{hf}$$

vanishes as $k\to 0$.

It follows from (18) that if we take the $k\to 0$ limit and observe, since all the terms but the first are nonsingular, then we have the condition

$$G(p)\left[G(p')+3ig^2\int\frac{d^4l}{(2\pi)^4}\frac{G(l)}{[(p'-l)^2-\mu^2](l^2-M^2)}\right]=0\,. \tag{19}$$

The two solutions to this equation correspond to the Goldstone alternative. If $G(p)=0$, then the Goldstone mode does not couple. Then the integral equation for the amplitude is from (18) as $k\to 0$

$$F^R(p,p',0)=\frac{1}{(p'-p)^2-\mu^2}-3ig^2\int\frac{d^4l}{(2\pi)^4}\frac{F^R(p,l,0)}{[(p'-l)^2-\mu^2](l^2-M^2)}\,. \tag{20}$$

This equation is the integral equation considered by Willey,[16] who showed that regular solutions existed.

Alternatively, if the Goldstone mode couples, then $G(p)\neq 0$ and hence from (19) we require

$$G(p')=-3ig^2\int\frac{d^4l}{(2\pi)^4}\frac{G(l)}{[(p'-l)^2-\mu^2](l^2-M^2)}\,. \tag{21}$$

The solution to this is given by the gap equation (7):

$$G(p)=A\Sigma(p)\,.$$

This is the Goldberger-Treiman formula where the constant A is specified in terms of the meson decay constant. If we use this Goldstone solution, then the integral equation for the regular part of the amplitude is as $k\to 0$

$$F^R(p,p')=\frac{1}{(p'-p)^2-\mu^2}-\frac{3i}{4}g^2G(p)\int\frac{d^4l}{(2\pi)^4}\frac{G(l)(3M^2-2l^2)}{[(p'-l)^2-\mu^2](l^2-M^2)^3}-3ig^2\int\frac{d^4l}{(2\pi)^4}\frac{F^R(p,l)}{[(p'-l)^2-\mu^2](l^2-M^2)}\,. \tag{22}$$

This equation with $G(p)\neq 0$ is completely distinct from the one considered by Willey, Eq. (20), and hence the existence of solutions to his equation in no way prejudices the existence or nonexistence of solutions to (22). Whether or not solutions exist to (22) with $G(p)\neq 0$ is not known; however, an iterative solution in a power series in g^2 can be constructed with convergent coefficients.

It is of interest to note that the coupling constant $G(p)$ of the Goldstone boson, to the leading-order BS approximation, is independent of the value of the gluon coupling constant g. It is, of course, related to the fermion mass M by the Goldberger-Treiman formula, but M is an input parameter to this same approximation. Consequently the coupling constant for the Goldstone bosons can be large and serve to regulate the strength of the strong interactions, while the gluon coupling is relatively much weaker.

The coupling of the Goldstone bosons may be the dominant strong-interaction force. Other bound states, like massive mesons and baryons, could be due to the larger coupling of the Goldstone particles and not the gluon, whose influence on real physics could be negligible, except to trigger the Goldstone mode.

III. SYMMETRY BREAKING

Symmetry breaking can also be introduced into the model. So far we have discussed only a single vector boson W_μ coupling to the total baryon number B. For this model we have a solution for which the quark triplet has a common mass M and there exists an octet of massless Nambu-Goldstone bound-state bosons. By coupling additional vector bosons to the hypercharge $\lambda_8/\sqrt{3}$ and isospin λ_3, we explicitly break the symmetry and the Goldstone bosons get a mass. However, since the currents to which the additional vector mesons couple are absolutely conserved, the model is still renormalizable.

Instead of coupling these fields to $\lambda_0, \lambda_8, \lambda_3$ we can diagonalize the problem by coupling to the charges

$$\begin{aligned} Q_1 &= (\tfrac{1}{3})^{1/2}[\lambda_0 + (\tfrac{1}{2})^{1/2}\lambda_8 - \tfrac{3}{2}\lambda_3],\\ Q_2 &= (\tfrac{1}{3})^{1/2}[\lambda_0 + (\tfrac{1}{2})^{1/2}\lambda_8 + \tfrac{3}{2}\lambda_3],\\ Q_3 &= (\tfrac{1}{6})^{1/2}(\lambda_0 - \sqrt{2}\lambda_8), \end{aligned} \tag{23}$$

with the orthonormal property

$$Q_i Q_j = \delta_{ij} Q_i . \tag{24}$$

If g_i are the coupling constants to the charges Q_i, and e, b, y are the couplings to $\lambda_3+\lambda_8/\sqrt{3}$, λ_0, $\lambda_8/\sqrt{3}$, respectively, then

$$\begin{aligned} e &= \tfrac{2}{3}g_2 - \tfrac{1}{3}(g_1+g_3),\\ b &= \tfrac{2}{3}(g_1+g_2+g_3),\\ y &= \tfrac{1}{3}(g_1+g_2-2g_3). \end{aligned} \tag{25}$$

The Lagrangian is

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_q + \mathcal{L}_G + \mathcal{L}_I ,\\ \mathcal{L}_q &= -i\bar{q}(x)\partial\!\!\!/ q(x) - m_i^0\bar{q}(x)Q_i q(x),\\ \mathcal{L}_G &= -\tfrac{1}{4}W^i_{\mu\nu}W^i_{\mu\nu} - \tfrac{1}{2}(\mu_i^0)^2[W^i_\mu W^i_\mu + (\partial_\mu W^i_\mu)^2/\lambda_i^2],\\ \mathcal{L}_I &= g_i^0\bar{q}W\!\!\!/^i Q_i q , \end{aligned} \tag{26}$$

from which one obtains the equations of motion

$$\begin{aligned} &(i\partial\!\!\!/ + g_i W\!\!\!/_i Q_i)q(x) = m_i^0 Q_i q(x),\\ &i\partial_\mu\bar{q}(x)\gamma_\mu - g_i\bar{q}(x)W\!\!\!/_i Q_i = -m_i^0\bar{q}(x)Q_i ,\\ &(\Box + \mu_0{}^2)W^i_\mu(x) = g_i^0 V^i_\mu(x),\\ &V^i_\mu(x) = \bar{q}(x)\gamma_\mu Q_i q(x),\\ &(\Box + \lambda_i{}^2)\partial_\mu W^i_\mu = 0 . \end{aligned} \tag{27}$$

For the divergences of the vector and axial-vector currents $A^a_\mu = \bar{q}\gamma_\mu\gamma_5\frac{1}{2}\lambda^a q$, $V^a_\mu = \bar{q}\gamma_\mu\frac{1}{2}\lambda^a q$, one has

$$\begin{aligned} \partial_\mu A^a_\mu &= \bar{q}i\gamma_5 m_i^0[Q_i, \tfrac{1}{2}\lambda^a]_+ q + g_i^0\bar{q}i\gamma_5 W\!\!\!/_i[Q_i, \tfrac{1}{2}\lambda^a]_- q ,\\ \partial_\mu V^a_\mu &= i m_i^0 q[Q_i, \tfrac{1}{2}\lambda^a]_- q - i g_i^0\bar{q}W\!\!\!/_i[Q_i, \tfrac{1}{2}\lambda^a]_- q . \end{aligned} \tag{28}$$

We have retained the bare masses m_i^0.

A. Eigenvalue Conditions

For such a model to be finite we require that the Baker-Johnson function vanish. As a consequence of the orthonormality (24), it is clear that only a single coupling constant g_i enters the vacuum polarization $\pi^i(q^2)$ of the gluon W^i_μ. Hence the three eigenvalue conditions are the same in this basis,

$$f(g_i{}^2) = 0. \tag{29}$$

This equation is the same function that appears in pure QED, but the condition is on different coupling constants. Only if we set $b=y=0$ in (25) do we recover the condition in pure QED, $f(e^2)=0$. Consequently the solution to the eigenvalue problem in pure QED may have nothing to do with the physical fine-structure constant. However, the solutions to (29) in conjunction with (25) can, in principle, determine the fine-structure constant.

The physical fine-structure constant is a small number. In principle, $g_i{}^2$, which are solutions to the eigenvalue problem, could be large since the eigenvalue condition is nonperturbative. It is easy to see that if there is only one nontrivial solution to the eigenvalue problem and this eigenvalue is large, then the fine-structure constant must either vanish or itself be large. However, if there are two nontrivial solutions to (29), both of which are large, then one can accommodate a small value for e as a difference between large eigenvalues. What we wish to suggest is that a small value of the fine-structure constant could emerge in spite of the nonperturbative nature of the eigenvalue condition, which seems to suggest that only large solutions may be possible. However, perturbation theory may be completely deceptive in this application.

B. Symmetry Breaking in the Hadrons

In order to study symmetry breaking in this model we will make some simplifying assumptions. First we will set the three gluon masses equal, which has as a particular consequence the vanishing of the photon mass $\mu_\gamma^2 = \frac{2}{3}\mu_2^2 - \frac{1}{3}(\mu_1^2 + \mu_3^2) = 0$. Second we will examine the fermion masses in the leading-order Bethe-Salpeter approximation. Writing for the self-energy

$$\Sigma(p^2) = \sum_{i=1}^{3} \Sigma_i(p^2) Q_i \ , \tag{30}$$

the gap equation (7) reads

$$\Sigma_i(p^2, g_i^2) = \frac{-3ig_i^2}{(2\pi)^4} \int \frac{d^4l}{l^2 - \mu^2} \frac{\Sigma_i((p-l)^2, g_i^2)}{(p-l)^2 - M_i^2} \ . \tag{31}$$

To specify the solution to this equation[17] we must impose a boundary condition

$$\Sigma_i(M_i^2, g_i^2) = M_i \ , \tag{32}$$

where we are completely free to pick M_i. Specifying the M_i amounts to stipulating the vacuum symmetry in this model. $M_i \neq 0$ corresponds to a broken vacuum symmetry. A. remarked previously, whether or not this leads to Goldstone bosons depends on whether or not the symmetry is broken explicitly or only in the vacuum.

In the Σ model one is not completely free to pick the vacuum symmetry. The solutions for the vacuum expectation values of the scalar field arise from minimizing a nonlinear potential function, and the minimum depends on coupling constants, etc. As remarked earlier, for the gluon model in the full nonlinear version one may perhaps not be able to pick the M_i arbitrarily.

To proceed we will make the assumption that if $g_i^2 = 0$, then $\Sigma_i(p^2, 0) = M$ independent of i. Although we are not free to vary the coupling constants g_i because of the eigenvalue conditions, it is helpful to think in such terms. Alternatively we could have also chosen $\Sigma_i(p^2, g_i^2) = M(p^2, b^2)$ if $y = e = 0$, but we will not pursue this here. The point is that we want an SU(3)-symmetric vacuum in the absence of explicit symmetry breaking so that there will be an octet of Goldstone bosons.[18] The SU(3)×SU(3) can be accommodated [rather than U(3)×U(3)] if we chose a nontrivial boundary condition for the divergence transforming like $\gamma_5\lambda_0$ consistent with the Ward identity (12) with $G(p) = 0$. Then the eight Goldstone bosons acquire mass when we turn on the explicit breaking.

With this assumption about the nature of the symmetry breaking, we write

$$\Sigma_i(p^2, g^2) = M + g_i^2 A(p^2, g_i^2) , \tag{33}$$

where $g_i^2 A(p^2, g_i^2)$ vanishes as $g_i^2 \to 0$ by assumption. Then we have for the quark masses

$$M_i = M + g_i^2 A(M_i^2, g_i^2) . \tag{34}$$

To estimate the shift in the pseudoscalar mesons due to explicit breaking, we will calculate only that due to the quark mass shift. This is effectively only the tadpole part. Setting $A(M_i^2, g_i^2) \simeq A(M^2, 0)$, we obtain from $\mu_{ab}^2 \propto \mathrm{Tr}[\lambda^a[\lambda^b, \Sigma - MI]_+]_+$, where μ_{ab}^2 is the pseudoscalar mass matrix,

$$\begin{aligned} C\mu_{ab}^2 &= (g_1^2 + g_2^2 + g_3^2)\tfrac{2}{3}\,\delta_{ab} \\ &\quad + [\tfrac{1}{2}(g_1^2 + g_2^2) - \sqrt{2}\, g_3^2]\, d_{ab8} \\ &\quad + (g_1^2 - g_2^2)\tfrac{3}{2}\, d_{ab3} , \end{aligned} \tag{35}$$

with C a constant.

The solutions to eigenvalue problem

$$f(g_i^2) = 0 \tag{36}$$

serve to regulate the structure of explicit symmetry breaking. If we choose the trivial solution (the only known solution) to (36) for $g_1^2 = g_2^2 = 0$ and assume $g_3^2 \neq 0$, then (35) implies a vanishing pion mass. This corresponds to SU(2)×SU(2) residual symmetry. With other assumptions about the solutions of (36) one can accommodate other symmetry-breaking schemes. Without further knowledge of solutions to the eigenvalue problem, however, one can say nothing.

All symmetry-breaking effects in this model are finite. In particular, radiative corrections are finite so that one can obtain finite results for electromagnetic mass shifts, etc. However, at the level of approximation that we are examining, the boundary conditions on the homogeneous Dyson-Schwinger equations are arbitrary inputs like the quark masses M_i. So one cannot calculate except in terms of the M_i which are not known *a priori*. To examine how, if at all, the M_i are determined takes one into the full nonlinear problem of the solutions to the Schwinger-Dyson equations. This we do not undertake here.

IV. CONCLUDING REMARKS

We have shown how it is possible to have Nambu-Goldstone bosons as bound states in the quark model. Then the quark model, properly interpreted, can incorporate the successful features of chiral current algebra. We have suggested that the solution to the gap equation for the fermion self-energy $\Sigma(p^2)$ is the analog to the problem of minimizing the potential for the Σ model. However, in the leading-order Bethe-Salpeter approximation, the vacuum symmetry is an arbitrary input into the model in the form of a boundary condition. To go beyond this takes us into confronting the nonlinear structure of the theory. For example, if we approximate in the gap equation (4) $\Gamma_\mu = \gamma_\mu$ and $S_F'(p) = \not p - \Sigma(p^2)$, then we have the equation ($\xi = +1$)

$$\Sigma(p^2) = \frac{3ig^2}{(2\pi)^4} \int \frac{d^4 l}{(l-p)^2 - \mu^2} \frac{\Sigma(l^2)}{l^2 - \Sigma^2(l^2)} . \qquad (37)$$

It is possible that the solutions to this equation can serve to specify the quark mass $M = \Sigma(M^2)$ in terms of g^2 and μ.

We have assumed that eigenvalue conditions on the coupling constants are satisfied. If this is so then the theory is finite. It could turn out that no solution exists to the eigenvalue problem. Some of the simple features we have described will be lost in that case; however, the Goldstone alternative will not be changed.

Even in the simplified model which ignores gluon self-energy insertions, the electroproduction structure functions measured at SLAC, $W_{1,2}(q^2, \nu)$, will not exhibit the scaling behavior.[19] This seems to be a rather general property of interacting field theories in which one does not invoke a cutoff. If, however, the gluon coupling is small, then we can have approximate scaling consistent with the experiments. One of the interesting features of the present model is that the gluon coupling could be small but the coupling of bound states to the quarks large. The strong interactions could proceed predominantly via the bound states. In fact all of the observed hadrons would be bound states in this model since one does not need elementary scalar and pseudoscalar fields to implement the chiral symmetry.

One can amend a gauge model of the weak interaction to this hadron model or a suitable generalization. An intriguing question that the present work raises in the context of gauge models is whether or not one can dispense with elementary Higgs scalars. Can the Higgs scalars be bound states of the leptons? This is a more difficult question to answer than the one examined in this article. We have made use of the renormalizability of the Green's functions in the gluon model to establish the Goldstone alternative. In gauge models, in the U gauge, only the S matrix is finite, not the Green's functions. Consequently it is difficult to see how one can discuss bound states in gauge models. This is not to preclude the possibility of bound-state Higgs mesons, but only to point out that the theoretical techniques usually used to study this question may not apply.

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Goldstone bosons. This symmetry can be further broken either in the boundary conditions or by the additional vector fields.

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Regge Trajectories in the ϕ^3 Multiperipheral Model with Strong Coupling

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The trajectory function $\alpha(t)$ for small t is studied in the ϕ^3 multiperipheral model as the coupling constant becomes very large. Special emphasis is paid to the analogy of the ϕ^3 model to a one-dimensional gas system. We discuss the upper and lower bounds for $\alpha(t)$ when 2,3,... up to N-body potentials in the gas model are taken into account. The power dependence of α on the coupling constant as well as its dependence on N is determined by our bounds. This power dependence changes as N approaches infinity; it also changes as the mass ratio of the produced final particles to the exchanged particles varies. We show that the equal-spacing approximation suggested by T. D. Lee gives the correct power dependence of α on the coupling constant with a coefficient in error by only a few percent.

I. INTRODUCTION

One of the important problems in hadron physics is to understand the regularities and the general features in the production processes. Feynman[1] and Wilson[2] suggested that the distribution properties of the final particles in the high-energy scattering process should be analogous to the behavior of a real gas contained in a finite volume. This can be understood qualitatively by studying the distribution properties in a multiperipheral model. Mueller[3] demonstrated further that many of the inclusive properties suggested by the gas model can be derived from the assumption of a factorizable leading Regge pole, and thus put the gas model on a rather sound theoretical footing.

Based on the properties of ϕ^3 ladder model, one may also postulate a factorization property of the exclusive amplitude for the process[4] $A+B\to A+B+n\pi$. This exclusive factorization property enables one to analyze the exclusive data in terms of a cluster decomposition and to relate the inclusive and the exclusive spectra in an energy-independent way.

Recently, Lee[5] has demonstrated that the ϕ^3 ladder model is in fact equivalent to a particular one-dimensional gas with only repulsive forces, which can be decomposed into multibody forces in a cyclic way.

In the gas model the trajectory function α corresponds to the pressure, and the coupling constant g^2 is proportional to the fugacity.[4,5] An interesting and important question is: "What is the pressure in the high-density limit?" In other words, how does α depend on g^2 as $g^2\to\infty$? This is the question we will study in this paper from the statistical-mechanical point of view.

This question has been studied before by different approaches.[6-11] In the special case of massless final particles ($m^2=0$ as defined in Sec. II), the solution is known analytically and given by Wick, Cutkosky, and Nakanishi.[6,8] Tiktopoulos and Treiman[7] have given upper and lower bounds in this case which both approach the exact result as $g^2\to\infty$. In the more general case $m^2\neq 0$, the upper bound obtained by Tiktopoulos and Treiman[7] is not optimal since their bound is proportional to g^2 while the exact result should be proportional to $(g^2)^{1/4}$. The correct power dependence on g^2 is obtained by Rosner,[9] by Wyld,[10] and by Cheng and Wu[11] through numerically solving the Bethe-Salpeter equation.

The method presented in this paper for obtaining bounds is very elementary and emphasizes the analogy to the gas model. Special attention is paid to the change of power dependence of α on g^2 as the mass ratio of the final particles to the exchanged particles varies and as all multiparticle potentials are included. Our method also treats the nonforward case when the momentum transfer