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[7]We consider only odd-parity amplitudes. For a discussion of the absence of anomalies in even-parity amplitudes, see, e.g., W. A. Bardeen, Phys. Rev. 184, 1848 (1969); K. G. Wilson, Phys. Rev. 181, 1909 (1969).

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[9]The anomaly will be finite, since $D = 0$.

[10]"More than quadratic" suffices to prove the result, but for simplicity, we use the fact that it is quartic.

[11]Details are given in Appendix A.

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Symmetry Breaking in the Meson-Baryon System and Triangular Representations of Chiral SU(3)

A. McDonald
Department of Theoretical Physics, Oxford University, England

S. P. Rosen
Purdue University, Lafayette, Indiana 47907

(Received 5 February 1973)

We assume that the chiral-symmetry-breaking Lagrangian is an admixture of SU(3) singlet and octet components contained in a *single* representation $(m, \overline{m}) + (\overline{m}, m)$ of SU(3) × SU(3). If we restrict ourselves to the case in which (m) is any triangular representation of SU(3), we find that, except for the triplet, all such symmetry-breaking Lagrangians are inconsistent with present data on meson-baryon σ terms. Thus the only irreducible representation that may be consistent is the $(3,\overline{3}) + (\overline{3},3)$.

Under fairly general conditions the present data on π-π scattering lengths can be used to rule out certain classes of representation for the chiral-SU(3)-breaking part of the mesonic Lagrangian.[1] The conditions are that the symmetry-breaking Lagrangian consist of the SU(3) singlet and octet components of a single representation $(m, \overline{m}) + (\overline{m}, m)$ of SU(3)×SU(3), and the classes ruled out are those in which m is either a self-adjoint representation of SU(3) or a triangular one other than the triplet $(m = 3)$.

Here we shall extend this result to the symmetry-breaking meson-baryon Lagrangian by showing that the σ terms predicted by triangular representations larger than the triplet are not consistent with the range of values obtained from experiment. We work with nonlinear, effective Lagrangians, and we make use of the technical apparatus developed in earlier papers.[1,2]

Let $S(m, \overline{m})$ be a singlet in an $(m, \overline{m})$ representation of SU(3)×SU(3). Then the two octets in this representation are given by[1]

$$M_i(m, \overline{m}) = [K_i, S(m, \overline{m})]$$

and (1)

$$\tilde{M}_i(m, \overline{m}) = d_{ijk}[K_j, [K_k, S(m, \overline{m})]],$$

where K_i $(i = 1, \ldots, 8)$ is the axial charge obeying the Gell-Mann algebra of charges.[3] The assumption that the symmetry-breaking Lagrangian is dominated by the singlet and octet contributions means that we can write it as follows:

$$\mathcal{L}_B((m, \overline{m}) + (\overline{m}, m)) = \tfrac{1}{2}[\mathcal{L}_B(m, \overline{m}) + \mathcal{L}_B(\overline{m}, m)],$$
$$\mathcal{L}_B(m, \overline{m}) = \epsilon_0 S(m, \overline{m}) + (\tfrac{3}{2})^{1/2}\epsilon_8 M_8(m, \overline{m}) + \tfrac{3}{5}(\tfrac{3}{2})^{1/2}\tilde{\epsilon}_8 \tilde{M}_8(m, \overline{m}). \quad (2)$$

These coefficients have been chosen so that in the $(3, \overline{3}) + (\overline{3}, 3)$ representation,

$$\mathcal{L}_B((3, \overline{3}) + (\overline{3}, 3)) = \epsilon_0 u_0 + (\epsilon_8 + \tilde{\epsilon}_8)u_8.$$

In the case of the meson-meson system, we have constructed $\mathcal{L}_B$ for the nonlinear realizations of SU(3)×SU(3) and have shown that up to second order in the meson field we get[1]

$$\mathcal{L}_B((m,\bar{m})+(\bar{m},m))$$

$$=-\tfrac{1}{2}\epsilon_0\pi_i\pi_i+d_{8jk}\pi_j\pi_k\times\left[\frac{3m_3}{5m_2}(\tfrac{3}{2})^{1/2}\epsilon_8-\frac{3(2m_2+3)}{50}(\tfrac{3}{2})^{1/2}\tilde{\epsilon}_8\right],\tag{3}$$

where π_i ($i=1,\ldots,8$) represents the octet of pseudoscalar mesons, and m_2 and m_3 are the Casimir eigenvalues:

$$m_2=\tfrac{2}{3}(\mu_1^2+\mu_2^2+\mu_1\mu_2+3\mu_1+3\mu_2),$$

$$m_3=\tfrac{1}{9}(\mu_2-\mu_1)[(\mu_1+2\mu_2)(\mu_2+2\mu_1)+9(\mu_1+\mu_2+1)],\tag{4}$$

where μ_1 and μ_2 can take all positive integer values.

Using Eq. (3) we can immediately write the following relationship between the meson masses and ϵ_0, ϵ_8, and $\tilde{\epsilon}_8$:

$$R=-\frac{6m_3}{5m_2}\frac{\epsilon_8}{\sqrt{2}\,\epsilon_0}+\frac{3(2m_2+3)}{25}\frac{\tilde{\epsilon}_8}{\sqrt{2}\,\epsilon_0}=\frac{2(m_\pi^2-m_K^2)}{m_\pi^2+2m_K^2}.\tag{5}$$

In the case of triangular representations $\mu_2=0$ in Eq. (4), the two octets M_8 and $\tilde{M}_8$ are proportional to each other, and so we can take $\tilde{\epsilon}_8=0$; thus

$$R=-\frac{6m_3}{5m_2}\frac{\epsilon_8}{\epsilon_0\sqrt{2}}=\frac{2(m_\pi^2-m_K^2)}{m_\pi^2+2m_K^2}.\tag{6}$$

We now turn to the meson-baryon system in triangular representations. The singlet S will contain three independent contributions,[2] and so will the octet M_8. ($\tilde{M}_8$ is proportional to M_8.) Expanding these terms up to second order in the meson field, we can write the symmetry-breaking Lagrangian as follows[2]:

$$-\mathcal{L}_B((m,\bar{m})+(\bar{m},m))$$

$$=\epsilon_0\left[\bar{\Psi}\Psi\left(1-\frac{m_2}{8}\frac{X}{F_\pi^2}\right)+(\tfrac{2}{3})^{1/2}\frac{3m_3}{5m_2}\bar{\Psi}(D_id+F_if)\Psi\frac{\Pi_i}{F_\pi^2}\right]$$
$$+(\tfrac{3}{2})^{1/2}\epsilon_8\left\{\frac{3m_3}{20}\bar{\Psi}\Psi\frac{\Pi_8}{F_\pi^2}+(\tfrac{2}{3})^{1/2}\bar{\Psi}(D_id+F_if)\Psi\left[\delta_{8i}+\frac{X}{F_\pi^2}\left(\tfrac{11}{60}-\frac{m_2}{10}\right)\delta_{8i}+\tfrac{1}{4}d_{8ij}\frac{\Pi_j}{F_\pi^2}+\frac{\pi_8\pi_i}{10F_\pi^2}(-2m_2+\tfrac{1}{3})\right]\right\},\tag{7}$$

where Ψ represents the octet of baryons, F_π is the pion decay constant (~95 MeV), $X=\pi_i\pi_i$, and $\Pi_i=d_{ijk}\pi_j\pi_k$.

The contributions from the above Lagrangian to the masses of the baryons along with the contribution from the kinetic-energy term, which we call m_0, are given by

$$\epsilon_8 d=\tfrac{1}{2}\sqrt{3}\,(m_\Sigma-m_\Lambda);\quad \epsilon_8 f=(\tfrac{1}{3})^{1/2}(m_N-m_\Xi);\quad m_0+\epsilon_0=\tfrac{1}{2}(m_\Sigma+m_\Lambda).\tag{8}$$

Substitution of the known masses of the mesons and baryons into Eqs. (6) and (8) allows us to write the meson-baryon Lagrangian in terms of a single parameter, m_0, the "average mass" of the baryon octet. We can now calculate the meson-baryon σ terms as a function of m_0.

In an effective-Lagrangian approach, the meson-baryon σ terms can be defined by [i, j, a, b are SU(3) indices and p_1, p_2, q_1, q_2 their respective four-momenta]

$$\frac{\delta(p_1+q_1-p_2-q_2)\sigma_{bj;ai}}{(2\pi)^2(2\omega_1 2\omega_2 E_1E_2/m_1m_2)^{1/2}F_\pi^2}=\int d^4x\,\langle B_j(p_2)\pi_b(q_2)|\mathcal{L}_B(x)|B_i(p_1)\pi_a(q_1)\rangle.\tag{9}$$

Substituting $\mathcal{L}_B$ from Eq. (7) we get the following values for the meson-baryon σ terms for which experimental values have been quoted. Because $\mathcal{L}_B$ conserves isospin, it is convenient to write $\sigma_{bj;ai}$ in terms of the isospin I of the initial state, that is, as $\sigma_{MB}^{(I)}$:

$$\sigma_{\pi N}^{(1/2)}=\sigma_{\pi N}^{(3/2)}=\sigma_{\pi N}=\frac{m_2}{4}\epsilon_0(1+R)+(\tfrac{1}{3})^{1/2}(d-3f)\epsilon_8\left[\tfrac{1}{10}(1-m_2)-\frac{(2m_2+3)}{50R}\right],$$

$$\sigma_{KN}^{(1)}=\frac{m_2}{8}\epsilon_0(2-R)+(\tfrac{1}{3})^{1/2}d\epsilon_8\left(\frac{1}{10}-\frac{m_2}{10}+\frac{2m_2+3}{25R}\right)+(\tfrac{1}{3})^{1/2}f\epsilon_8\left(\frac{3m_2}{10}-\frac{4}{5}\right),$$

$$\sigma_{KN}^{(0)}=\frac{m_2}{8}\epsilon_0(2-R)+(\tfrac{1}{3})^{1/2}d\epsilon_8\left[\frac{3}{5}-\frac{m_2}{10}-\frac{2(2m_2+3)}{25R}\right]+(\tfrac{1}{3})^{1/2}f\epsilon_8\left[\frac{3m_2}{10}-\frac{3}{10}-\frac{3(2m_2+3)}{25R}\right], \quad (10)$$

$$\sigma_{\pi\Sigma}^{(2)}=\sigma_{\pi\Sigma}^{(1)}=\sigma_{\pi\Sigma}^{(0)}=\sigma_{\pi\Sigma}=\frac{m_2}{4}\epsilon_0(1+R)+(\tfrac{1}{3})^{1/2}d\epsilon_8\left(-\frac{1}{5}+\frac{m_2}{5}+\frac{2m_2+3}{25R}\right).$$

From Eqs. (6) and (8) and the empirical masses of mesons and baryons, we obtain

$$\begin{aligned}\sigma_{\pi N}&\approx\tfrac{1}{50}\mu(\mu+3)(395-m_0)+70\text{ MeV},\\ \sigma_{KN}^{(1)}&\approx\tfrac{6}{25}\mu(\mu+3)(1028-m_0)+100\text{ MeV},\\ \sigma_{KN}^{(0)}&\approx\tfrac{6}{25}\mu(\mu+3)(960-m_0)+18\text{ MeV},\\ \sigma_{\pi\Sigma}&\approx\tfrac{1}{50}\mu(\mu+3)(1288-m_0)-13\text{ MeV},\end{aligned} \quad (11)$$

where we have set $\mu_1=\mu$ and $\mu_2=0$ in Eq. (4).

Attempts to extract these σ terms from experimental data have led to a variety of results. Values ranging from 20 to 110 MeV have been obtained for the pion-nucleon terms,[4-10] and an even wider range of values have been claimed for the KN terms[11-14] and $\pi\Sigma$ terms.[4,5] These ranges are listed in Table I along with the predictions of Eq. (11) for a number of different cases.

In columns III, IV, and V of this table, we list the values of the σ terms for the three lowest triangular representations under the assumption that $m_0=940$ MeV. As can be seen, present analysis of the πN data already excludes the $(6,\bar{6})+(\bar{6},6)$ representation ($\mu=2$) as a possible alternative[15] to the $(3,\bar{3})+(\bar{3},3)$. Because of the dependence of the σ terms upon the parameter μ, matters get worse as we go to higher representations.[16] So with $m_0=940$ MeV, the $(3,\bar{3})+(\bar{3},3)$ type of breaking[17] is consistent with experimental data and favors the small value for the $\pi N\,\sigma$ commutator.[18,19]

In column VI we assume that the $\pi N\,\sigma$ commutator is small and that the symmetry breaking belongs to the $(6,\bar{6})+(\bar{6},6)$ representation. This means that $m_0\approx620$ MeV and that the $KN\,\sigma$ terms are outside the presently quoted experimental values. As in the previous case, matters are not improved in the higher representations.

Columns VII and VIII list what happens if we assume that $\sigma_{\pi N}$ is large. Again m_0 is far from the nucleon mass, and the $KN\,\sigma$ terms are too large. In the higher representations they become even larger.

The conclusion is that the present crude experimental information seems to rule out all triangular representations of SU(3)×SU(3) except the $(3,\bar{3})+(\bar{3},3)$ as possibilities for describing symmetry breaking in the meson-baryon system. Of course, a large number of nontriangular representations have not been considered, and these have enough free parameters to fit all the present data. Also, reducible representations will give a satisfactory description of these phenomena.

Nevertheless, these results, along with those on the meson-meson scattering lengths,[1] tempt one to speculate that the higher irreducible representations are undesirable.

Interestingly, there exists a theoretical mechanism for restricting the representations to which the symmetry-breaking Hamiltonian can belong to the low-lying ones. General arguments[20] have been given to the effect that the symmetry-breaking Hamiltonian should be invariant under the transformation

TABLE I. Experimental and theoretical values for meson-baryon σ terms.

I m_0 (MeV)	II Expt.	III 940 $(3,\bar{3})+(\bar{3},3)$	IV 940 $(6,\bar{6})+(\bar{6},6)$	V 940 $(10,\overline{10})+(\overline{10},10)$	VI 620 $(6,\bar{6})+(\bar{6},6)$	VII −100 $(3,\bar{3})+(\bar{3},3)$	VIII 200 $(6,\bar{6})+(\bar{6},6)$
$\sigma_{\pi N}$ (MeV)	20–110	26	−40	−126	25	110	110
$\sigma_{KN}^{(1)}$ (MeV)	100–650	185	310	480	1080	1180	2090
$\sigma_{KN}^{(0)}$ (MeV)	−100–250	37	65	105	835	1030	1840
$\sigma_{\pi\Sigma}$ (MeV)	−225–870	16	57	115	120	100	205

$$W = \exp(i3\pi Y_-). \quad (12)$$

This restriction, combining with the assumption of octet dominance, limits the symmetry breaking to the (1, 8), (8, 1), (3, $\bar{3}$), ($\bar{3}$, 3), (6, $\bar{6}$), ($\bar{6}$, 6), and (8, 8) representations.[21]

Unfortunately, a W-invariant irreducible Hamiltonian gives unpleasant results in the (3, $\bar{3}$) + ($\bar{3}$, 3) system ($m_\pi^2 = 0$ and $\sigma_{\pi N} = 0$).[19] However, a reducible, W-invariant Hamiltonian certainly contains sufficient free parameters to fit all the presently known data on meson-meson and meson-baryon scattering.

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PHYSICAL REVIEW D VOLUME 7, NUMBER 12 15 JUNE 1973

Goldstone Bosons as Bound States in the Quark-Gluon Model*

Heinz Pagels

Rockefeller University, New York, New York 10021

(Received 5 March 1973)

We examine the implications of a Nambu-Goldstone realization of chiral symmetry in the quark-gluon model. The context of this examination is that of a renormalizable, finite theory, so eigenvalue conditions are assumed to be satisfied. We discuss the solutions to the Schwinger-Dyson gap equation for the fermion self-energy $\Sigma(p^2)$ that exhibit spontaneous breaking of the vacuum symmetry. In the leading-order Bethe-Salpeter approximation the boundary conditions to the homogeneous, linear integral equations stipulate the vacuum symmetry. It is shown how the Goldstone bosons emerge as bound states, as suggested by Nambu and Jona-Lasinio. We also examine the Goldstone alternative in the Bethe-Salpeter equation for fermion-fermion scattering. Explicit symmetry breaking is introduced by additional Abelian vector gluons coupling to hypercharge and isospin besides baryon number. The eigenvalue condition for the fine-structure constant is consequently model-dependent but takes a simple form. We also consider the influence of explicit symmetry breaking on the ground-state mesons and indicate how the solutions to the eigenvalue problem regulate the structure of symmetry breaking.

I. INTRODUCTION

The standard model for implementing the Nambu-Goldstone realization of chiral symmetry has been the renormalized Σ model.[1,2] This model has the property that to lowest order in the coupling constants and in the tree approximation one may examine the spontaneous symmetry breaking of the ground state. In particular the mechanism of the Goldstone theorem is explicit in this model, and most of the consequences of chiral symmetry like soft-pion theorems can be examined in the tree approximation.

In this article we will examine how a Nambu-