[1]See, for example, Y. Aharonov and D. Bohm, Phys. Rev. 122, 1649 (1961); V. A. Fock, Zh. Eksp. Teor. Fiz. 42, 1135 (1962) [Sov. Phys. JETP 15, 784 (1962)]; Y. Aharonov and D. Bohm, Phys. Rev. 134, B1417 (1964).

[2]A review of the phase-number and angle–angular-momentum studies, as well as references to the energy-time controversy, are given by P. Carruthers and M. M. Nieto, Rev. Mod. Phys. 40, 411 (1968).

[3]L. Mandelstam and I. Tamm, J. Phys. U.S.S.R. 9, 249 (1945). See also A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1961), pp. 319–320.

[4]F. T. Smith, Phys. Rev. 118, 349 (1960).

[5]B. A. Lippmann, Phys. Rev. 151, 1023 (1966).

[6]The conventional understanding is essentially dichotomous. That is, the uncertainty times associated with wave-packet spreading and with excited-state decay are regarded as unrelated consequences of the uncertainty principle. This point of view is apparent in every quantum mechanics text known to the authors.

[7]Y. Kano, Ann. Phys. (N.Y.) 30, 127 (1964).

[8]J. H. Eberly and A. Kujawski, Phys. Letters 24A, 426 (1967); see also D. Dialetis and C. L. Mehta, Nuovo Cimento 56B, 89 (1968).

[9]The closest approach which we have found to the ideas embodied in our discussion is in the text by D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, N. J., 1951), pp. 227–228.

[10]V. F. Weisskopf and E. Wigner, Z. Physik 63, 54 (1930); G. Källén, *Encyclopedia of Physics*, edited by S. Flügge (Springer, Berlin, 1958), Vol. V. Although the use of the rotating-wave approximation in these well-known discussions of spontaneous emission makes them incomplete [see J. R. Ackerhalt, J. H. Eberly, and P. L. Knight, in Proceedings of the Third Rochester Conference on Coherence and Quantum Optics, edited by L. Mandel and E. Wolf (Plenum, to be published)], the predicted frequency shift but not the line width is affected, so the present discussion remains unaltered.

PHYSICAL REVIEW D VOLUME 7, NUMBER 2 15 JANUARY 1973

Baryon-Antibaryon Phase Transition at High Temperature*

Arturo Cisneros
California Institute of Technology, Pasadena, California 91109
(Received 9 March 1972)

Present experimental data on nucleon-antinucleon scattering allow a study of the possibility of a phase transition in a nucleon-antinucleon gas at high temperature. Estimates can be made of the general behavior of the elastic phase shifts without resorting to theoretical derivation. A phase transition which separates nucleons from antinucleons is found at about 280 MeV in the approximation of the second virial coefficient to the free energy of the gas. A rigorous treatment of the contribution of the inelastic channels remains, however, a difficulty.

I. INTRODUCTION

Harrison[1] has suggested that if baryon-antibaryon inhomogeneities existed in the early universe, several problems of galaxy formation could be solved. Harrison's suggestion and the earlier conjecture[2] of a charge-symmetrical boundary condition between baryons and antibaryons has led to the proposal[3] for mechanisms to separate baryons and antibaryons at high temperature. Statistical fluctuations in the baryon number density are not adequate to explain the present baryon density in the universe. Dynamical mechanisms are therefore required to separate baryons and antibaryons if a symmetrical boundary condition is assumed. Apart from the necessity of finding a separation mechanism, symmetrical models must explain various observational data such as the present ratio of the number of photons to baryons and the absence of any appreciable mixture of matter and antimatter[4] in interstellar gas.

A model has been proposed by Omnès[3] which gives baryon-antibaryon separation in the black-body radiation at a temperature of 350 MeV. The system under consideration is a gas of pions, nucleons, and antinucleons at constant volume and temperature. To obtain the equilibrium configuration, the free energy is minimized with respect to variations in the numbers of nucleons and antinucleons. The free energy is expanded in powers of the numbers of nucleons and antinucleons. It is found in minimizing the free energy that if the second virial coefficient has a large enough positive value (corresponding to an effective repulsion between nucleons and antinucleons) separation is possible.

An effective repulsion between nucleons and antinucleons arises in the Omnès model from the assumption of validity of Levinson's theorem, and considering that the corresponding bound states of the $N\bar{N}$ system ($\pi, \rho, \ldots$) are an independent component of the radiation. The approximation is made that only S waves are important with Levinson's theorem holding for scattering states with the quantum numbers of the π, η, ρ, and ω mesons. The $N\bar{N}$ phase shifts therefore fall from π to 0 as momentum goes from 0 to ∞.

To understand how a falling phase shift causes repulsion it is sufficient to look at the modification of the number of states in a range of momentum due to the interaction. The asymptotic wave function in spherical coordinates is proportional to

$$\sin(pr + \delta + \tfrac{1}{2}l\pi) .$$

We assume the particle is contained in a spherical volume of radius R. The condition that the wave function vanishes at the boundary gives

$$pR + \delta + \tfrac{1}{2}l\pi = n\pi .$$

The number of states dn in the range of momentum dp is given by

$$\frac{dn}{dp} = \frac{R}{\pi} + \frac{1}{\pi}\frac{d\delta}{dp} .$$

We therefore find that if $d\delta/dp$ is negative, the number of states in the range dp is reduced below that in the absence of interactions.

We find a falling phase shift, for example, in a system in which there is one bound state and Levinson's theorem holds. In this case the phase space which is excluded from the scattering states has gone into the formation of the bound state, as pointed out by Omnès. The presence of the bound state must ordinarily be taken into account in the calculation of the second virial coefficient for a gas of such particles. The second virial coefficient for a gas of particles interacting through an attractive potential is in fact negative, corresponding as expected to an attraction. This coefficient consists of two terms, one due to the bound state and the other depending on $d\delta/dp$. At high temperatures the two terms approach opposite values giving zero for the second virial coefficient.

In Omnès's model the bound states of the nucleon-antinucleon system are assumed to be the π, η, ρ, and ω mesons. The phase shifts are taken to be monotonically decreasing from π to 0 in a range of momentum of the order of the ω mass. Since the π, η, ρ, and ω are considered to be independent components of the radiation they are *not* included in the calculation of the second virial coefficient B. As a result a positive value of B is obtained corresponding to an effective repulsion. Separation is possible if there is a large enough number of nucleons and antinucleons interacting with momenta of a few hundred MeV. In the blackbody radiation the density of particles is a rapidly increasing function of temperature. Increasing the temperature eventually produces a density of nucleons and antinucleons large enough that it becomes more profitable (for lowering the free energy) to have different numbers of nucleons and antinucleons. For these statements to have any relevance it is necessary, of course, that the separation temperature occur within the range in which the original assumptions are valid.

It is the purpose of this paper to point out that the present experimental data on low-energy nucleon-antinucleon scattering are adequate to make good estimates of the general behavior of the phase shifts without resorting to *theoretical* derivations such as the one made by Omnès. Every known model of the nucleon-antinucleon interaction which makes an attempt to fit the data contains an absorptive potential[5] that causes some of the real phase shifts to attain negative values of the order of $-\frac{1}{2}\pi$ at 600-MeV center-of-mass momentum, whereas the phase shifts that take positive values are small. We take the simplest model of the nucleon-antinucleon interaction which consists of a purely absorptive potential. This simple model gives good fits to the low-energy total inelastic and differential cross sections. We find that the phase shifts in this model fall fast enough that separation is again possible at 280 MeV. If we had used any of the more sophisticated models of the nucleon-antinucleon interaction which include spin-dependent interactions, the answer would not be changed in any essential way. In all these models the phase shift falls fast enough to give a second virial coefficient large enough to cause separation in the blackbody radiation at around 300 MeV.

II. FORMALISM

Thermodynamic quantities are calculated for the high-temperature radiation assuming thermal equilibrium. The system considered is a gas of pions, nucleons, and antinucleons at constant volume and temperature which can exchange particles with the surroundings. The various particle densities and the configuration of this system will be such as to minimize the free energy.

The contribution to the free energy coming from the interaction among the various particles is expanded in a power series in the densities of nucleons and antinucleons N/V and $\bar{N}/V$. Only terms up to quadratic order are kept in this expansion. The term linear in N and $\bar{N}$ is due to the pion-nucleon interaction. The term of order $N\bar{N}$ is due to the nucleon-antinucleon interaction. Terms of order N^2 and $\bar{N}^2$ are due to nucleon-nucleon interactions. If the effects of Fermi statistics are taken in the first approximation, they provide additional terms in the free energy proportional to N^2 and $\bar{N}^2$.

Bouchiat[6] has analyzed the modifications to the free energy of the nucleon gas due to the presence of pions. At temperatures around 200 MeV, the approximation is made that the nucleon-pion interaction occurs only in the Δ state. The zero-width limit is taken for the Δ resonance. With these simplifying assumptions the baryon gas consists of nucleons and Δ resonances, the Δ being considered as an excited state of the nucleon. The free energy of the baryons is found to be

$$F_1 = -NT\ln\frac{eN^0}{N} - \bar{N}T\ln\frac{eN^0}{\bar{N}}, \tag{1}$$

where

$$\frac{N^0}{V} = 4\left(\frac{M_N T}{2\pi}\right)^{3/2}\exp\left(-\frac{M_N}{T}\right) + 16\left(\frac{M_\Delta T}{2\pi}\right)^{3/2}\exp\left(-\frac{M_\Delta}{T}\right). \tag{2}$$

M_N and M_Δ are the masses of the nucleon and Δ resonance, respectively. V is the volume of the gas. $\hbar$, c, and k have been placed equal to one in this expression and throughout this paper. In the absence of other interactions the equilibrium state will be that with a density of nucleons equal to N^0/V. The presence of pions permits a larger density of nucleons at equilibrium given by the second term on the right-hand side of Eq. (2).

The contribution to the free energy due to the nucleon-antinucleon interaction is given by

$$F_2 = 2TBN\bar{N}/V, \tag{3}$$

where B is determined by the Beth-Uhlenbeck formula

$$B = -8\left(\frac{\pi}{M_N T}\right)^{3/2}\sum g_n \exp\left(\frac{E_n}{T}\right) - 8\left(\frac{\pi}{M_N T}\right)^{3/2}\sum_{IJ}\frac{(2I+1)(2J+1)}{16\pi} \times \int_0^\infty \frac{d\delta^{IJ}}{dp}\exp\left(-\frac{p^2}{M_N T}\right)dp; \tag{4}$$

p is the center-of-mass momentum of the nucleon; E_n is the binding energy of the bound state of the system and g_n is its degeneracy.

The nucleon-nucleon interaction contribution to the free energy is

$$F_3 = TB'(N^2+\bar{N}^2)/V, \tag{5}$$

where B' is again given by an expression of the form (4) with the sum over J and I subject to those states allowed by the exclusion principle.

Corrections due to Fermi statistics for the nucleons and antinucleons can be taken into account in first approximation by adding an appropriate term to the virial coefficient in F_3. This correction is given by[7]

$$F_4 = TB''(N^2+\bar{N}^2)/V, \tag{6}$$

where

$$B'' = \frac{1}{8}\left(\frac{\pi}{M_N T}\right)^{3/2}. \tag{7}$$

The part of the free energy of the gas which depends on the density of nucleons and antinucleons is therefore given by

$$F = -NT\ln\frac{eN^0}{N} - \bar{N}T\ln\frac{eN^0}{\bar{N}} + 2TB\frac{N\bar{N}}{V} + \frac{T(B'+B'')(N^2+\bar{N}^2)}{V}. \tag{8}$$

The free energy F is minimized with respect to the number of nucleons and antinucleons. It is found that for a positive and greater than $\exp[(a+b)/(a-b)]+b$ the minimum of free energy is achieved for a state with unequal numbers of nucleons and antinucleons; where $a = 2BN_0/V$, $b = 2(B'+B'')N_0/V$. For negative a, which corresponds to an effective attraction between nucleons and antinucleons, the minimum of F is at $N=\bar{N}$; the same holds when $a=b$ or when $a<\exp[(a+b)/(a-b)]+b$. In the case where all nucleon-nucleon interactions are neglected or cancel out, the condition for separation becomes simply $2BN_0/V > e$.

The temperature dependence of N_0/V is dominated by the exponential factors in (2) at the temperatures under consideration. For this reason the critical temperature for separation is not sensitive to the value of B. To achieve a separation temperature of a few hundred MeV it is only necessary for B to have a value of the order of 1 F^3.

III. THE OMNÈS MODEL

The main assumptions in the Omnès model which give separation are (a) that nucleon-antinucleon interactions occur mainly in $l=0$ states and the corresponding bound states which have the quantum numbers of the π, η, ω, and ρ mesons are an independent component of the radiation, and (b) that Levinson's theorem holds for scattering in states of the corresponding quantum numbers.

Assumption (a) allows Omnès to drop the first term on the right-hand side of (4), which is the contribution of the N-$\bar{N}$ bound states to the second virial coefficient. Levinson's theorem states that the phase shifts fall from π to 0 as p goes from 0 to ∞. If the phase shifts fall to zero in a sufficiently small range of momentum, a sufficiently large positive value of B is obtained.

Omnès takes for the N-$\bar{N}$ phase shifts in all the $l=0$ states

$$\delta=\begin{cases}\pi(1-p^2/p_0^2), & p\leq p_0\\ 0, & p>p_0\,.\end{cases} \tag{9}$$

The value Omnès used for p_0 ($\sim\frac{1}{2}M_\omega$) leads to a violation of the Wigner bound ($d\delta/dp>-$ range of forces) provided we take the range of the forces to be 1.4 F. This can be corrected by taking a suitable value for p_0. The smallest possible value of p_0 consistent with the Wigner bound is $p_0=885$ MeV, corresponding to $(d\delta/dp)_{\text{minimum}}=-1.4$ F. The results of calculations using $p_0=885$ MeV give a value of the critical temperature for separation of $T_c=378$ MeV. Fermi statistics taken in the approximation of the second virial coefficient raise the critical temperature to 381 MeV; this justifies the additional approximation made by Omnès that the effect of Fermi statistics is small.

IV. ABSORPTIVE MODEL FOR THE N-$\bar{N}$ INTERACTION

Phillips[5] has discussed various models of the N-$\bar{N}$ interaction in the region of a few hundred MeV. The simplest model which gives reasonable fits to the data is the pure absorptive model. The interaction is due to a pure imaginary Woods-Saxon potential

$$W=-iW_0/(1-A\,e^{Dr})\,. \tag{10}$$

Good fits to the differential, total, and reaction cross sections are obtained with the parameters $A=1$, $D=3$ F^{-1}, and $W_0=3.3$ GeV.

We have calculated the scattering phase shifts due to the potential given in (10); the real parts are shown in Table I for S, P, and D waves. The phase shifts of all partial waves, except S waves, show the same qualitative features. Reδ is small and positive near threshold; it becomes negative at a value of momentum which is higher for higher partial waves. Reδ for S waves is always negative. The significant result, as far as the problem of separation is concerned, is that the S and P phase shifts fall through an angle of the order of $\frac{1}{2}\pi$ when p varies from 0 to 600 MeV.

TABLE I. Phase shifts due to the potential of Eq. (10).

E_{lab} (MeV)	Reδ_0 (deg)	Reδ_1 (deg)	Reδ_2 (deg)
10	−7.2	+2.1	+0.4
20	−27.7	+0.9	+0.7
60	−49.2	−6.0	+1.8
100	−62.5	−12.5	+0.06
300	−93.6	−32.0	−5.8
500	−103.8	−39.9	−9.7

Nucleon-antinucleon separation is again possible in this purely absorptive model of N-$\bar{N}$ interactions. We get a positive value for B of the order of 1 F^3 at temperatures of a few hundred MeV due to the falling phase shifts. Numerical calculations give a value of 280 MeV for the critical temperature for separation. Including the effect of Fermi statistics to the approximation of the second virial coefficient raises T_c to 283 MeV.

Although we have used the simplest theoretical model of the nucleon-nucleon interactions, it is important to note that the feature which gives negative phase shifts is present in other more sophisticated models. All models discussed by Phillips[5] which make an attempt to fit the total and differential cross sections contain an absorptive core. In particular, Bryan and Phillips[8] take the model of nucleon-nucleon interactions of Bryan and Scott[9] consisting of various one-boson exchanges, change the sign of negative-G-parity exchanges, and add an imaginary Woods-Saxon potential. They state that due to the absorptive core negative real parts are obtained for the low partial amplitudes; all spin and isospin dependence is contained in the one-boson-exchange terms. The absorptive potential gives the short-range interaction, while the long-range interactions are mainly contributed by the exchange terms. Another example is the model of Ball and Chew.[10] Ball and Chew take the nucleon-nucleon model of Signell and Marshak[11]; they adapt it to the nucleon-antinucleon system by changing the sign of the one-pion-exchange term and adding an absorptive core. They give a table for their theoretical phase shifts at 140-MeV laboratory energy. The same features of the absorption-only model are again noticed: Low partial waves have sizable negative values for the real parts of the phase shifts, high partial waves have positive but small values for their phase shifts.

V. FROISSART PHASE SHIFTS

It is necessary to justify our use of the real parts of the phase shifts in the Beth-Uhlenbeck formula (4). To do this, we reexamine the validity of the formula in the presence of inelastic channels. According to Dashen, Ma, and Bernstein,[12] the general expression for the second virial coefficient in an S wave is

$$b_2=-\frac{1}{2^{3/2}}\frac{1}{4\pi i}\int_0^\infty dE\,e^{-E/T}\left\langle N\bar{N}\left|S^{-1}\overleftrightarrow{\frac{\partial}{\partial E}}S\right|N\bar{N}\right\rangle, \tag{11}$$

where S is now a matrix and

$$\frac{\overleftrightarrow{\partial}}{\partial E}=\frac{\overrightarrow{\partial}}{\partial E}-\frac{\overleftarrow{\partial}}{\partial E}.$$

The one-channel case gives $[S=\exp(2i\delta)]$

$$S^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S=4i\frac{d\delta}{dE},$$

which leads to the Beth-Uhlenbeck formula. However, in the multichannel case

$$\langle n|S|i\rangle=\eta^{in}\exp(2i\delta^{in}),$$

$$\langle n|S|i\rangle^{\dagger}=\eta^{in}\exp(-2i\delta^{in}),$$

η and δ are real and depend on E. Putting a complete set of states $|n\rangle\langle n|$ and working out the derivatives, one finds

$$\left\langle N\bar{N}\left|S^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S\right|N\bar{N}\right\rangle=4i\sum_n|\eta^{in}|^2\frac{d\delta^{in}}{dE}.$$

One recovers the Beth-Uhlenbeck formula only by ignoring all inelastic channels and setting $\eta^{\text{elastic}}=1$. To proceed we follow an argument given by Omnès.[13] Define

$$S=\Sigma S_F,$$

where S_F is a diagonal matrix satisfying inelastic unitarity in each channel; it follows that Σ is also unitary. S_F is

$$S_F=\exp(2i\delta_F),$$

where

$$\delta_F=\delta-\delta_\alpha,$$
$$\delta_\alpha=\frac{q}{\pi}\int_0^\infty\frac{\mathrm{Im}\,\delta(\nu')\,d\nu'}{q'(\nu'-\nu-i\epsilon)},\qquad(12)$$

which is defined so that $\mathrm{Im}\,\delta_\alpha=\mathrm{Im}\,\delta$, leaving δ_F purely real; δ is the physical phase shift and δ_F are called the Froissart phase shifts.[14] Now write

$$\langle n|S_F|i\rangle=\exp(2i\delta_{Fii})\ \text{(diagonal)},$$

$$\langle n|\Sigma|i\rangle=\eta^{in}\exp(2i\alpha_{in}),$$

and we find

$$\left\langle N\bar{N}\left|S^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S\right|N\bar{N}\right\rangle=4i\frac{d\delta_F^{N\bar{N}\to N\bar{N}}}{dE}+4i\sum_n|\eta^{N\bar{N}\to n}|^2\frac{d\alpha_{N\bar{N}\to n}}{dE}.\qquad(13)$$

Omnès[13] argues that the second term on the right-hand side of the above equation vanishes in the statistical model and in the Veneziano model. It should be recognized, however, that this does not show rigorously that annihilation does not contribute to the second virial coefficient; this is only an argument that suggests that the contribution is small. If we can drop the second term in Eq. (13) we recover the Beth-Uhlenbeck formula with δ replaced by δ_F.

The results we have derived using δ (physical) in (4) are nevertheless not essentially changed by using δ_F. This is due to the behavior of $\mathrm{Im}\,\delta$ in the region of a few hundred MeV. The principal part of the integral in (12) gives $\mathrm{Re}\,\delta_\alpha$, which in the nonrelativistic limit becomes

$$\mathrm{Re}\,\delta_\alpha=\frac{E^{1/2}}{\pi}\,\mathrm{P}\int_0^\infty\frac{\mathrm{Im}\,\delta dE'}{(E')^{1/2}(E'-E)},\qquad(14)$$

where E is the laboratory energy of the nucleon. In the absorptive model $\mathrm{Im}\,\delta$ is very closely equal to $CE^{1/2}$ where C is a constant. The result of this is that all the negative contributions to the integral (14) will cancel out in taking the principal part. Since $\delta_F=\mathrm{Re}\,\delta-\mathrm{Re}\,\delta_\alpha$ and $\mathrm{Re}\,\delta_\alpha$ is nonnegative it follows that δ_F will show the same falloff with energy characteristic of $\mathrm{Re}\,\delta$; this is the essential feature of our calculations as indicated in the previous section. The model cannot be used to get a reliable numerical value for $\mathrm{Re}\,\delta_\alpha$, because it would have to be pushed to excessively large energies. The negative contributions are nevertheless clearly canceled out because $\mathrm{Im}\,\delta$ does look like $CE^{1/2}$ in the region where we want δ_F; this is where the main contributions to the integral come from.

Lower Bound on T_c Due to the Wigner Bound on $d\delta/dp$

From causality arguments it is possible to establish that $d\delta/dp>-$range of the forces. This limit on how fast the phase shifts can fall places a lower bound on the critical temperature for separation. We assume the radiation is at a temperature where only S and P waves are important in $N\cdot\bar{N}$ scattering. If we let δ fall linearly as fast as possible ($d\delta/dp=-1.4$ F) through as many multiples of π as we wish, we find that T_c cannot be lower than 247 MeV. The answer justifies the assumption that only S and P waves are important.

Effect of a Possible Resonance in the Nucleon-Nucleon System

A resonance in the N-N system could provide an attractive force among nucleons and among antinucleons. This attraction would lower the critical temperature for separation in the presence of a mechanism which separates nucleons from antinucleons. Phase-shift analyses have been performed for nucleon-nucleon scattering up to energies of a few hundred MeV. No resonances have been observed; this permits us to estimate a limit on the effect of a possible resonance near the energy limits of the phase-shift analyses. The change in the critical temperature is found not to be significant. For definiteness, if we assume a zero-width resonance in the $^1D_2$ state at 450-MeV center-of-mass momentum, the critical temperature for separation is not lowered by more than 50 MeV in either the Omnès or the absorptive model. Resonances in states of higher angular momentum would enter with a larger statistical weight, but they would be expected to occur at a higher energy, which would make their effect small.

Pion Exchange

Pion exchange affects the scattering in high partial waves. High-angular-momentum phase shifts contribute with high statistical weight to the second virial coefficient, but the individual phase shifts are small. Numerically, it is found that the pion-exchange phase shifts are small and contribute little to B in spite of their high statistical weights.

Note. Since this work was completed the author has learned of a new derivation of the phase transition temperature using theoretical phase shifts by Aldrovandi and Caser.[15]

ACKNOWLEDGMENTS

The author is indebted to Professor S. C. Frautschi and G. Steigman for valuable discussions and suggestions.

---

*Work supported in part by the U. S. Atomic Energy Commission. Prepared under Contract No. AT(11-1)-68 for the San Francisco Operations Office, U. S. Atomic Energy Commission. The author has received scholarship support from the Latin American Scholarship Program of American Universities during the preparation of this work.

[1]E. R. Harrison, Phys. Rev. Letters 18, 1011 (1967); Phys. Rev. 167, 1170 (1968).

[2]M. Goldhaber, Science 124, 218 (1958).

[3]R. Omnès, Phys. Rev. Letters 23, 38 (1969).

[4]G. Steigman, Nature 224, 477 (1969).

[5]R. J. N. Phillips, Rev. Mod. Phys. 39, 681 (1967).

[6]C. Bouchiat, Lett. Nuovo Cimento 2, 243 (1969).

[7]L. Landau and E. Lifshitz, *Statistical Physics* (Pergamon, New York, 1969).

[8]R. A. Bryan and R. J. N. Phillips, Nucl. Phys. B5, 201 (1968).

[9]R. A. Bryan and B. L. Scott, Phys. Rev. 135, B434 (1964).

[10]J. S. Ball and G. F. Chew, Phys. Rev. 109, 1385 (1958).

[11]P. S. Signell and R. E. Marshak, Phys. Rev. 106, 832 (1957).

[12]R. Dashen, S. Ma, and H. J. Bernstein, Phys. Rev. 187, 345 (1969).

[13]R. Omnès, Phys. Reports 3C, 1 (1972).

[14]M. Froissart, Nuovo Cimento 22, 19 (1961).

[15]R. Aldrovandi and S. Caser, Nucl. Phys. B38, 593 (1972).