PHYSICAL REVIEW D

PARTICLES AND FIELDS

THIRD SERIES, VOL. 7, NO. 12 — 15 June 1973

Quantized Longitudinal and Transverse Shifts Associated with Total Internal Reflection

O. Costa de Beauregard

Institut Henri Poincaré, Université de Paris VI, Paris, France

C. Imbert

Institut d'Optique, Université de Paris Sud, 91-Orsay, France

(Received 21 August 1972)

In additional measurements following their experimental demonstration of the now well-known longitudinal shift of a light beam undergoing total reflection, Goos and Hänchen have observed that an incident beam of natural light is separated into two beams with polarizations normal and parallel to the incidence plane. Although an explanation of this filtering effect of orthogonal modes is easy in terms of either the Artmann or the Renard formulas of the Goos-Hänchen shift, which are deduced from a stationary phase and an energy-conservation argument, respectively, it seems that this explanation has not yet been produced. We describe here associated theoretical and experimental work showing that both the longitudinal Goos-Hänchen shift and the new transverse shift are quantized, each having two eigenvalues and two eigenfunctions, which are the principal linear polarization states in the first case and the circularly polarized states in the second case. Thus the longitudinal and the transverse shifts should not be simultaneously observable. This can be justified in terms of the so-called tangential and sagittal focal lines produced by total reflection from a point source. Our theoretical reasoning is based on formal properties of Poynting's energy-flux vector. Our experiments consist of (1) a confirmation, with photographic recording, of the Goos-Hänchen "polarization effect," (2) a similar demonstration of the filtering of the two circular polarization modes by observation of the transverse shift. In the latter case our apparatus is similar to the one we have used for demonstrating the transverse shift, but with the circular polarization analyzer placed after rather than before the totally reflecting prism. Our new measurements also comprise an improved evaluation of the transverse shift.

I. INTRODUCTION

Let us recall that the now well-known Goos-Hänchen[1] longitudinal shift in total reflection had been qualitatively predicted.[2] However, the first closed and rigorously deduced formula, based on a stationary phase argument, was given later, by Artmann.[3] Still later a slightly different formula, based on an energy-flux conservation argument and presenting (as we will explain) excellent theoretical properties, was produced by Renard.[4]

Both the Artmann and the Renard formulas are polarization-dependent, and predict that the shift is minimum or maximum for linear polarization perpendicular or parallel to the incidence plane, respectively. It is quite clear that a little more thinking using either of these formulas[5] would have shown that the above statement refers to the *mean value* of the longitudinal displacement, which is in fact quantized, with its minimum and maximum values as eigenvalues, and the transverse electric and magnetic modes as corresponding eigenfunctions. Such a reasoning is part of the present paper.

The so-called "polarization effect" was indeed observed by Goos and Hänchen[6] and later photographed by one of us and co-workers.[7] These experiments show that observation of the longitudinal

Goos-Hänchen shift acts as a filter of polarization states, very much like ordinary double refraction does: It separates from an incident beam of arbitrary (coherent or incoherent) polarization two reflected beams that are linearly polarized, one normal and one parallel to the incidence plane. In quantum-mechanical terminology, observation of the longitudinal Goos-Hänchen shift throws the incoming photons into either of the two principal linear-polarization states that form a complete orthogonal set for describing the general polarization state. In other words, the longitudinal shift is quantized with two different eigenvalues, the corresponding eigenfunctions being the transverse electric and the transverse magnetic modes.

The new transverse shift in total reflection, calculated[8] and measured[9] by one of us, had also been qualitatively predicted.[10] Apart from the fine structure that will be discussed in this paper, the transverse shift is also found, both theoretically and experimentally,[11] to be polarization-dependent, with a sign corresponding to that of the elliptic polarization inside the evanescent wave. It reaches its maximum (given the incidence angle) when the evanescent wave is circularly polarized, and is zero for linear polarization either perpendicular or parallel to the incidence plane.

Now we come to the fine structure of the transverse shift. After our experimental confirmation[7] of the observation by Goos and Hänchen[6] of the eigenvalues and eigenfunctions of the longitudinal shift, it was remarked[12] that our formula for the transverse shift can also be brought into the canonical form of a quantum-mechanical mean value, displaying two symmetric eigenvalues, with the two circular polarization modes inside the evanescent wave as the corresponding eigenfunctions. Then an experimental proof that the transverse shift is quantized, with the two circular polarization modes as eigenfunctions, was obtained.[13] To this end our transverse shift apparatus[11] was modified, with the circular polarization analyzer placed *after* (rather than before) the totally reflecting prism.

Now, if both the longitudinal and the transverse shift turn out to be quantized, it is obvious that they cannot be simultaneously observable, as the corresponding eigenfunctions belong to two different representations of the general polarization state. Therefore we must discuss the fundamentals for measuring shifts of the beam.

Let us recall that, in geometrical optics,[14] the light rays form a *normal congruence* orthogonal to the wave fronts, which in homogeneous media *is a rectilinear normal congruence*. When diverging from, or converging to, a point, such a congruence is called *homocentric*, the wave fronts then being spheres. In the general case where the wave fronts are not spheres, their common evolute is a two-sheet surface, the *caustic surface*. The reason for this is that two adjacent light rays will intersect to first order if, and only if, they emanate from one of the two *principal curvature lines* of a wave front, which are orthogonal to each other. Therefore each ray contains *two foci*, which are the *principal centers of curvature* of the wave fronts, and the generating points of the two-sheet caustic surface. Thus a *thin pencil* of rays (Fig. 1) contains *two orthogonal focal lines* (which merge together and into a point when the wave fronts are spheres). All this remains approximately true in the domain of wave optics, with, of course, diffraction patterns replacing the two focal lines.

In order to observe and measure a shift of a light beam, we must obviously have it "marked" or "focalized." If we can use a homocentric beam, then we can measure its shift in any direction. But this is definitely *not* our case: By hypothesis we are working near the limiting case of total reflection, that is, with a pencil of rays making a wide angle with the axis of revolution of our optical system (the plane interface). Therefore our beam is necessarily marked by *two* orthogonal focal lines; this means that *two, and only two* (orthogonal) shifts of our beam will be observable and measurable. If the beam emanates (Fig. 2) from a point source S, the focal lines of the reflected beam are known as the *sagittal focal line* F lying along the axis of revolution, that is, in our case, the normal to the reflecting plane drawn from S (first sheet of the caustic surface); and the *tangential focal line* T, which is orthogonal to the incidence plane, and generates the second sheet of the caustic surface. Therefore *there are two* and *only two* shifts of the beam that we can measure with some precision: the longitudinal, or Goos-Hänchen one, which we can measure by bringing the tangential focal line into focus, and the transverse shift, which we can measure by bringing the sagittal focal line into focus. All other positions of a (real or virtual) receiving screen will only yield a blurred section of the beam, allowing no position measurement.

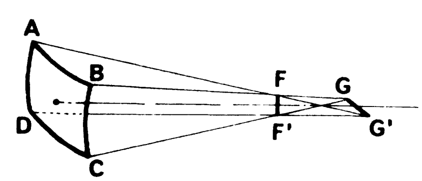

FIG. 1. Nonspherical wave front, curvilinear square made of principal curvature lines, and thin pencil of rays with the two orthogonal focal lines.

Of course, the two principal linear polarization modes and the two circular polarization modes are *not* the only orthogonal modes allowing a representation of the general polarization state; any two elliptic polarization modes termed "opposite" by Fresnel and Arago[15] are indeed orthogonal to each other and provide a possible representation of the general polarization state. However, as explained above, they are not the eigenfunctions of a physical magnitude that would be an "oblique shift" of the beam.

Keeping these remarks in mind, we recall in Sec. II that Renard's formula for the Goos-Hänchen shift and our formula for the transverse shift have the form of a diagonalized canonical mean value, with eigenvalues and eigenfunctions displayed, these being the principal linear polarization modes in the first case and the circular polarization modes in the second one. However, due to the simplifying assumptions used in the derivation of these two formulas, a general demonstration that the two shifts are quantized, with their two eigenfunctions as previously described, is still lacking.

This demonstration is given in Sec. III. Postulating that the significant magnitude for calculating the image of a point source is Poynting's energy-flux vector, we give closed formulas for the solutions of Maxwell's equations in the vacuum of the evanescent wave in the two cases previously selected as significant: t-harmonic and z-independent solution with (possibly) a focal line at $x=\text{const}$ and $y=\text{const}$; t-harmonic and y-dependent solution through $\exp[(n^2\alpha^2-1)^{1/2}\omega y]$ with (possibly) a focal line at $x=\text{const}$ and $z=\text{const}$.

We show that in the first case the S_x and S_y components of the Poynting vector $\vec{S}$ are diagonal in the transverse electric and transverse magnetic modes, and that in the second case the S_x and S_z components of $\vec{S}$ are diagonal in the two circular polarization modes (of the evanescent wave).

Sections IV and V contain descriptions of our experimental arrangements for displaying the quantization of the longitudinal and the transverse shifts. Section VI presents an improved measurement of the transverse shift, as allowed by an improved optical quality.

II. QUANTUM-MECHANICAL INTERPRETATION OF THE LONGITUDINAL AND THE TRANSVERSE SHIFT FORMULAS

Renard's[4,8] formula, based on an energy-flux conservation argument, for the longitudinal Goos-Hänchen shift X is

$$\langle X\rangle = C_1(\tau_\perp^*\tau_\perp E_\perp^*E_\perp + \tau_\parallel^*\tau_\parallel E_\parallel^*E_\parallel), \tag{1}$$

with

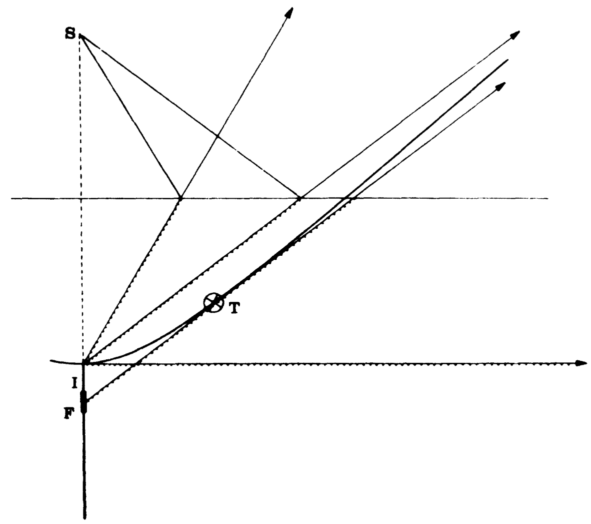

FIG. 2. The caustic surface of a point source S in total reflection: T (tangential) and F (sagittal) focal lines.

$$C_1 = c\alpha/\{2\omega[(n^2\alpha^2-1)(1-\alpha^2)]^{1/2}\} \tag{2}$$

and the normalization condition

$$E_\perp^*E_\perp + E_\parallel^*E_\parallel = 1\,; \tag{3}$$

c denotes the velocity of light *in vacuo*, n the index of the refracting medium above the plane interface $y=0$ separating it from the vacuum where the evanescent wave propagates, $\omega=2\pi\nu$ the angular frequency of the plane incident wave, $\alpha>1/n$ the sine of the incidence angle, and $\tau_\perp$ and $\tau_\parallel$ Fresnel's transmission coefficients.

Formula (1) together with (2) and (3) displays $\langle X\rangle$ as a continuous function of the ratio $E_\parallel^*E_\parallel/E_\perp^*E_\perp$. However, based as it is on classical, macroscopic reasoning, what it basically expresses is the *mean value* $\langle X\rangle$ of a possibly quantized displacement X. If so, the two eigenfunctions are clearly displayed in (1) as the transverse electric and the transverse magnetic modes of the polarization state function, the corresponding eigenvalues being

$$X_E = C_1\tau_\perp^*\tau_\perp\,,\qquad X_H = C_1\tau_\parallel^*\tau_\parallel\,. \tag{4}$$

Our[8,11] formula for the transverse shift Z, also based on energy flux conservation, is

$$\langle Z\rangle = iC_2(\tau_\perp^*\tau_\parallel E_\perp^*E_\parallel - \tau_\parallel^*\tau_\perp E_\parallel^*E_\perp), \tag{5}$$

with

$$C_2 = cn\alpha/[2\omega(1-\alpha^2)^{1/2}]\,. \tag{6}$$

Denoting by $e_\perp$ and $e_\parallel$ the phase factors in $\tau_\perp$ and $\tau_\parallel$, the substitution

$$\sqrt{2}\, L = e_\perp E_\perp + i e_\parallel E_\parallel ,$$
$$\sqrt{2}\, R = e_\perp E_\perp - i e_\parallel E_\parallel , \tag{7}$$

or conversely

$$\sqrt{2}\, e_\perp E_\perp = L + R ,$$
$$i\sqrt{2}\, e_\parallel E_\parallel = L - R , \tag{8}$$

confers on (5) the form[16]

$$\langle Z \rangle = C_2 \,|\, \tau_\perp^* \tau_\parallel \,|\, (R^*R - L^*L) \tag{9}$$

and *formally* conserves the scalar product

$$E_{1\perp}^* E_{2\perp} + E_{1\parallel}^* E_{2\parallel} = L_1^* L_2 + R_1^* R_2 \tag{10}$$

and the norm

$$E_\perp^* E_\perp + E_\parallel^* E_\parallel = L^*L + R^*R = 1 . \tag{11}$$

However, there is an orthogonality condition for the L and R modes:

$$R^*L = 0 \Longleftrightarrow e_\perp E_\perp = \pm i e_\parallel E_\parallel . \tag{12}$$

Then (as will be more obvious later) L and R are the left and right circularly polarized modes (positive and negative helicity states) *inside the evanescent wave*. Conversely, starting from (8) and (12), the orthogonality condition for the $E_\perp$ and $E_\parallel$ modes is

$$E_\perp^* E_\parallel = 0 \Longleftrightarrow R^*R = L^*L . \tag{13}$$

Formula (9), together with (6) and (11), displays the transverse shift $\langle Z \rangle$ in the form of the mean value of a quantized displacement Z, with L and R as the two eigenfunctions, and

$$Z_L = -C_2 \,|\, \tau_\perp^* \tau_\parallel \,| ,$$
$$Z_R = +C_2 \,|\, \tau_\perp^* \tau_\parallel \,| \tag{14}$$

as the two eigenvalues.

It should be kept in mind, however, that although the Renard formula (1), and our formula (5) or (9), turn out to be experimentally quite good, and although each of them has the theoretically attractive features we have just explained, neither of them has been derived in a truly rigorous fashion. This is true also with Artmann's[3] formula deduced from a stationary-phase argument. Up to now there are, strictly speaking, no completely rigorous calculations of the X or the Z shift. It is even doubtful that one can be given without the use of a computer; that is, that any *rigorously deduced* closed formula can exist for a realistic situation.

This being said, it is of course a routine matter to write down the unitary transformation between the

$$|\,\mathfrak{L}\,\rangle \equiv \begin{pmatrix} E_\perp \\ E_\parallel \end{pmatrix}$$

and the

$$|\,\mathfrak{C}\,\rangle \equiv \begin{pmatrix} L \\ R \end{pmatrix}$$

pictures. And, since we know the matrix representations of X and Z in their diagonalized forms, it is a straightforward matter to find their commutator and to write down the corresponding uncertainty relation. However, our motivation for beginning along these lines will become stronger when we have a more definite idea of what to look for afterwards.

So, for the present, we will satisfy ourselves by showing which are the mechanical magnitudes that are quantized by formulas (4) and (14) [together with (2) and (6)]. Multiplying (4) and (14) by $m \equiv c^{-2}\omega\hbar$, where m denotes the photon's kinematical mass, we see that the longitudinal and transverse quantities that are quantized as multiples of $c^{-1}h$ are the "boosts" mX and mZ associated with the photon's tunneling inside the evanescent wave. That the eigenvalues of these quantities are expressed in terms of not only h and c but also n and α (as $\tau_\perp$ and $\tau_\parallel$ are expressible in terms of n and α) is somewhat reminiscent of the fact that the hydrogen-atom energy eigenvalues are expressed in terms of h, c, e, and m.

III. DEDUCTION OF THE QUANTIZATION OF THE LONGITUDINAL AND TRANSVERSE SHIFTS

We take as our basic assumption that the physically significant quantity for calculating the illumination of a plane where either the tangential or the sagittal focal line of Fig. 1 is brought into focus is the Poynting vector $\vec{S}$. For studying the tangential focal line T and its images one should obviously work with Fourier expansions on the k_x and k_y components of the propagation vector $\vec{k}$, while for studying the sagittal focal line F and its images one should work with Fourier expansions on the k_x and k_z components of $\vec{k}$. Or, more efficiently in an abstract deduction, one should work in the first case with the general z-independent evanescent wave *in vacuo* satisfying the Helmholtz equation

$$(\partial_x{}^2 + \partial_y{}^2 + \omega^2)\,|\,\mathfrak{L}(x, y)\rangle = 0 , \tag{15}$$

and in the second case with the general evanescent wave having a given damping factor $\exp[(n^2\alpha^2 - 1)^{1/2}\omega y]$, that is, obeying the Helmholtz equation

$$(\partial_x{}^2 + \partial_z{}^2 + n^2\alpha^2\omega^2)\,|\,\mathfrak{C}\,(x, z)\rangle = 0 ; \tag{16}$$

for simplicity we now use units such that $c = 1$; as before, n denotes the index of the refracting medium, α the sine of the incidence angle, and $|\,\mathfrak{L}\rangle$ the linear and $|\,\mathfrak{C}\rangle$ the circular polarization mode representation of the evanescent wave.

In the first case the general solution $|\mathfrak{L}\rangle$ of Maxwell's equations *in vacuo* is a superposition of the transverse electric (TE) mode

$$E_z(x, y) \equiv E(x, y), \quad H_x = i\omega^{-1}\partial_y E, \quad H_y = -i\omega^{-1}\partial_x E \tag{17}$$

and of the transverse magnetic (TH) mode

$$H_z(x, y) \equiv H(x, y), \quad E_x = -i\omega^{-1}\partial_y H, \quad E_y = i\omega^{-1}\partial_x H. \tag{18}$$

Inserting these expressions into the Poynting vector formula

$$\vec{S} = \tfrac{1}{4}(\vec{E}^* \times \vec{H} + \vec{E} \times \vec{H}^*) \tag{19}$$

yields

$$S_x = (i/4\omega)(E^*[\partial_x]E + H^*[\partial_x]H), \quad S_y = (i/4\omega)(E^*[\partial_y]E + H^*[\partial_y]H), \tag{20}$$

with $i[\partial]$ denoting the well-known Schrödinger or Gordon current operator.

In the second case the general solution $|\mathfrak{C}\rangle$ of Maxwell's equations *in vacuo* is a superposition of the left circular mode

$$L_y(x, z) \equiv L, \quad L_x = p(q\partial_x + \partial_z)L, \quad L_z = p(q\partial_z - \partial_x)L \tag{21}$$

and the right circular mode

$$R_y(x, y) \equiv R, \quad R_x = p(q\partial_x - \partial_z)R, \quad R_z = p(q\partial_z + \partial_x)R, \tag{22}$$

where

$$p \equiv (n^2\alpha^2\omega)^{-1}, \quad q \equiv (n^2\alpha^2 - 1)^{1/2}; \tag{23}$$

L and R are defined as[16]

$$\sqrt{2}\,\vec{L} = \vec{E} + i\vec{H}, \quad \sqrt{2}\,\vec{R} = \vec{E} - i\vec{H}, \tag{24}$$

or, reciprocally, through

$$\sqrt{2}\,\vec{E} = \vec{L} + \vec{R}, \quad i\sqrt{2}\,\vec{H} = \vec{L} - \vec{R}. \tag{25}$$

Inserting these expressions into the Poynting vector formula

$$\vec{S} = \tfrac{1}{4}i(\vec{R}^* \times \vec{R} - \vec{L}^* \times \vec{L}) \tag{26}$$

yields .

$$\begin{aligned} S_x &= \tfrac{1}{4}ip\{L^*[\partial_x]L + R^*[\partial_x]R - q(L^*[\partial_z]L - R^*[\partial_z]R)\}, \\ S_z &= \tfrac{1}{4}ip\{L^*[\partial_z]L + R^*[\partial_z]R + q(L^*[\partial_x]L - R^*[\partial_x]R)\}. \end{aligned} \tag{27}$$

Thus the vector $(S_x, 0, S_z)$ appears as the sum of a longitudinal and a transverse Schrödinger-like current, the latter (with a minus sign inserted) explaining the transverse shift effect.

The essential point, from which our conclusion follows, is that S_x and S_y in (20) are diagonal in the two eigensolutions (17) and (18) of (15), while S_x and S_z in (27) are diagonal in the two eigensolutions (21) and (22) of (16). We thus conclude that the tangential and the sagittal focal lines produced by total reflection from a source with arbitrary polarization both consist of a doublet, the corresponding eigenfunctions being the principal linear polarization states for the former and the circular polarization states for the latter.

IV. OTHER CLOSED FORMULAS FOR THE TWO PRECEDING CASES

The transverse S_z component in the preceding "first case" and the vertical S_y component in the "second case" have not yet been written down. Compact closed formulas exist for them which might be useful in computer calculations of the longitudinal and transverse shifts in more realistic cases than those highly symbolic cases which have been treated up to now.[3,4,8] Thus we will write down these formulas.

The S_z component of the Poynting vector (19) calculated from (17) and (18) reads

$$S_z = (1/4\omega^2)(\partial_x E^* \partial_y H - \partial_x H^* \partial_y E) + \text{c.c.} \tag{28}$$

It is in general nonzero, though the k_z component of the photon's momentum is identically zero in the $|\mathfrak{L}\rangle$ solutions of (15), and it depends critically on the relative phase between E and H. From (20) one deduces

$$\partial_x S_y - \partial_y S_x = (i/2\omega)(\partial_x E^* \partial_y E + \partial_x H^* \partial_y H) + \text{c.c.} \tag{29}$$

Thus, if the boundary conditions allow the condition

$$H(x, y) = a\,E(x, y), \tag{30}$$

with a denoting a real constant (and this is the case with Fresnel's evanescent wave), then

$$S_z = \left(\frac{i}{2\omega}\right)\frac{H^*E - E^*H}{E^*E + H^*H}\,(\partial_x S_y - \partial_y S_x) \tag{31}$$

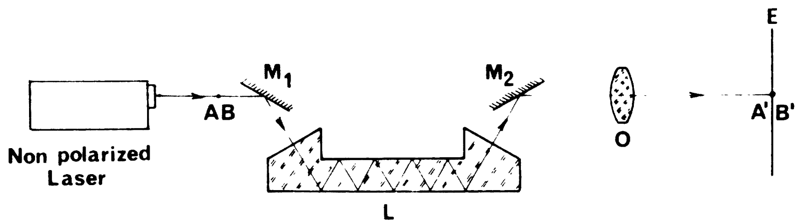

FIG. 3. Our version of the Goos-Hänchen-type device for amplifying the longitudinal shift in total reflection.

or, in integral form,

$$\iint S_z dx dy = \left(\frac{i}{2\omega}\right) \frac{H^*E - E^*H}{E^*E + H^*H} \oint (S_x dx + S_y dy); \tag{32}$$

$a = \pm 1$ correspond to pure positive and negative helicity states.

The latter expression of the transverse energy flux may facilitate a general calculation of the transverse shift.

Similarly, the S_y component of the Poynting vector (26) calculated from (21) and (22) reads

$$S_y = (i/4n^2\alpha^2\omega^2)(\partial_x L^* \partial_z L - \partial_x R^* \partial_z R) + \text{c.c.} \tag{33}$$

From (27) one deduces

$$\partial_z S_x - \partial_x S_z = -(i/2n^2\alpha^2\omega)(\partial_x L^* \partial_z L + \partial_x R^* \partial_z R) + \text{c.c.} \tag{34}$$

Thus, imposing the condition

$$R = aL, \tag{35}$$

with a denoting a complex constant, we see that

$$S_y = \left(\frac{1}{2\omega}\right) \frac{L^*L - R^*R}{L^*L + R^*R} (\partial_z S_x - \partial_x S_z), \tag{36}$$

or, in integral form,

$$\iint S_y dx dz = -\left(\frac{1}{2\omega}\right) \frac{L^*L - R^*R}{L^*L + R^*R} \oint (S_x dx + S_z dz). \tag{37}$$

The latter expression of the energy flux through the reflecting plane may help in calculating both the longitudinal and the transverse shifts in realistic cases.

V. EXPERIMENTAL PROOF OF QUANTIZATION OF THE LONGITUDINAL SHIFT

Goos and Hänchen[6] have explained how they have observed that the longitudinal shift they previously discovered[1] separates an incident beam of arbitrary polarization into two beams that are linearly polarized, one perpendicular and the other parallel to the incidence plane. This separation is only perceptible near the critical value of the incidence angle, and the transverse magnetic mode is then the more shifted one. However, Goos and Hänchen did not publish a photographic record of their observation.

Mazet, Imbert, and Huard[7] have demonstrated anew this filtering effect by using (as Goos and Hänchen had) a parallel-face plate for producing total reflection very near the critical angle and adding the longitudinal shifts (Fig. 3). Our parallel-face plate had an index $n = 1.8$, and we have used 31 reflections. The light source was an unpolarized laser, and the beam was marked by a rectilinear object (a thread) AB normal to the plane of incidence. A lens O projected the image $A'B'$ of AB on a plane screen. Figure 4 is a photographic reproduction of this image, which is seen to be a doublet. By interposing a linear polarization analyzer between the lens and the screen we have observed that the components of the doublet are polarized, the more displaced one parallel and the less displaced one perpendicular to the incidence plane.

We have also verified that the same final configuration is obtained when the incident light is either linearly polarized at 45° or circularly polarized.

Thus, in full accord with Goos and Hänchen,[6] we have observed the filtering effect of the longitudinal shift, that is, its quantization, with its two

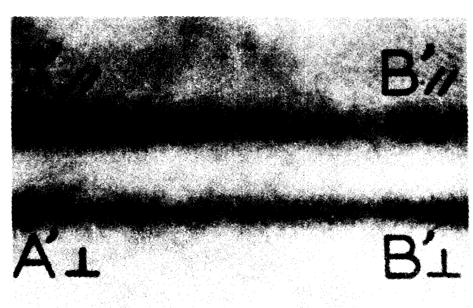

FIG. 4. Photographic record of the twin tangential focal lines in total reflection, displaying the two eigenvalues and the two eigenfunctions (linear polarization states) of the longitudinal shift.

eigenvalues and its two eigenfunctions.

We have not attempted to duplicate the very precise measurements by Goos and Hänchen[6] of the two shifts as functions of the incidence angle.

VI. EXPERIMENTAL PROOF OF QUANTIZATION OF THE TRANSVERSE SHIFT

The difficulty with the transverse shift is that it is so much smaller than the longitudinal one. Therefore we could not hope to obtain, as a result of adding many transverse shifts, two nonoverlapping images of a rectilinear object with opposite circular polarizations. For this reason we have used the same trick as in our previous[9,11] measurements of the transverse shift.

Let us suppose for a moment (Fig. 5) that we obtain as a final image of a rectilinear object two nonoverlapping straight lines, with opposite circular polarizations [Fig. 5(a)]. Then, by interposing a half-wave plate covering half of the beam, we can exchange the circular polarizations on half of the two images [Fig. 5(b)]. And then by interposing afterwards a circular polarization analyzer suppressing, say, the left circular polarization, we will obtain a broken image of the rectilinear object [Fig. 5(c)]. Of course we can invert the broken figure by suppressing the other circular polarization state, and this is conveniently done by rotating the quarter-wave plate of the circular polarization analyzer.

Figure 6 displays the whole experimental arrangement. It is very much the same as in our previous measurements of the transverse shift,[11] except that now the half-wave plate covering half of the beam and the circular polarization analyzer

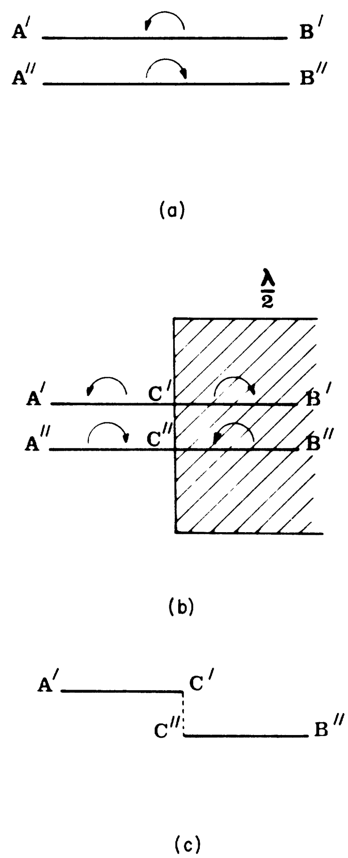

FIG. 5. The "trick" for unraveling the two overlapping circularly polarized eigenfunctions (a) of the transverse shift: (b) a half-wave plate covering half of the beam reverses the circular polarizations; then (c) a circular polarization analyzer suppresses one of the two states.

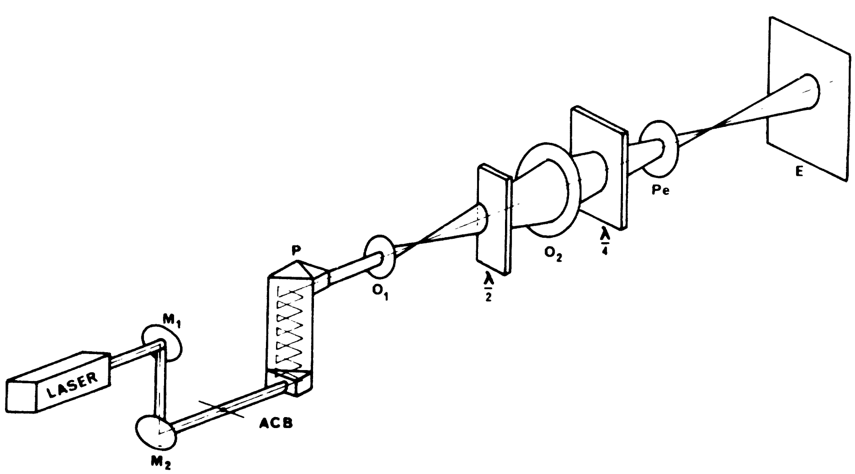

FIG. 6. The over-all experimental arrangement: unpolarized laser, rectilinear object, multiplying prism where the beam undergoes 28 near-to-limit total reflections, half-wave plate covering half of the beam, circular polarization analyzer (quarter-wave plate and linear polarizer), amplifying lenses, and screen receiving the image of the rectilinear object.

are placed after rather than before the prism P where the beam undergoes 28 near-to-limit total reflections. The beam is marked by a rectilinear object ACB parallel to the cross-section planes of the prism. The light source is an unpolarized laser (with of course the possibility of interposing before the object any sort of polarizer).

Figure 7 is a photographic record of the final image of the object ACB on the screen E with either left [Fig. 7(a)] or right [Fig. 7(b)] circular polarization suppressed.

The very same aspect is obtained when the incident light is linearly polarized either perpendicular or parallel to the first incidence plane inside the prism. It should be remembered that due to the helical path of the light inside our prism of index 1.8, the incidence plane is rotated at each successive reflection. It thus turns out that *any* linear polarization state is separated, by total reflection, into two opposite circular polarization states which (as shown by Fresnel's formulas) are conserved by near-limit total reflection, and are transversely shifted, one up and the other down, at each consecutive reflection.[17]

We have verified that the same final configurations are obtained when the laser beam is linearly polarized in a direction that is oblique on the first incidence plane; however, in this case, due to interaction between the initial polarizer and the final analyzer, the two halves of the beam have unequal luminosities.

Finally we conclude that quantization of the transverse shift, with its two eigenvalues and its two eigenfunctions as theoretically predicted,[12] has been observed.

VII. NEW MEASUREMENT OF THE TRANSVERSE SHIFT

The improved quality of our images has allowed us a more precise measurement of the new transverse shift.

We have recorded on a photographic plate (1) the broken image of the rectilinear object CAB (Fig. 7), and (2) the image of a micrometer placed in the same plane.

Taking into account the angle θ between the incidence planes and the cross section plane of the prism,[11] which is such that $\cos\theta = 0.865 \pm 0.005$, we obtain for the calculated value of the transverse shift in one reflection

$$\Delta Z_{\text{calc}} = 0.244 \pm 0.004\ \mu .$$

Experimentally we have found

$$\Delta Z_{\text{exp}} = 0.24 \pm 0.02\ \mu .$$

This confirms our previous verification of our calculations.[11]

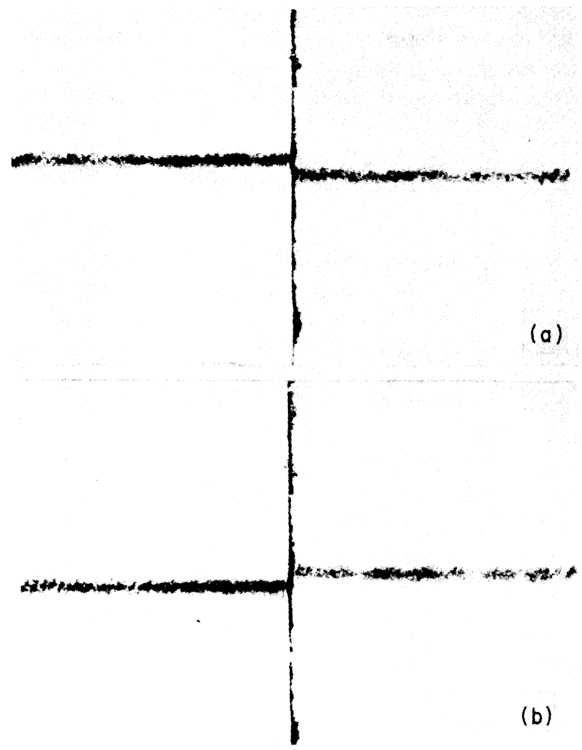

FIG. 7. The two symmetric broken images of the rectilinear object displaying the two eigenvalues and the two eigenfunctions (circular polarization states) of the transverse shift. (a) left circular polarization suppressed; (b) right circular polarization suppressed.

VIII. CONCLUSIONS

There is a complete consistency between our calculations and measurements as reported in the present paper.

Both the longitudinal and the transverse shifts associated with total internal reflection are quantized. We have calculated and measured the two eigenvalues and the two eigenfunctions of each of them (the latter being also eigenvalues of the polarization state).

Finally we come back to the impossibility of simultaneously measuring the longitudinal and the transverse shifts. It could be objected that, by using a cylindrical lens, one could bring simultaneously into focus the tangential and the sagittal focal lines.

The point is that it is impossible to amplify (for observing and measuring them) *both* the longitudinal and the transverse shifts. An apparatus amplifying the longitudinal shift will *filter* and *conserve* the principal linear polarization states, while an apparatus amplifying the transverse shift will *filter* and *conserve* the circular polarization states.

There is thus no polarization state that can go through a longitudinal and then a transverse shift amplifier (or the contrary). In other words, as these two shifts have no common eigenfunctions, their eigenvalues cannot be simultaneously displayed.

ACKNOWLEDGMENTS

We express our thanks to Professor A. Kastler and Professor A. Maréchal for their continued interest and support in this research, and to Dr. J. Ricard for his always constructive remarks or criticism.

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[17]*Note added in proof.* Of course, when using our equilateral multiplying prism where the light follows a helical polygonal path, there is a component of the Goos-Hänchen shift parallel to the edges. As previously explained (Ref. 11) and due to the rotation of the incidence plane at each consecutive reflexion, this shift is common to the two circular polarization modes and is not recorded by our apparatus.

PHYSICAL REVIEW D VOLUME 7, NUMBER 12 15 JUNE 1973

Foundations for a Theory of Gravitation Theories*

Kip S. Thorne, David L. Lee,[†] and Alan P. Lightman[‡]
California Institute of Technology, Pasadena, California 91109
(Received 10 January 1973)

A foundation is laid for future analyses of gravitation theories. This foundation is applicable to any theory formulated in terms of geometric objects defined on a 4-dimensional spacetime manifold. The foundation consists of (i) a glossary of fundamental concepts, (ii) a theorem that delineates the overlap between Lagrangian-based theories and metric theories; (iii) a conjecture (due to Schiff) that the weak equivalence principle implies the Einstein equivalence principle; and (iv) a plausibility argument supporting this conjecture for the special case of relativistic, Lagrangian-based theories.

I. INTRODUCTION

Several years ago our group initiated[1] a project of constructing theoretical foundations for experimental tests of gravitation theories. The results of that project to date (largely due to Will and Ni) and the results of a similar project being carried out by the group of Nordtvedt at Montana State University are summarized in several recent review articles.[2-4] Those results have focused almost entirely on "metric theories of gravity" (relativistic theories that embody the Einstein equivalence principle; see Sec. III below).

By January 1972, metric theories were suffi-

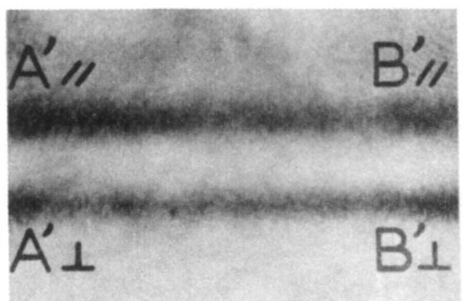

FIG. 4. Photographic record of the twin tangential focal lines in total reflection, displaying the two eigenvalues and the two eigenfunctions (linear polarization states) of the longitudinal shift.

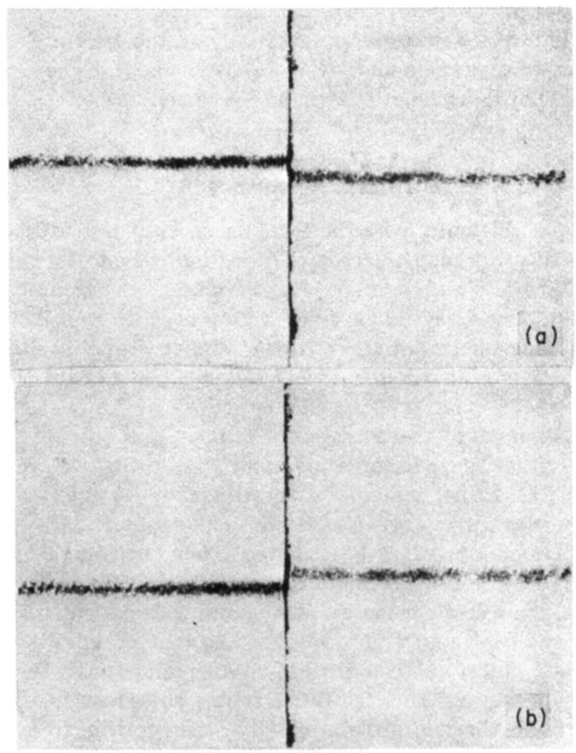

FIG. 7. The two symmetric broken images of the rectilinear object displaying the two eigenvalues and the two eigenfunctions (circular polarization states) of the transverse shift. (a) left circular polarization suppressed; (b) right circular polarization suppressed.