# Kinematics and Dynamics of $K_{l3}$ Form Factors*

Ephraim Fischbach and Michael Martin Nieto†
*Physics Department, Purdue University, Lafayette, Indiana 47907*

C. K. Scott
*Physics Department, McMaster University, Hamilton, Ontario, Canada*

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We discuss how the question of a zero at $t=(m_K+m_\pi)^2$ in the Duffin-Kemmer-Petiau $K_{l3}$ divergence matrix element [the effective form factor $\hat{f}_0(t)$] is a combined question of kinematics and dynamics.

The authors of the preceding note,[1] hereafter referred to as WWY, claim to show that the zero we predicted[2-6] in the effective $K_{l3}$ divergence form factor $\hat{f}_0(t)$ is not of kinematical origin. It is clear that WWY are employing the term "kinematic" in a different sense than we are, and we would like to clarify this point. We call the dependence of the physical matrix element on the choice of wave functions kinematic, and the dependence on the currents and interactions dynamic. WWY combine both the dependences into the word dynamic. If we were to use WWY's definition of the words kinematic and dynamic, then we would agree that indeed the zero of $\hat{f}_0(t)$ is not kinematic. Whether or not the zero exists is a question of physics. In the models we have studied,[5,6] it does, and, in the event that it is found experimentally, it would indicate that the Duffin-Kemmer-Petiau (DKP) equation is superior to the Klein-Gordon (KG) equation for the description of scalar particles.

To be more explicit, let us explain the differences between our approach and that of WWY. We both agree, of course, that there are only two independent form factors $f_+$ and $f_-$ in the KG treatment of $K_{l3}$ decays. These form factors are free of kinematic zeros in the conventional (WWY) sense. When we discuss $K_{l3}$ decays in the DKP formalism the situation is slightly more complicated. WWY claim that there is a proliferation of terms in the general Lorentz-covariant form of the matrix element of the current. In fact, they begin with six terms $g_1, \ldots, g_6$ in Eq. (11a), reduce the matrices, and end up with two equations connecting $f_+$ and $f_-$ with the six $g_i$'s. The $f$'s are free of kinematic singularities in the WWY sense. Hence if one introduces a common factor of $t-(m_K+m_\pi)^2$ among the $g_i$'s (by some elimination procedure), it must be canceled by a corresponding pole. Thus, with such a factor, to ensure that the $f$'s have no kinematic singularities, WWY argue that the $g$'s cannot be smooth functions.

However, nothing in WWY's analysis should be construed as implying that $\hat{f}_0(t)$ cannot, for *dynamical* reasons, have the form

$$\hat{f}_0(t)=\frac{[(m_K+m_\pi)^2-t]g_0(t)}{2(m_Km_\pi)^{1/2}(m_K+m_\pi)} \tag{1}$$

[where $g_0(t)$ is an arbitrary smooth function of $t$], such as has been shown to be the case in the particular models of Refs. 2–6. We can clarify this further as follows. As we have shown elsewhere,[7] by use of Eqs. (3) and (4) of WWY, all of the DKP invariants in their Eq. (10a) can be reduced to a *single* invariant in Eq. (10b), not to two as WWY assert. Similarly, the six form factors in Eq. (11a) of WWY can be reduced to leave only two form factors, by use of the same algebraic techniques that WWY have by implication used in arriving at the six form factors in the first place. [Thus the physical significance of the redundant expressions in Eqs. (10b) and (11a) of WWY is not clear.] The reduction is, of course, not unique, but our particular form can be motivated by a consideration of special models. We then identify these two terms as the physical form factors in the problem, so that our matrix element reads

$$\langle \pi(p')|V_\lambda(0)|K(p)\rangle = i\left(\frac{m_K m_\pi}{p_0 p_0' V^2}\right)^{1/2}\bar{u}_\pi(p')\left[\beta_\lambda g_V(t)+i\,\frac{q_\lambda}{(m_K+m_\pi)}\,g_S(t)\right]u_K(p). \tag{2}$$

Now, excluding the possibility that $g_V$ and $g_S$ have poles at $t=(m_K+m_\pi)^2$, which we say is a question of dynamics, we conclude that the divergence of the vector current is

$$\langle \pi(p')|\partial_\lambda V_\lambda(0)|K(p)\rangle = i\,(m_K - m_\pi)\left(\frac{m_K m_\pi}{p_0 p_0' V^2}\right)^{1/2} \frac{t-(m_K+m_\pi)^2}{4m_K m_\pi}\, g_0(t), \tag{3}$$

where

$$g_0(t) = g_V(t) - t\,g_S(t)/(m_K^2 - m_\pi^2). \tag{4}$$

From an analysis of $K_{e3}$ decay in the dilepton rest frame we find

$$\hat{f}_+(t) = (m_K + m_\pi) g_V(t)/2(m_K m_\pi)^{1/2}, \tag{5}$$

so the usual assumption that $\hat{f}_+(t)$ is smooth implies that $g_V(t)$ is smooth. If we now assume $g_S(t)$ to be smooth then the zero arising from the DKP wave functions is responsible for the zero in the scalar form factor. We have consistently referred to this zero as a kinematical zero in our previous papers, but we emphasize that it is dynamical in the sense of WWY.

In fact the analysis can be made immediately for the scalar matrix element without discussing the vector matrix element. Again we contend that a careful analysis of the matrix element $\langle\pi|\partial_\lambda V_\lambda(0)|K\rangle$ in the DKP formalism shows that it depends only on $g_0(t)$ as defined above.

We conclude from the above that the question of whether $\langle\pi|\partial_\lambda V_\lambda(0)|K\rangle$ has a zero is a combined one of kinematics and dynamics. In our previously considered $K^*$-pole model, which is the only model for $g_0(t)$ that has been studied to date in detail, the predicted zero is certainly present. This model has already demonstrated the assertion made by WWY (which we of course agree with) that starting from a fundamental Lagrangian containing only the primitive coupling $\bar{\psi}\beta_\lambda\psi$ one can induce a term containing $q_\lambda\bar{\psi}\psi$. However, this therefore means that by writing the matrix element as in Eq. (2), the effects of all possible radiative corrections are automatically taken into account.

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†Present address: Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544.

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