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Analysis of Proton Fragmentation into Two π^- Mesons

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We report on the reactions $A + p \to \pi^- + \cdots$ and $A + p \to \pi^- + \pi^- + \cdots$, where A is either a 13-GeV/c K^+ or K^- meson or a 7-GeV/c π^+ meson. We compare the distribution of final-state pions in the proton-fragmentation region with several theoretical predictions concerning limiting behavior and factorization.

Mueller[1] has shown that the invariant cross section $E\, d\sigma/d^3p = f(\vec{p}, s)$ for production of the particle c in the single-particle inclusive reaction A+p→ c+ anything can be related to the discontinuity in the imaginary part of the forward $A\bar{c}p \to A\bar{c}p$ elastic scattering amplitude. Chan *et al.*[2] have stressed the application of dual Regge-pole ideas in conjunction with Mueller's observation. In Fig. 1(a) we show what Chan *et al.* call the single-Regge limit of the Mueller diagram. These authors argue on the basis of exchange degeneracy that, when the $(A\bar{c}p)$ quantum numbers are exotic, Pomeranchuk exchange (P) dominates the scattering, and limiting behavior consequently sets in at relatively low energies. In addition, assuming factorization at the APA vertex, the ratio of $f(\vec{p}, s)$ for different beam particles (A) is given by the ratios of the corresponding asymptotic Ap total cross sections (σ_T). Although the predictions of Chan *et al.* (CHQW) have been tested experimentally, and in general have been found to be valid,[3] there remain theoretical questions concerning the proper application of the duality hypothesis. Ellis *et al.*[4] (EFFJ) argue that $(A\bar{c}p)$ exotic is not a sufficient condition for the dominance of vacuum exchange, but that it is also necessary for (Ap) to be exotic. On the other hand, Einhorn *et al.*[5] (EGV) propose that (Ap) and $(A\bar{c})$ must both be exotic; finally, Tye and Veneziano[6] (TV) claim that all channels must be exotic if P exchange is to dominate.[7]

In order to further examine some of these questions, we compare the following reactions in the proton-fragmentation region:

$K^+p \to \pi^- + \cdots$ 13 GeV/c (1)

$K^-p \to \pi^- + \cdots$ 13 GeV/c (2)

$\pi^+p \to \pi^- + \cdots$ 7 GeV/c (3)

$K^+p \to \pi^- + \pi^- + \cdots$ 13 GeV/c (4)

$K^-p \to \pi^- + \pi^- + \cdots$ 13 GeV/c (5)

$\pi^+p \to \pi^- + \pi^- + \cdots$ 7 GeV/c (6)

The K^+p data (Rochester), K^-p data (Yale), and π^+p data (Rochester-Yale) are from exposures of the BNL 80-in. hydrogen bubble chamber to high-energy separated beams.[8] We estimate a maximum K^- background of 5% for reactions (1)–(6).[8,9]

Reactions (4)–(6) are examples of two-particle inclusive reactions. Investigations of correlations between the two detected final-state particles have been discussed in the literature.[10] Here we are only interested in applying the arguments of Chan *et al.* to the single-Regge limit of the Mueller diagram shown in Fig. 1(b). Although extending the application of the duality hypothesis to more complicated fragmentation reactions [Fig. 1(b)] may not be a trivial matter, it is nevertheless instructive to investigate this additional part of the multiparticle cross section. We retain the previous definitions of exoticity by replacing c by the sum of the quantum numbers of the two π^- mesons.

In Fig. 2(a) we display the quantity

$$g(p_l) = \frac{1}{\sigma_T} \int E \frac{d^2\sigma}{dp_l\, dp_T^2}\, dp_T^2$$

for reactions (1)–(3), where the variables p_l (longitudinal momentum) and E are evaluated in the laboratory frame.[11] We also give ratios of these normalized invariant cross sections relative to reaction (1) in Figs. 2(b) and 2(c). The error bars in Fig. 2(a) are only statistical; however, the error bars on the ratios [Figs. 2(b) and 2(c)] include systematic uncertainties as well. Since reaction (2) is not exotic according to any of the previously mentioned authors,[2,4-6] we expect the distribution for reaction (2) to be different from that for reaction (1). (This is, of course, not a very strong statement, since meson exchanges can be present in the t channel for the K^--induced reaction but not for the K^+p channel.) This is, indeed, what is observed. Reactions (1) and (3), on the other hand, agree very well for $p_l < 0.5$ GeV/c. This result reiterates the fact that in this instance the criteria of EFFJ, EGV, and TV are not necessary, and that the criteria of CHQW suffice for the rapid onset of limiting behavior.[12]

Next we turn to reactions (4)-(6), which are all exotic in the sence of CHQW; reactions (5) and (6), however, are not exotic in the sense of EFFJ, EGV, and TV. In Fig. 3(a) we display the quantities

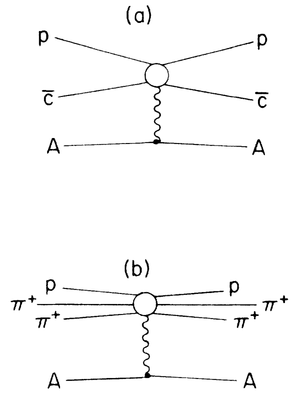

FIG. 1. (a) Single-Regge limit of the Mueller diagram for the single-particle inclusive reaction $A+p \rightarrow c+\cdots$; (b) same as (a) for the two-particle inclusive reaction $A+p \rightarrow \pi^- + \pi^- + \cdots$.

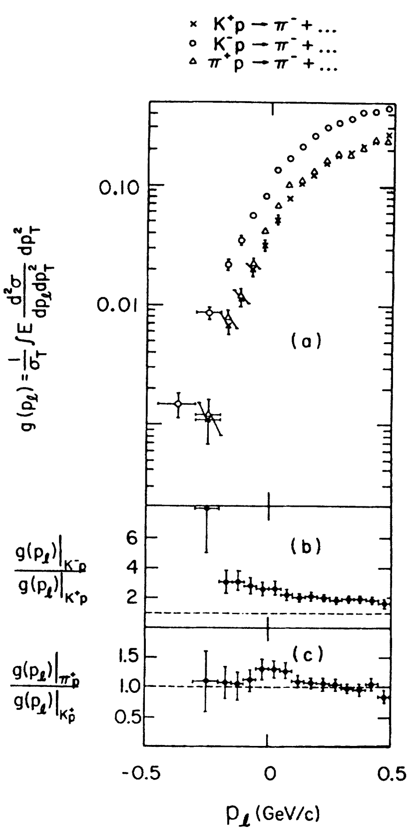

FIG. 2. (a) $g(p_l) = (1/\sigma_T) \int E(d^2\sigma/dp_l\,dp_T^2)dp_T^2$ for reactions (1)-(3). The errors shown on the data points include only statistical uncertainties. Systematic errors are less than 8%. (b) The ratio of $g(p_l)$ for reactions (2) to $g(p_l)$ for reaction (1). (c) The ratio of $g(p_l)$ for reaction (3) to $g(p_l)$ for reaction (1). The errors shown in (b) and (c) include statistical as well as systematic uncertainties.

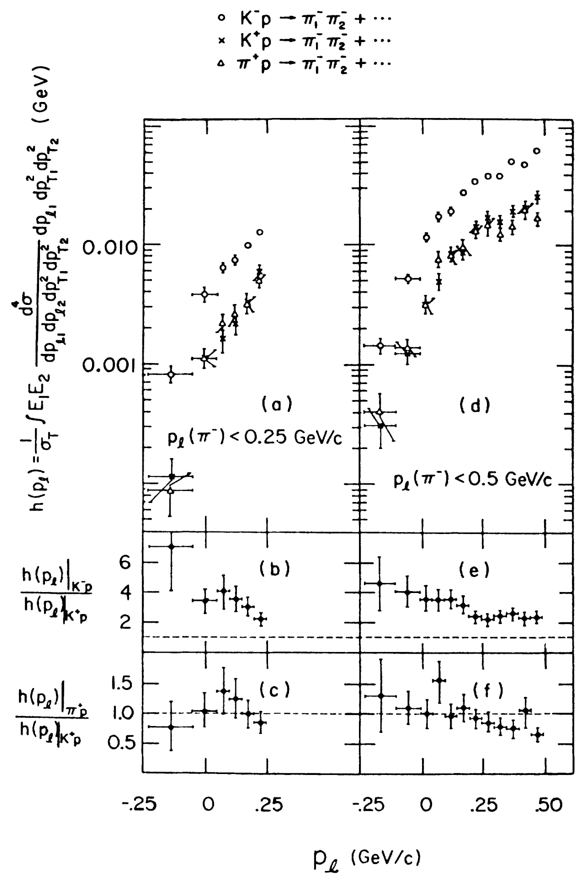

FIG. 3. (a) $h(p_l) = (1/\sigma_T)\int E_1E_2(d^4\sigma/dp_{l1}dp_{l2}dp_{T1}{}^2dp_{T2}{}^2) \times dp_{l2}dp_{T1}{}^2dp_{T2}{}^2$ for reactions (4)-(6) with the reaction tion that both final-state π^- mesons have $p_l < 0.25$ GeV/c. The errors shown on the data points include only statistical uncertainties. (b) The ratio of $h(p_l)$ for reaction (5) to $h(p_l)$ for reaction (4). (c) The ratio of $h(p_l)$ for reaction (6) to $h(p_l)$ for reaction (4). The errors shown in (b) and (c) include systematic as well as statistical uncertainties (d)-(f). The same as (a)-(c) with the restriction that both final state π^- mesons have $p_l < 0.05$ GeV/c.

TABLE I. Integrated invariant cross sections.[a]

Limits on both p_t (GeV/c)	$\sigma_E(K^+p)$ (μb GeV^2)	$\sigma_E(K^-p)$ (μb GeV^2)	$\sigma_E(\pi^+p)$ (μb GeV^2)	$\frac{\sigma_E(K^-p)}{\sigma_E(K^+p)}$	$\frac{\sigma_E(\pi^+p)\sigma_T(K^+p)}{\sigma_E(K^+p)\sigma_T(\pi^+p)}$
<0.1	1.1 ± 0.2	5.8 ± 0.7	1.9 ± 0.4	5.1 ± 1.1	1.3 ± 0.4
<0.2	5.0 ± 0.6	22.0 ± 2.3	8.6 ± 0.1	4.4 ± 0.7	1.1 ± 0.2
<0.3	23 ± 2	67 ± 7	31 ± 3	3.0 ± 0.4	1.0 ± 0.1
<0.4	55 ± 4	150 ± 15	70 ± 6	2.7 ± 0.3	0.9 ± 0.1
<0.5	124 ± 5	320 ± 30	148 ± 13	2.6 ± 0.3	0.9 ± 0.1

[a] Errors shown contain systematic as well as statistical uncertainties.

$$h(p_t) = \frac{1}{\sigma_T} \int E_1 E_2 \frac{d^4\sigma}{dp_{l1}dp_{l2}dp_{T1}^2dp_{T2}^2}\, dp_{l2}dp_{T1}^2dp_{T2}^2$$

for events where both final-state π^- mesons have $p_t < 0.25$ GeV/c in the laboratory; in Fig. 3(d) we display $h(p_t)$ for events in which both $p_t < 0.5$ GeV/c. The ratios of these cross sections are also displayed in this figure. While the agreement between reactions (4) and (6) is quite good, the cross section for reaction (5) is clearly larger than that for reactions (4) and (6). This may reflect a dependence of proton fragmentation on the charge of the incident beam particle. In Table I we show the quantity

$$\sigma_E = \sigma_T \int h(p_t) dp_t$$

for increasing upper limits on the p_t of each of the two π^- mesons. It is apparent that the excess cross section for reaction (4) persists down to the smallest values of p_t.

In Table II we summarize the contributions of different multiplicities to $h(p_t)$; we present the values of σ_E for different topologies in reactions (4) and (5), each with the restriction that both π^- have $p_t < 0.5$ GeV/c.

In conclusion, our data do not appear to yield a consistent over-all picture for the applicability of any particular duality scheme in inclusive reactions. However, we note that although the agreement between the normalized invariant cross sections for reactions (4) and (6) is not really surprising considering the previously observed success of the factorization hypothesis found for reactions (1) and (3),[12] these two-particle processes are nevertheless different from the single-particle reactions.[13] The fact that reaction (5) appears to be at odds with the predictions of the CHQW hypothesis may mean that the degree of exoticity of a reaction or perhaps just the charge of the incident system may have important bearing on the rate of approach of two-body inclusive reactions to limiting behavior; alternatively, the rate of approach to scaling of any reaction may simply be strongly mass-dependent, as suggested by the behavior of the reaction $pp \to \bar{p} + \cdots$, which, while exotic by each of the aforementioned criteria, nevertheless approaches limiting behavior very slowly.

We thank Dr. M. Jacob and Dr. C. Quigg for useful discussions. After completing the described analysis, we became aware of a similar study by W. S. Lam [Phys. Lett. 40B, 466 (1972)].

TABLE II. Invariant topological cross sections for both $p_t < 0.5$ GeV/c.[a]

Number of neg. tracks	K^-p Topology	K^-p σ_E(μb GeV^2)	K^+p Topology	K^+p σ_E (μb GeV^2)
2	4-prong	73 ± 7	6-prong	68 ± 4
3	6-prong	170 ± 17	8-prong	49 ± 3
4	8-prong	78 ± 8	10-prong	7 ± 1

[a] Errors shown contain systematic as well as statistical uncertainties.

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[3]M.-S. Chen *et al.*, Phys. Rev. Lett. 26, 1585 (1971); see also *Proceedings of the International Conference on Inclusive Reactions, University of California, Davis, 1972,* edited by R. L. Lander (Univ. of Calif. Press, Davis, 1972); and *Proceedings of the Fourth International Conference on High Energy Collisions, Oxford, 1972* (Rutherford High Energy Laboratory, Chilton, Didcot, Berkshire, U. K., 1972); and Proceedings of the Zakopane Colloquium on Multiparticle Reactions, 1972 (unpublished).

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[7]See also Chan Hong-Mo and P. Hoyer, Phys. Lett. 36B, 79 (1971); M. Kugler *et al.*, *ibid.* 38B, 423 (1972); E. L. Berger, ANL Report No. ANL/HEP 7148, 1971 (unpublished); M. Jacob, NAL Report No. NAL-THY-47, 1972 (unpublished).

[8]The K^+p and π^+p experiments have been discussed elsewhere. See S. L. Stone *et al.*, Nucl. Phys. B32, 19 (1971); S. L. Stone, Ph.D. thesis, University of Rochester, 1972 (unpublished). The data presented here represent a larger sample of events than previously published.

[9]The background in the K^--induced reactions was estimated from neutral kaon production in the backward direction.

[10]W. Ko, Phys. Rev. Lett. 28, 935 (1972); W. D. Shephard *et al.*, *ibid.* 28, 703 (1972); S. L. Stone, *et al.*, Phys. Rev. D 5, 1621 (1972); J. Hanlon *et al.*, Vanderbilt University Report (unpublished); J. V. Beaupre *et al.*, Phys. Lett. 40B, 510 (1972).

[11]For σ_T, we use 17.4 mb for $K^\pm p$ collisions and 23.4 mb for π^+p collisions.

[12]See, for example, M.-S. Chen *et al.*, in Ref. 3; J. V. Beaupre *et al.*, Phys. Lett. 37B, 432 (1971); D. J. Crennell *et al.*, Phys. Rev. Lett. 28, 643 (1972).

[13]We wish to point out that at present energies reactions (4)–(6) account for about 50% of the contributions to reactions (1)–(3), and as energies and produced-particle multiplicities increase we expect that reactions (4)–(6) will contribute ever-increasing fractions to reactions (1)–(3). In a specific model, it is of course possible to make predictions about the two-particle spectra from knowledge of the single-particle spectra. See, for example, E. Berger and M. Jacob, Phys. Rev. D 6, 1930 (1972); E. Berger, M. Jacob, and R. Slansky, *ibid.* 6, 2580 (1972).

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Average Multiplicity, Secondary Trajectory, and Mueller Analysis*

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The correction to the average multiplicity is calculated in a general Mueller analysis when a secondary Regge trajectory with intercept $\frac{1}{2}$ is also included. $\langle n\rangle$ is then approximately given by $A\ln s+B+Cs^{-1/2}$, which fits well all existing accelerator multiplicity data from $s=25$ to 2800 GeV^2. We show that part of the coefficient C may be estimated from the 90° production data. We comment on the sensitivity (or insensitivity) of $\langle n\rangle$ as a test of production models.

From general Regge behavior for the six-line connected part, Mueller derived[1] the $(A\ln s+B)$ formula for the average multiplicity by keeping only the leading Regge contribution. Recent multiplicity data[2] seem to suggest that the low-laboratory-energy data may not lie on a straight line with the new CERN Intersecting Storage Rings (ISR) data when average multiplicity is plotted against $\ln s$. The purpose of this paper is to calculate the correction to the $(A\ln s+B)$ formula when a secondary Regge trajectory with intercept $\frac{1}{2}$ is also included in Mueller's analysis. We find that the average multiplicity is given by

$$\langle n\rangle=A\ln s+B+(C'+C''\ln s)s^{-1/2}, \tag{1a}$$

where C' and C'' depend on the type of incident particles. Since we are interested in explaining with as few parameters as possible an upward curvature of the multiplicity data, in our phenomenological analysis we group together the last two terms of (1a) and approximate (1a) by