K_{l3} and π_{e3} Decays in Terms of the Kemmer β Current

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Energy spectra of leptons and partial decay rates in K_{l3} and π_{e3} decays are evaluated within the framework of single Kemmer currents. The branching ratio of the two K_{l3} decay modes and the π_{e3} decay rate are found to be in agreement with experiment. The formalism brings in mass ratios naturally and hence considerably simplifies the problem.

I. INTRODUCTION

The use of Kemmer currents for spin-0 particles in the problem of K_{l3} and π_{e3} decays has been recently reintroduced.[1] There has also been considerable controversy about the equivalence of the use of the one-component Klein-Gordon wave function or of the five-component Kemmer wave function.[2] On the general S-matrix level there is, *a priori,* complete equivalence, but for dynamical reasons a particular Kemmer current may be more appropriate and provide a simpler description of the physical phenomena.[3] In this latter sense, the Kemmer formalism had been used long ago by Barut and Zeleny[4] in a comprehensive treatment of all weak interactions. In that work all boson currents were taken into account and "a fine-structure splitting of the weak coupling constants as a consequence of the observation of the different symmetries involved in the weak processes" was introduced. The idea of giving a fine structure to the weak coupling constants was a forerunner of the introduction of the Cabibbo angle. In Cabibbo's theory, the coupling constant is G for purely leptonic decays, $G\cos\theta \simeq G$ for the hypercharge-conserving semileptonic decays, and $G\sin\theta \simeq 0.22G$ for the hypercharge-changing semileptonic decays. In Ref. 4 all leptonic and hypercharge-conserving *currents* were given a constant factor $G_2^{1/2}$ in front, and the hypercharge-changing *currents* were given a different factor, $G_3^{1/2}$. Therefore, every purely leptonic and hypercharge-conserving semileptonic decay has the coupling constant G_2, whereas the hypercharge-changing semileptonic decays have the coupling constant $(G_2G_3)^{1/2}$. We shall also see in this note that $(G_2G_3)^{1/2}$ is found to be of the order of about $(0.1-0.2)G_2$. In addition, Barut and Zeleny constructed the currents in isotopic spin space in such a way that the $|\Delta I| = \frac{1}{2}$ selection rule automatically follows. At this stage they introduced one more coupling constant, G_1, in order to explain the violation of the $|\Delta I| = \frac{1}{2}$ rule.

In this work we use the two simplest Kemmer currents for bosons [Eqs. (1) and (2) below]. The current (2), which is the only current used in Ref. 4, gives, in spite of generally reasonable results, a higher branching ratio for K_{l3} decays and an anomalously small partial rate for π_{e3}. In this paper we shall use the electromagnetic-type current (1), show that the ratio $\Gamma(K_{e3})/\Gamma(K_{\mu3})$ is equal to 1.7 compared to 1.5, experimentally, and obtain $\Gamma(\pi_{e3}) = 0.395\ \text{sec}^{-1}$, whose experimental value is $(0.396 \pm 0.027)\ \text{sec}^{-1}$. This formalism also gives the order of magnitude of the weak coupling constant correctly.

II. CURRENTS IN K_{l3} DECAY

Let us write down, first of all, some vector currents for K_{l3} decay in the Kemmer formalism:

$$J_1^\lambda(x) = \overline{\psi}_\pi(x)\beta^\lambda\psi_K(x)\,, \tag{1}$$

$$J_2^\lambda(x) = \overline{\psi}_\pi(x)\beta^\lambda C\psi_K(x)\,, \tag{2}$$

$$J_3^\lambda(x) = i\partial^\lambda[\overline{\psi}_\pi(x)\psi_K(x)]\,. \tag{3}$$

Here $\psi(x)$ has five components, and β^λ and C are 5×5 matrices given explicitly in Ref. 4. It is interesting to express the first two currents in terms of the scalar Klein-Gordon fields $\phi_\pi(x)$ and $\phi_K(x)$ in the x space. We have

$$J^\lambda(x) = i\left[\left(\frac{m_K}{m_\pi}\right)^{1/2}(\partial^\lambda\phi_\pi^\dagger)\phi_K \mp \left(\frac{m_\pi}{m_K}\right)^{1/2}\phi_\pi^\dagger(\partial^\lambda\phi_K)\right], \tag{4}$$

where (and hereafter throughout the paper) the upper sign stands for the current J_1 and the lower sign for J_2. Both differ from the usual current[5]

$$J^\lambda = i[(\partial^\lambda\phi_\pi^\dagger)\phi_K - \phi_\pi^\dagger(\partial^\lambda\phi_K)] \tag{5}$$

by mass ratios. Finally, the currents (1) and (2) can be expressed in the momentum space as follows:

$$J^\lambda = \frac{1}{2}\left(\frac{m_\pi m_K}{E_\pi E_K}\right)^{1/2}\left(\frac{p_K^\lambda}{m_K} \pm \frac{p_\pi^\lambda}{m_\pi}\right). \tag{6}$$

Now before going into the calculational details, we should like to remark that the general vector current V^{λ} in Kemmer form is the linear combination of J_1 and J_2, for example. [Notice that only two of the three currents (1)–(3) are linearly independent.] The usual form factors $f_{\pm}$ are defined by the most general form of the current

$$(4E_{\pi}E_K)^{1/2}V^{\lambda}=f_+(t)(p_K+p_{\pi})^{\lambda}+f_-(t)(p_K-p_{\pi})^{\lambda}. \tag{7}$$

If we assume that the (π, K) current, for dynamical reasons, coincides with one of the J_i's, then we have only one free parameter, namely the weak coupling constant. From this assumption we find immediately[3] for the parameter $\xi=f_-(t)/f_+(t)$

$$\xi=-\frac{m_K-m_{\pi}}{m_K+m_{\pi}}$$
$$=-0.57 \quad (f_+=1.19,\ f_-=-0.7) \quad \text{for } J_1, \tag{8}$$

$$\xi=-\frac{m_K+m_{\pi}}{m_K-m_{\pi}}$$
$$=-1.75 \quad \text{for } J_2, \tag{9}$$
$$\xi=\infty \quad (\text{or } f_+\equiv 0) \quad \text{for } J_3,$$

independent of t. Because the form factors $f_+(t)$ and $f_-(t)$ themselves do not seem to vary much with t, and the value of the ξ parameter seems to be between 0 and -2 experimentally,[6] it seems reasonable to make a field-theoretical calculation with the interaction Lagrangian of the form

$$\mathcal{L}_{\text{int}}=GJ_a^{\lambda}[\bar{\psi}_l\gamma_{\lambda}(1-i\gamma_5)\psi_{\nu}], \quad a=1 \text{ or } 2. \tag{10}$$

We should like to emphasize that we interpret $\mathcal{L}_{\text{int}}$ in (10) as a phenomenological Lagrangian to be used in first order, and therefore G is the physical coupling constant. In view of practical and conceptual difficulties of a field theory of weak interactions (mainly because of the unstable character of the decaying state) this is a sensible attitude.

III. ENERGY SPECTRA OF LEPTONS AND DECAY RATES

The square of the lowest-order matrix elements, summed over lepton spins, for K_{l3} decay is given by

$$\sum_{\text{spins}}|M|^2=\frac{G_2G_3}{2E_lE_{\nu}E_{\pi}E_Km_{\pi}m_K}(-P\cdot Pp_l\cdot p_{\nu}+2P\cdot p_lP\cdot p_{\nu}), \tag{11}$$

where $P=m_Kp_{\pi}\pm m_{\pi}p_K$, and the coefficient G_2G_3 is the square of the coupling constant, which turns out to be $G^2\sin^2\theta_c$, numerically. The transition rate is

$$d\Gamma=(2\pi)^4\delta^4(p_K-p_{\pi}-p_l-p_{\nu})\sum_{\text{spins}}|M|^2\frac{d^3p_{\pi}}{(2\pi)^3}\frac{d^3p_l}{(2\pi)^3}\frac{d^3p_{\nu}}{(2\pi)^3}. \tag{12}$$

Integrating Eq. (12) over the final momenta and evaluating the result in the center-of-mass frame of the decaying particle, we obtain the lepton energy spectrum in the form

$$d\Gamma=\frac{G_2G_3}{(2\pi)^3m_{\pi}}(E^2-m^2)^{1/2}\frac{(E_0-E)^2}{(E_0-E+m_{\pi}^2/2m_K)^2}$$
$$\times[m_{\pi}^2E(m_K-E)+m_K(m_KE-m^2)(E_0-E)\pm 2m_{\pi}m_KE(E_0-E+m_{\pi}^2/2m_K)]\,dE, \tag{13}$$

where m is the lepton mass, E its (total) energy, and E_0 the maximum value of E: $E_0=(m_K^2+m^2-m_{\pi}^2)/2m_K$. We plot the energy spectra of the electron for the currents J_1 and J_2, separately, in Fig. 1, and those of the muon in Fig. 2. The general shapes of the curves coincide with those of Marshak, Riazuddin, and Ryan.[7] Integrating Eq. (13) over dE, we find the first-order decay rates as

$$\Gamma_1(K_{e3}^{\pm})=3.23\times10^{30}G_2G_3\,\text{MeV}^4\,\text{sec}^{-1},$$
$$\Gamma_1(K_{\mu3}^{\pm})=1.90\times10^{30}G_2G_3\,\text{MeV}^4\,\text{sec}^{-1} \tag{14}$$

for the current J_1, and

$$\Gamma_2(K_{e3}^{\pm})=1.05\times10^{30}G_2G_3\,\text{MeV}^4\,\text{sec}^{-1},$$
$$\Gamma_2(K_{\mu3}^{\pm})=0.50\times10^{30}G_2G_3\,\text{MeV}^4\,\text{sec}^{-1} \tag{15}$$

for the current J_2. The theoretical branching ratio $\Gamma_2(K_{e3}^{\pm})/\Gamma_2(K_{\mu3}^{\pm})=2.1$ obtained from (15) is higher than the experimental ratio of about 1.5, but note that the ratio $\Gamma_1(K_{e3}^{\pm})/\Gamma_1(K_{\mu3}^{\pm})=1.7$ obtained from (14) is pretty close to its experimental value. Furthermore, if we compare the theoretical partial decay rates (14) and (15) with the experimental rates

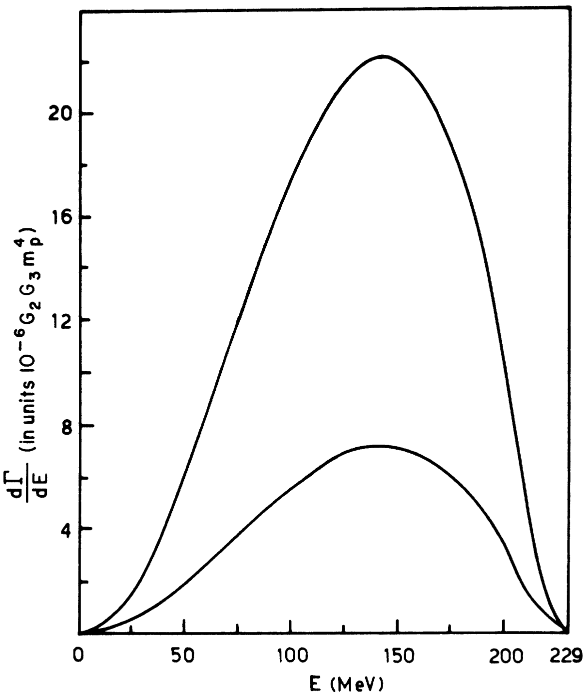

FIG. 1. Energy spectrum of electron in K_{l3} decay. Upper curve is for current J_1, and lower curve for J_2.

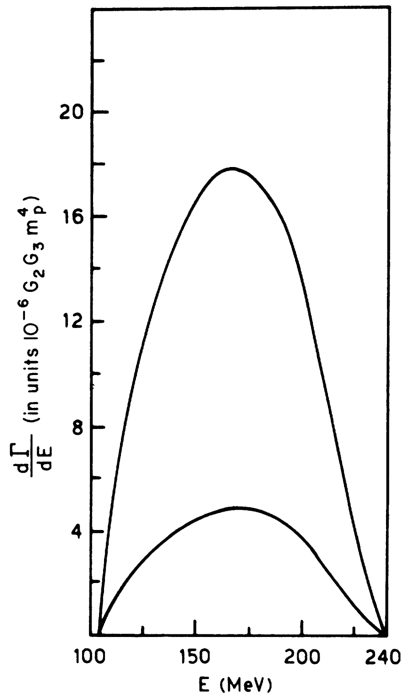

FIG. 2. Energy spectrum of muon in $K_{\mu 3}$ decay. Upper curve is for current J_1, and lower curve for J_2.

$$\Gamma(K_{e3}^{\pm}) = (3.92 \pm 0.06) \times 10^6 \text{ sec}^{-1},$$
$$\Gamma(K_{\mu 3}^{\pm}) = (2.59 \pm 0.09) \times 10^6 \text{ sec}^{-1}, \quad (16)$$

we obtain the values for the decay coupling constant given in Table I, where we have also evaluated a coupling constant G defined by

$$(G_2 G_3)^{1/2} m_p^2 = G m_p^2 \sin\theta_c \simeq 0.22 G m_p^2 .$$

We see that the values of Gm_p^2 found in Table I are compatible with the commonly accepted value of $\sim 10^{-5}$, and the current J_1 requires a slightly smaller value of G than the current J_2.

IV. π_{e3}^{+} DECAY

In order to derive the quantities pertaining to π_{e3}^{+} decay, it is sufficient to replace m_K by the mass of the charged pion in Eqs. (11)–(13). We obtain in this way the decay rate

$$\Gamma_1(\pi_{e3}^{+}) = 3.04 \times 10^{21} G_2^2 \text{ MeV}^4 \text{ sec}^{-1} \quad (17)$$

for the current J_1, and

$$\Gamma_2(\pi_{e3}^{+}) = 8.36 \times 10^{17} G_2^2 \text{ MeV}^4 \text{ sec}^{-1} \quad (18)$$

for J_2. With $G_2 = 10^{-5}\, m_p^{-2}$, Eq. (18) gives an anomalously small number, but from Eq. (17) we obtain $\Gamma_1(\pi_{e3}^{+}) = 0.395 \text{ sec}^{-1}$, which is in excellent agreement with the experimental value[7] of $(0.396 \pm 0.027) \text{ sec}^{-1}$.

V. DISCUSSION

The current J_2 was the only current used in Ref. 4, which gives rise to some discrepancies with experiment on the branching ratio of K_{l3} decays and the value of the π_{e3}^{+} partial rate, as we have here reexhibited. On the other hand, the current J_1 gives very reasonable results, Eqs. (14) and (17). In addition, notice that the current J_1 is similar to the usual current (5), except for the coefficients $(m_K/m_\pi)^{1/2}$ or $(m_\pi/m_K)^{1/2}$. Indeed, it is obvious that the form (5) can be interpreted as the probability (mass) current only for equal-mass bosons, i.e., for (π, π) or (K, K) currents, but not for the (π, K) and (π^0, π^+) currents. For the latter the mass transfer must be exhibited in

TABLE I. Decay coupling constants evaluated from the decay rate for currents J_1 and J_2, respectively.

Decay mode	Type of interaction current	$(G_2G_3)^{1/2}m_p^2$	Gm_p^2
$K_{l3}^{\pm}$	J_1	0.97×10^{-6}	0.44×10^{-5}
$K_{\mu 3}^{\pm}$	J_1	1.02×10^{-6}	0.46×10^{-5}
$K_{l3}^{\pm}$	J_2	1.70×10^{-6}	0.77×10^{-5}
$K_{\mu 3}^{\pm}$	J_2	2.00×10^{-6}	0.91×10^{-5}

some way, in our case by coefficients like $(m_K/m_\pi)^{1/2}$ and $(m_\pi/m_K)^{1/2}$. This is achieved naturally in the Kemmer formalism by choosing J_1 as the appropriate current. Another strong support for the choice of the current J_1 comes from the fact that the form of J_1 is identical to the electromagnetic current in the Kemmer formalism, and hence we restore the symmetry between the weak and electromagnetic currents.

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Estimate of the Magnitude of Triple-Pomeranchukon Coupling from the Observed Energy Dependence of Total and Elastic pp Cross Sections*

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The measured total and elastic pp cross sections are used to give an estimate, to within a factor 2, of triple-Pomeranchukon coupling. The assumption underlying the estimate is that single-fireball formation is asymptotically controlled by an isolated Regge pole.

A two-component model of high-energy particle production has been showing promise of correlating the observations emerging from the National Accelerator Laboratory (NAL) and the CERN Intersecting Storage Rings (ISR).[1–6] For the purposes of this paper we shall characterize the two-component model in terms of "fireballs," where the number of fireballs in a given event is defined through the rapidity distribution of produced particles.[7] Events where no large gaps[8] appear in the rapidity distribution will be described as "single-fireball."[9] If one large gap appears we shall speak of two fireballs, and so on. The two-component model ignores the possibility of more than one large gap and supposes each of the two fireballs in a one-gap event to be of low mass. The model furthermore supposes the collection of single-fireball events to have an aggregate (i.e., inclusive) energy dependence that corresponds to isolated factorizable Regge poles (short-range order in rapidity) and thus to be susceptible to the Mueller treatment of inclusive cross sections.[10] The two- (low-mass) fireball events, on the other hand, are supposed to be described in the exclusive sense by Pomeranchukon exchange in the same way as elastic scattering, which in fact represents about half of this category. The energy dependence of this "fragmentation" component thus corresponds to the Amati-Stanghellini-Fubini (ASF) two-Pomeranchukon branch point[11] (long-range order in rapidity).

An experimental difficulty for the two-component model is the observed near-constancy of the high-energy pp total cross section, which is observed to vary by less than 0.5 mb between