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Some Consequences of a Model Based on the Gell-Mann Hyperweak Theory of CP Nonconservation*

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We propose a simple model which parametrizes the main features of a hyperweak-type theory due to Gell-Mann in which the source of CP nonconservation is attributed to strongly interacting $X^{\pm}$ particles (sthenons) which, however, have no strong interactions with the normal hadrons. The model suggests that it is not unreasonable to expect CP effects at the level of (α/π) in amplitude in muon decay, though (contrary to earlier expectation) CP effects are negligible in neutrino production of these X particles when they are identified with the charged intermediate vector boson of weak interactions.

I. INTRODUCTION

A few years back, Gell-Mann[1] postulated that there exists a group of strongly interacting particles, strongly interacting among themselves, which communicate with the usual world of hadrons and leptons only electromagnetically or through the usual (CP-conserving) semiweak interactions. The strong interactions of these so-called $X^{\pm}$ particles conserve a quantum number n_X, basically the difference between the numbers of X^+ and X^- particles; however, these $X^{\pm}$ particles have *no* strong interactions with the normal hadrons. The strong interactions of $X^{\pm}$ are postulated by Gell-Mann to be strongly CP-nonconserving. In modern terminology, the X particles are a subclass of the large family of new particles which interact strongly with each other, characterized as sthenons by Appelquist and Bjorken.[2]

It can be shown explicitly[1] that in order G (the weak coupling), where the weak interaction is essentially

$$GJ_\mu^{\text{wk}}(J_\nu^{\text{wk}})^*\langle(\phi_\mu^*\phi_\nu)\rangle_{\text{vac}}, \tag{1.1}$$

no CP-nonconserving effects are present. Here $J_\mu^{\text{wk}} = J_\mu^{\text{wk}}(l) + J_\mu^{\text{wk}}(h)$ is the usual weak current with leptonic (l) and hadronic (h) parts; ϕ_μ (ϕ_μ^*) changes n_X by $+1$ (-1) and is like a creation (annihilation) operator for X^+ particles.

To see CP violation, we must go to some higher-order process, and the largest effects are obtained via electromagnetic perturbations. A real photon gives order Ge; a virtual photon order Ge^2. If we write the electromagnetic current as $J_\lambda^\gamma + \mathcal{J}_\lambda^\gamma$, where J_λ^γ is the ordinary electromagnetic current for hadrons and leptons and $\mathcal{J}_\lambda^\gamma$ refers to the current of the world of $X^{\pm}$ particles, then in order Ge^2 the effective interaction is

$$Gj_\mu^{\text{wk}}(J_\nu^{\text{wk}})^* J_\lambda^\gamma\langle(\phi_\mu^*\phi_\nu\mathcal{J}_\lambda^\gamma)\rangle_{\text{vac}}\ . \tag{1.2}$$

Lorentz invariance allows (1.2) to have many more terms than (1.1), and a number of these give CP violation. Hence, in order Ge^2, we obtain the Fitch-Cronin (FC) effect via electromagnetic-weak cooperation.[1] The theory thus predicts CP effects at the level of (α/π) in weak processes, but none in strong and electromagnetic processes (as is common to the class of all hyperweak theories of CP violation).

In order to suppress a neutron electric dipole ($E1$) moment of order Ge, two additional assumptions are made: The strong interactions of $X^{\pm}$ are P-conserving and the creation (annihilation) operators for single emission (absorption) of $X^{\pm}$ particles satisfy

$$\partial_\mu\phi_\mu = 0, \quad \partial_\mu\phi_\mu^* = 0\ . \tag{1.3}$$

It is customary to identify $X^{\pm}$ with the $W^{\pm}$ intermediate boson of weak interactions, though the Gell-Mann theory can also be discussed in terms of just quantum numbers so that n_X is an abstract quantity. For definiteness of discussion, we shall henceforth make the identification $X \equiv W$. The CP nonconservation in (1.2) occurs through the presence of the electromagnetic $\mathcal{J}_\lambda^\gamma$ current ($\gamma W\overline{W}$) primitive vertex, depicted schematically as follows:

$$\gamma \xrightarrow[C,P\text{ conserving}]{\text{electromagnetic}} (W+\overline{W}) \xrightarrow[C\text{ violating, }P\text{ conserving}]{\text{strong}} (W+\overline{W})\ . \tag{1.4}$$

Here CP violation occurs at the second stage of (1.4) via the strong $(W\overline{W}W\overline{W})$ vertex. This suggests that a useful parametrization of the effective current $\mathcal{J}_\mu^\gamma$ describing (1.4) can be written as

$$\mathcal{J}_\mu^\gamma = fJ_\mu^\gamma + f'K_\mu^\gamma , \tag{1.5}$$

where f and f' are strong coupling constants associated with the $(W\overline{W}W\overline{W})$ strong vertex, while J_μ^γ and K_μ^γ are, respectively, the CP (and P) conserving and CP (and C) violating (but P conserving) conserved "electromagnetic" currents. To wit, we require

$$\begin{aligned} CJ_\mu^\gamma C^{-1} &= -J_\mu^\gamma , \\ CK_\mu^\gamma C^{-1} &= +K_\mu^\gamma . \end{aligned} \tag{1.6}$$

The charge of W is associated with J_μ^γ, and since the strong interactions of W's do not renormalize the charge, the scale provided by charge conservation requires that $f = 1$ and

$$\begin{aligned} Q &\equiv -i\int J_4^\gamma d^3r , \\ Q|W^\pm\rangle &= \pm e|W^\pm\rangle . \end{aligned} \tag{1.7}$$

No charge is associated with K_μ^γ, hence

$$0 \equiv -i\int K_4^\gamma d^3r \tag{1.8}$$

and f' is left undetermined other than it represents a "strong" coupling constant. For convenience we shall assume

$$f' = f = 1. \tag{1.9}$$

In an earlier work[3] it was pointed out that Gell-Mann's CP proposals can lead in principle to rather unique predictions (uncommon to other hyperweak CP theories) such as CP effects at the level of 10^{-3} in amplitude in $\mu \to e + \overline{\nu}_e + \nu_\mu$ and gross CP effects as well as enhanced cross sections in near-forward neutrino production of $W^\pm$ mesons. The purpose of the present paper is to construct a specific model which incorporates the principal features of the Gell-Mann program, and hence test through detailed calculations whether the above-mentioned predictions are realized in the model under consideration.

As a first step in this construction, we need to identify the simplest form of K_μ^γ ($\neq 0$) which is Hermitian and satisfies (1.6) and (1.8) and the current-conservation condition

$$\partial_\mu K_\mu^\gamma = 0 . \tag{1.10}$$

Furthermore, it must lead to vanishing dipole moment to order Ge. In general, a conserved nonvanishing K_μ^γ which nevertheless satisfies (1.8) and may lead to suppression of neutron dipole moment to order Ge [at least in the *specific* case of zero four-momentum transfer ($q \to 0$)] can be derived by double differentiation from a third-rank tensor $T_{\mu\alpha\beta}$ according to[4]

$$\begin{aligned} K_\mu(x) &= \partial_\alpha\partial_\beta T_{\mu\alpha\beta}(x) , \\ T_{\mu\alpha\beta} &= T_{\mu\beta\alpha}, \quad T_{\mu\alpha\beta} = -T_{\alpha\mu\beta} . \end{aligned} \tag{1.11}$$

For our purposes, the current K_μ^γ needs to be explicitly constructed out of the W fields[5] satisfying (1.11) and the required Hermiticity and C properties. The end product is

$$K_\mu^\gamma = \frac{1}{M_W^2}[\partial_\alpha\partial_\beta\partial_\mu(\phi_\alpha^*\phi_\beta + \phi_\alpha\phi_\beta^*) - \Box^2\partial_\rho(\phi_\mu^*\phi_\rho + \phi_\mu\phi_\rho^*)] . \tag{1.12}$$

In order to eliminate rigorously the possibility of a neutron $E1$ moment to order Ge (independent of momentum transfer), we impose the Gell-Mann dipole condition[1] (1.3) on (1.12). This leads to

$$\begin{aligned} K_\mu = \frac{1}{M_W^2}\{&\partial_\mu(\partial_\beta\phi_\alpha\partial_\alpha\phi_\beta^* + \partial_\beta\phi_\alpha^*\partial_\alpha\phi_\beta) - [\partial_\rho(\partial_\sigma^2\phi_\mu)\phi_\rho^* + \partial_\rho\phi_\mu\partial_\sigma^2\phi_\rho^* + \partial_\rho(\partial_\sigma^2\phi_\mu^*)\phi_\rho + \partial_\rho\phi_\mu^*\partial_\sigma^2\phi_\rho \\ &+ 2\partial_\sigma\phi_\rho\partial_{\rho,\sigma}^2\phi_\mu^* + 2\partial_\sigma\phi_\rho^*\partial_{\rho,\sigma}^2\phi_\mu]\} . \end{aligned} \tag{1.13}$$

Note that in arriving at Eq. (1.13), we have not used at any stage the on-mass-shell Klein-Gordon equations, since to tackle the problem of μ decay and neutrino production of W it is important to maintain current conservation (1.10) for off-mass-shell amplitudes as well.

The diagrams contributing to CP violation in muon decay at level Ge^2 are shown in Figs. 1(a) and 1(b). The self-energy diagram [Fig. 1(c)] will not contribute to CP since CPT invariance plus Lorentz invariance will guarantee that symmetry information will not be forthcoming from this diagram. The lowest-order diagram is shown in Fig. 2. For the neutrino production of W mesons, the relevant CP-conserving diagrams are shown in Figs. 3(a) and 3(b), while the CP-violating diagram is shown in Fig. 3(c). Note that in both muon decay and neutrino production of W, the CP effects enter via the $(W\overline{W}\gamma)$ vertex, which we describe by the effective K_μ^γ current given in (1.13).

For the CP-violating $W\overline{W}\gamma$ vertex, we take as the interaction Lagrangian

$$L_{\rm int} = eA_\mu K_\mu^\gamma . \tag{1.14}$$

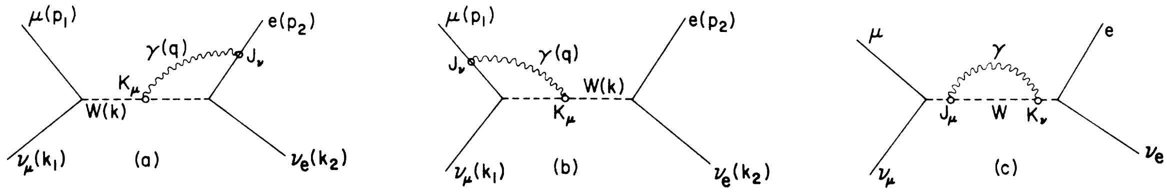

FIG. 1. (a) Lowest-order diagram contributing to CP violation in muon decay via the $(\gamma W \overline{W})$ vertex. The virtual photon is coupled to the electron line at the other end. (b) Lowest-order diagram contributing to CP violation in muon decay via the $(\gamma W \overline{W})$ vertex. The virtual photon is coupled to the muon line at the other end. (c) Self-energy diagram in muon decay for the W intermediate boson. It does not contribute to CP violation.

The vertex function for the primitive $W\overline{W}\gamma$ vertex can then be constructed, using momentum expansions for the ϕ_α fields

$$\phi_\alpha = \sum_\lambda \int \frac{d^4p}{(2\pi)^4}\epsilon_\alpha^{(\lambda)} a^\dagger(p) e^{ip\cdot x},$$
$$\phi_\alpha^* = \sum_\lambda \int \frac{d^4p}{(2\pi)^4}\epsilon_\alpha^{(\lambda)} a(p) e^{-ip\cdot x}. \tag{1.15}$$

This leads to

$$L_{\rm int}(p,p',q) = [eA_\mu(q)/M_W^2]\{-iq_\mu e^{-iq\cdot x}[p_\beta p'_\alpha \epsilon_\alpha^{(\lambda)}(p)\epsilon_\beta^{(\lambda)}(p') + p_\alpha p'_\beta \epsilon_\alpha^{(\lambda)}(p')\epsilon_\beta^{(\lambda)}(p)] - ie^{-iq\cdot x}q^2[p_\rho \epsilon_\mu^{(\lambda)}(p)\epsilon_\rho^{(\lambda)}(p') - p'_\rho \epsilon_\mu^{(\lambda)}(p')\epsilon_\rho^{(\lambda)}(p)]\}. \tag{1.16}$$

The advantage of (1.16) is that it is manifestly gauge-invariant upon making the substitution $A_\mu(q) \to A_\mu(q) + \lambda q_\mu$ and a consistent use of the Gell-Mann dipole conditions in momentum space [cf. (1.3)]. That our model Lagrangian is manifestly gauge-invariant means that we will preserve covariance and maintain the Ward identity to all orders.

Since the current K_μ^γ (and J_μ^γ) is conserved for off-mass-shell behavior as well, it is legitimate to take advantage of the simplicity which obtains by imposing the Lorentz condition[6]

$$q_\mu A_\mu(q) = 0. \tag{1.17}$$

Thus, the first term in the interaction Lagrangian (1.16) will yield zero when contracted. We are left with

$$L_{\rm int} = (eA_\mu/M_W^2)q^2(-ie^{-iq\cdot x})[p_\rho \epsilon_\mu^{(\lambda)}(p)\epsilon_\rho^{(\lambda)}(p') - p'_\rho \epsilon_\mu^{(\lambda)}(p')\epsilon_\rho^{(\lambda)}(p)]. \tag{1.18}$$

It is seen at once that as the photon goes on the mass shell $(q^2=0)$, $L_{\rm int}$ vanishes.[7] Hence, our constructed model for K_μ^γ is entirely consistent with the Gell-Mann theory[1] that the dipole moment vanishes to order Ge.

The vertex function for *internal* lines is from (1.18)

$$\Delta_{\alpha\beta\mu} = \left(\frac{-ie}{M_W^2}\right)(p_\beta \delta_{\alpha\mu} - p'_\alpha \delta_{\beta\mu})q^2 \tag{1.19}$$

and forms the basis of a detailed calculation of CP effects in our model. Also, since strongly interacting W's (sthenons) are involved, we need not concern ourselves with subtleties such as the unitarity limit problem, nor with renormalization, in our calculations.

In Sec. II, the two Feynman diagrams [Figs. 1(a) and 1(b)], with CP-nonconserving $W\overline{W}\gamma$ vertex given by (1.19) for muon decay, are calculated. The result suggests that CP effects can be present in the muon-decay amplitude at the level of

$$(\alpha/\pi)\,\mathfrak{A}(\Lambda^2). \tag{1.20}$$

Here $\mathfrak{A}(\Lambda^2)$ is formally a quadratically divergent constant. Hence, depending on the value we assign to the cutoff Λ, (1.20) can be consistent with the known violation $\sim 2\times 10^{-3}$ in K_L^0 decay.

Contrary to earlier expectations,[3] CP effects are negligible in neutrino production of W's. Indeed, the cross section $\sigma_{(c)}$ due to CP-violating diagram Fig. 3(c) is much smaller than the cross section $\sigma_{(ab)}$ due to the usual CP-conserving diagrams [Fig. 3(a) plus Fig. 3(b)]. This is true for the entire range of possible W masses between 5 and 15 GeV and for incident neutrino energy from threshold to 400 GeV. Since the ratio

$$[\sigma_{(c)}/\sigma_{(ab)}]^{1/2} \tag{1.21}$$

gives a rough measure of the CP-violating to CP-conserving amplitudes, while the dominant contribution to both cross sections will come from small q^2 (because of nucleon form factor suppression at large q^2), CP effects in the near-forward production of W's are small (<1% in amplitude) in our model. The cross-section calculations for Fig. 3(c) are presented in Sec. III. In Fig. 4, $\sigma_{(c)}$ versus incident neutrino energy is plotted for typical W masses between 5 and 15 GeV.

Finally, in Sec. IV, we discuss the implications to be drawn from our model representation of the Gell-Mann hyperweak theory.[1]

II. MUON DECAY

There are two Feynman diagrams involving the $(W\overline{W}\gamma)$ CP-violating vertex which can contribute to muon decay in order Ge^2, as illustrated by Figs. 1(a) and 1(b).

The contribution to the matrix element from Fig. 1(a) is

$$M(1\mathrm{a}) = \frac{e^2 g_W^{\,2}}{M_W^{\,4}}[i m_\mu \bar{u}(p_1)(1+\gamma_5)\nu(k_1)\bar{u}(p_2)\Sigma_\mathrm{I}(1+\gamma_5)\nu(k_2) - \bar{u}(p_1)\gamma_\alpha(1+\gamma_5)\nu(k_1)\bar{u}(p_2)\gamma_\alpha\Sigma_\mathrm{II}(1+\gamma_5)\nu(k_2)], \tag{2.1}$$

where Σ_I and Σ_II are 4×4 matrices given by

$$\Sigma_\mathrm{I} = \int \frac{d^4q}{(2\pi)^4} \frac{\{\gamma_\beta[-i\gamma\cdot(p_2-q)+m_e]\gamma_\beta + \gamma\cdot(k-q)[-i\gamma\cdot(p_2-q)+m_e]\gamma\cdot(k-q)/M_W^{\,2}\}}{[(p_2-q)^2+m_e^{\,2}][(k-q)^2+M_W^{\,2}]},$$

$$\Sigma_\mathrm{II} = \int \frac{d^4q}{(2\pi)^4} \frac{\{[-i\gamma\cdot(p_2-q)+m_e]\gamma\cdot(k-q)[1+(k-q)^2/M_W^{\,2}]\}}{[(p_2-q)^2+m_e^{\,2}][(k-q)^2+M_W^{\,2}]}.$$

Although these integrals are formally divergent, we can introduce a (large) cutoff in momentum transfer, Λ, and obtain an explicit result, as discussed in Appendix A.

We have

$$M(1\mathrm{a}) = \frac{e^2 g_W^{\,2}}{M_W^{\,4}}[m_e m_\mu A(\Lambda^2)\bar{u}(p_1)(1+\gamma_5)\nu(k_1)\bar{u}(p_2)(1+\gamma_5)\nu(k_2) + B(\Lambda^2)\bar{u}(p_1)\gamma_\alpha(1+\gamma_5)\nu(k_1)\bar{u}(p_2)\gamma_\alpha(1+\gamma_5)\nu(k_2)$$
$$+ m_e C(\Lambda^2)\bar{u}(p_1)i\gamma\cdot p_2(1+\gamma_5)\nu(k_1)\bar{u}(p_2)(1+\gamma_5)\nu(k_2) + O(m_e^{\,2})], \tag{2.2}$$

where we have neglected a term which is small compared to the rest. By inspection of Eq. (2.2), we can readily see that only the last term explicitly violates CP, and only that term can contribute to CP violation in μ decay.

The CP-violating part of the matrix element which is obtained from Fig. 1(a) is thus

$$M^{CPV}(1\mathrm{a}) = \frac{-e^2 g_W^{\,2}}{M_W^{\,4}}\, m_e C(\Lambda^2)\bar{u}(p_1)i\gamma\cdot p_2(1+\gamma_5)\nu(k_1)\bar{u}(p_2)(1+\gamma_5)\nu(k_2), \tag{2.3}$$

where

$$C(\Lambda^2) = -\frac{1}{8\pi^2}\left[\frac{25}{12}\ln\left(\frac{4\Lambda^2}{M_W^{\,2}}\right) - \frac{233}{72} + \frac{2\Lambda^2}{M_W^{\,2}}\right]. \tag{2.4}$$

We obtain a similar contribution from Fig. 1(b). Denoting the CP-conserving part of the matrix element by M^{CP} and the CP-violating part by M^{CPV}, we have

$$M(1\mathrm{a}) + M(1\mathrm{b}) = M_\mathrm{II}^{CP} + M_\mathrm{II}^{CPV}, \tag{2.5}$$

where

$$M_\mathrm{II}^{CPV} = -\frac{g_W^{\,2}e^2}{M_W^{\,4}}\, C(\Lambda^2)[m_e\bar{u}(p_1)i\gamma\cdot p_2(1+\gamma_5)\nu(k_1)\bar{u}(p_2)(1+\gamma_5)\nu(k_2) + m_\mu\bar{u}(p_1)(1+\gamma_5)\nu(k_1)\bar{u}(p_2)i\gamma\cdot p_1(1+\gamma_5)\nu(k_2)], \tag{2.6}$$

and we have the interesting result that (as far as those terms which violate CP are concerned) Fig. 1(b) dominates the matrix element, since it is proportional to the muon mass while Fig. 1(a) is proportional to m_e.

Now the largest amount of CP violation will arise from the interference of the CP-violating terms of order α with the usual first-order CP-*conserving* amplitude, whose matrix element is obtained from Fig. 2.

If we calculate the differential rate from the interference of these two terms, we will obtain the magnitude of CP violation we expect to be present in μ decay.

The differential rate obtained from the interference of Fig. 2 with the CP-violating part of Fig. 1 is (see Appendix B)

$$dW=\frac{\mathfrak{a}(\Lambda^2)G^2}{(2\pi)^3}\alpha m_\mu P_e\,dE_e\,d(\cos\theta)\left\{(W-E_e)-\frac{m_e{}^2}{m_\mu{}^2}(W-E_e)+\frac{1}{3}\frac{P_e}{E_e}\cos\theta\left[2(W-E_e)-\frac{E_e}{m_\mu}(4W-E_e)\right]\right\},\qquad(2.7)$$

where P_e and E_e are the three-momentum and energy, respectively, of the electron; $W=\frac{1}{2}m_\mu$ is the maximum electron energy; and

$$G=\frac{\sqrt{2}\,g_W{}^2}{M_W{}^2}\ .$$

The differential rate for the first-order process (Fig. 2) is the familiar expression

$$dW=\frac{G^2m_\mu P_eE_e\,dE_e\,d(\cos\theta)}{6\pi^3}\left\{3h(W-E_e)+2\rho\left(\frac{4}{3}E_e-W-\frac{1}{3}\frac{M_e{}^2}{E_e}\right)+\frac{3M_e}{E_e}\eta(W-E_e)\right.$$
$$\left.-\frac{P_e}{E_e}\cos\theta\,\xi\left[(W-E_e)+2\delta\left(\frac{4}{3}E_e-W-\frac{1}{3}\frac{m_e{}^2}{m_\mu}\right)\right]\right\}\ ,\qquad(2.8)$$

where the $V-A$ theory plus the assumption of the two-component neutrino predicts

$$\rho=\delta=\tfrac{3}{4},\quad \eta=0,\quad \xi=1,\quad h=1.$$

Adding the two contributions, we obtain the differential rate for $1+2$:

$$dW=\frac{M_\mu G^2P_e\,dE_e\,d(\cos\theta)}{6\pi^2}\left(3(W-E_e)\left[h+\frac{3\mathfrak{a}(\Lambda^2)\alpha}{4\pi}\right]+2\rho\left(\frac{4}{3}E_e-W-\frac{1}{3}\frac{M_e{}^2}{E_e{}^2}\right)+\frac{3M_e}{E_e}(W-E_e)\left[\eta-\frac{M_eE_e\alpha\,\mathfrak{a}(\Lambda^2)}{4\pi M_\mu{}^2}\right]\right.$$
$$-\frac{P_e}{E_e}\cos\theta\left\{(W-E_e)\left[\xi-\frac{\alpha\mathfrak{a}(\Lambda^2)}{2\pi}\right]+2\delta\left[\xi-\frac{\alpha\mathfrak{a}(\Lambda^2)}{2\pi}\right]\left(\frac{4}{3}E_e-W-\frac{1}{3}\frac{M_e{}^2}{M_\mu}\right)\right.$$
$$\left.\left.-\frac{2\alpha}{\pi}\mathfrak{a}(\Lambda^2)\frac{E_e}{M_\mu}(1+\tfrac{1}{2}E_e)\right\}\right)\qquad,\qquad(2.9)$$

where

$$\mathfrak{a}(\Lambda^2)=\frac{M_\mu{}^2}{\pi M_W{}^2}\left[\frac{25}{12}\ln\left(\frac{4\Lambda^2}{M_W{}^2}\right)-\frac{233}{72}+\frac{2\Lambda^2}{M_W{}^2}\right].\qquad(2.10)$$

Although $\mathfrak{a}(\Lambda^2)$ is a formally divergent constant, experience leads us to expect the electromagnetic corrections to occur at the level $\alpha/\pi\sim\frac{1}{500}$.

We see that the μ-decay parameters are changed in the following way:

$$h\to h+\frac{3\alpha\mathfrak{a}(\Lambda^2)}{4\pi}\ ,$$
$$\rho\to\rho\ ,$$
$$\eta\to\eta-\frac{\alpha M_eE_e\mathfrak{a}(\Lambda^2)}{4\pi M_\mu{}^2}\ ,\qquad(2.11)$$
$$\xi\to\xi-\frac{\alpha\mathfrak{a}(\Lambda^2)}{2\pi}\ ,$$
$$\delta\to\delta\ .$$

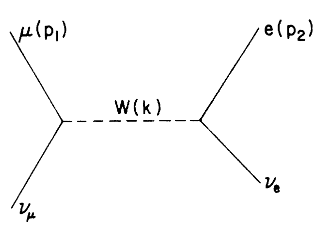

FIG. 2. The lowest-order CP-conserving diagram in muon decay.

The latest values for the μ-decay parameters are [9]

$$h=1.00\pm0.13\ ,$$
$$\rho=0.752\pm0.003\ ,$$
$$\eta=-0.12\pm0.21\ ,\qquad(2.12)$$
$$\xi=0.972\pm0.013\ ,$$
$$\delta=0.755\pm0.009\ .$$

With reference to Eq. (2.11), we, of course,

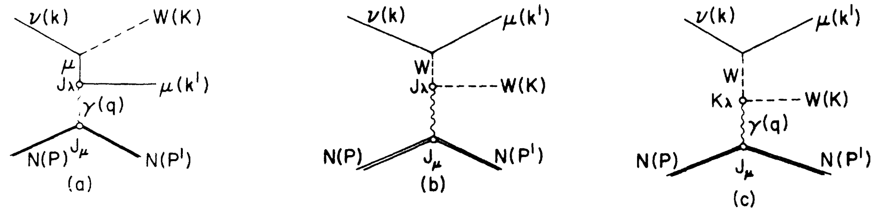

FIG. 3. (a) CP-conserving Feynman diagram for production of W by neutrinos via exchange of a virtual photon coupled to the nucleon source and the muon line. (b) CP-conserving Feynman diagram for neutrino production of W via exchange of a virtual photon coupled to the nucleon source and the CP-conserving part of the $(\gamma W \overline{W})$ vertex. (c) CP-violating Feynman diagram for neutrino production of W via exchange of a virtual photon coupled to the nucleon source and the CP-violating part of the $(\gamma W \overline{W})$ vertex.

recognize that the CP-conserving second-order electromagnetic corrections to muon decay will also contribute (α/π)-type terms to the decay parameters. However, assuming CPT invariance, CP violation implies T violation. Hence, we expect T violation to be present also at the level of (α/π) for the muon-decay parameters.

If we allow the constant $\mathfrak{a}(\Lambda^2)$ to be of the order of unity, then the corrections to the μ-decay parameters are

$$\Delta h \sim \frac{3\alpha}{4\pi},$$

$$\Delta\xi \sim -\frac{\alpha}{2\pi},$$

$$\Delta\eta \sim \frac{\alpha M_e E_e}{4\pi M_\mu{}^2} \cong \frac{\alpha M_e}{8\pi M_\mu},$$

and we see that the correction to η is suppressed by the additional factor of M_e/M_μ. This does not mean, however, that CP-violating effects are similarly suppressed. This is easy to demonstrate.

The more familiar parametrization of CP violation (T violation) in weak interactions is via the $V - \epsilon A$ prescription,[10] where ϵ is complex. The connection between the two equivalent parametrizations is

$$\xi = -\frac{2\,\mathrm{Re}\epsilon}{1+|\epsilon|^2}, \tag{2.13}$$

$$\eta = \frac{|\epsilon|^2 - 1}{2(1+|\epsilon|^2)}. \tag{2.14}$$

If we write

$$\xi = 1 - \Delta\eta,$$

$$\eta = -\Delta\eta,$$

then we are able to solve Eqs. (2.13) and (2.14) exactly for ϵ; i.e.,

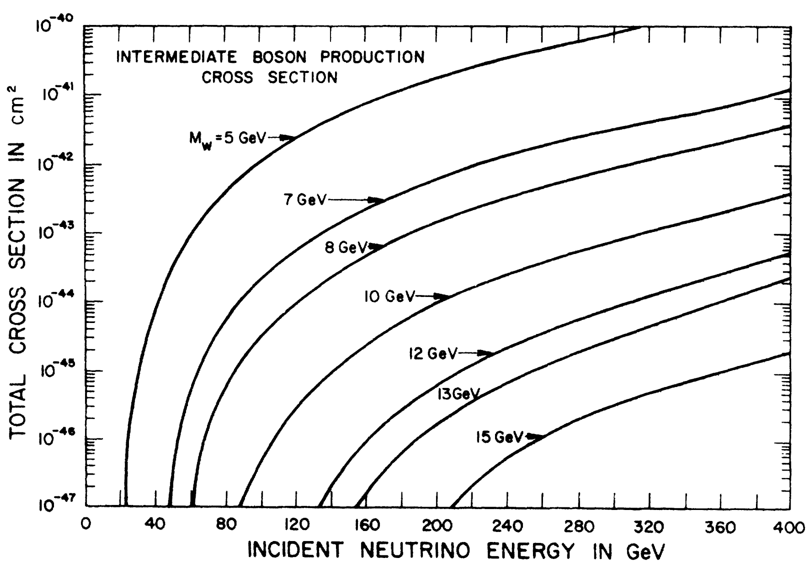

FIG. 4. Plot of the total cross section for production of an intermediate boson W in $\nu + p \to \mu + p + W$ for the CP-violating diagram [Fig. 3 (c)] versus incident neutrino energy from threshold to 400 GeV.

$$\mathrm{Re}\epsilon = -(1-\Delta\xi),$$
$$(\mathrm{Im}\epsilon)^2 = 2[\Delta\xi - (\Delta\eta)^2],$$
$$|\epsilon|^2 = 1 - 2(\Delta\eta)^2.$$

Thus, even though $\Delta\eta$ is suppressed by the additional M_e/M_μ factor, ϵ is not, since even if η were exactly zero we would have

$$\mathrm{Re}\epsilon = -(1-\Delta\xi),$$
$$\mathrm{Im}\epsilon = (2\Delta\xi)^{1/2}$$

and CP violation (T violation) would still occur at the level α/π.

III. W PRODUCTION

So far we have considered only the low-energy μ decay. We now examine the W-boson production process in high-energy ν-p collisions as a possible test of gross CP violation due to the presence of the virtually strong $(W\overline{W}W\overline{W})$ CP-violating interaction. The W-production reaction is

$$\nu + p \rightarrow \mu + p + W. \tag{3.1}$$

The traditional diagrams[8] for electromagnetic W production from this reaction are shown in Figs. 3(a) and 3(b). The total cross section for process (3.1) is quite small (e.g., $\sigma_{(ab)} \sim 10^{-38}$ cm^2 for $E_\nu = 40$ GeV and $M_W = 5$ GeV). The reason for this, as pointed out by several authors,[8,11] is the large cancellation between Figs. 3(a) and 3(b) for small q^2 due to gauge invariance. The Gell-Mann theory,[3] however, changes the situation considerably because now Fig. 3(c) is maximally CP-violating relative to Figs. 3(a) and 3(b) due to the CP-violating $(\gamma W\overline{W})$ vertex,[12] and hence does not participate in the cancellation between Figs. 3(a) and 3(b). We might, therefore, expect Fig. 3(c) to result in a cross section which is considerably enhanced over the traditional estimates.

Using the CP-violating model Lagrangian, (1.18), the matrix element for Fig. 3(c) is

$$M = \frac{-e^2 g_W \epsilon_\beta(K)\bar{u}(k')\gamma_\lambda(1+\gamma_5)\nu(k)}{M_W^2[(K+q)^2+M_W^2]}\left[\delta_{\lambda\alpha} + \frac{(K+q)_\lambda(K+q)_\alpha}{M_W^2}\right][(K+q)_\alpha - K_\beta]\langle P'|J_\mu^\gamma(0)|P\rangle\,, \tag{3.2}$$

where $\epsilon_\beta(K)$ is the W-polarization vector and $\langle P'|J_\mu^\gamma(0)|P\rangle$ is the electromagnetic current of the target nucleon. For production from a free proton, we take

$$\langle P'|J_\mu^\gamma(0)|P\rangle = i\bar{u}(P')[F_1^p(q^2)\gamma_\mu + F_2^p(q^2)\sigma_{\mu\rho}q_\rho]u(P)\,, \tag{3.3}$$

where $F_1^p(q^2)$ and $F_2^p(q^2)$ are the proton form factors which we take from the Stanford experiments.[13]

The differential cross section for reaction (3.1) is easily shown to be

$$d\sigma = (32\pi^5)^{-1}|M|^2 d^3k'\,d^3p'\,d^3K \times \delta(E_\mu + E_W + E_{P'} - E_\nu - E_P)\,. \tag{3.4}$$

The calculation was made with an IBM 360/65 computer. Following the technique presented by Brown and Smith,[8] we were able to perform two integrations by hand, leaving two numerical integrations to be done by machine. Since we were interested only in the magnitude of the total cross section, we summed over the various spins and did not calculate energy spectra.

In Fig. 4, we present the surprising results obtained for the total cross section, $\sigma_{(c)}$, for the production process on free protons as a function of the W mass.

Quite contrary to our original expectations, the total cross section in our model is considerably *smaller* than the traditional estimates for $\sigma_{(ab)}$.[8]

The *physical* reason for this is not clear, but the technical reasons are quite simple. First, in contrast to the "usual" production cross section, our cross section is proportional to the muon mass (squared); whether this is a general property of all CP-nonconserving interactions is not known, but it is quite likely. Secondly, due to the "nonminimal" nature of our model interaction Lagrangian (1.18) (it contains many powers of q, the momentum transfer), we must divide the Lagrangian by M_W^2 to ensure correct dimensionality. This (large) constant factor is not compensated for by the integration over phase space, and a further reduction in the cross section occurs.

Typically, the most favorable situation in terms of the largest production cross section, $\sigma_{(c)}$, for a given M_W occurs at the highest incident neutrino energy. For $M_W = 5$, 10, and 15 GeV and incident $E_\nu = 400$ GeV, we have

$$\left(\frac{\sigma_{(c)}}{\sigma_{(ab)}}\right)^{1/2} \cong \begin{cases} 1.4\times10^{-2}\ (M_W = 5\ \text{GeV}) \\ 1.1\times10^{-3}\ (M_W = 10\ \text{GeV}) \\ 2.45\times10^{-4}\ (M_W = 15\ \text{GeV})\,. \end{cases} \tag{3.5}$$

Hence, unless $f' \gg 1$ [cf. Eq. (1.5)], the CP-nonconserving effects in near-forward production of W are likely to be smaller than 1% for the entire gamut of W masses from 5 to 15 GeV.

IV. DISCUSSIONS

Our specific model Lagrangian (1.18) makes it not unreasonable to expect CP (and T) effects at the level of (α/π) in muon decay. It does not, however, support the earlier suggestion[3] that gross CP effects and enhanced cross sections for near-forward production of W's in neutrino-initiated processes can be realized in the Gell-Mann theory.[1] Hence, to the extent that our model is representative of the theory, the latter is indeed not easy to differentiate from other hyperweak CP theories, though accurate measurements in muon decay to one part in 10^3 in amplitude can be a meaningful test.

A word of caution is appropriate, however. The Lagrangian which we have used is only a *model* Lagrangian which appears to give the simplest concrete representation to the underlying theory.[1] There is no guarantee that our Lagrangian effectively describes the complex dynamics, since we are essentially *parametrizing* something very complicated (e.g., the strong interactions of sthenon W's) by something very simple. Given a better understanding of the dynamics, there may be a more appropriate CP-violating Lagrangian that leads to a completely *different* $\sigma_{\rm tot}$ for W production.

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APPENDIX A

We handle the divergent integrals in the following way: Consider, for example, the integral

$$I=\int\frac{d^4q}{(2\pi)^4}\,\frac{(q+k)^2}{q^2+s-i\epsilon}\;,$$

$$I=\int\frac{d^4q}{(2\pi)^4}\,\frac{|\vec{q}|^2-q_0^{\,2}+2k_\alpha q_\alpha+k^2}{|\vec{q}|^2-q_0^{\,2}+s-i\epsilon}\;.$$

Now, the integral

$$k_\alpha\int\frac{d^4q}{(2\pi)^4}\,\frac{q_\alpha}{q^2+s-i\epsilon}=0\,,$$

since it is an *odd* function of q_α and the integration is over all values of $|\vec{q}|$ and q_0. Therefore,

$$I=\int\frac{d^3|\vec{q}|}{(2\pi)^4}\,(|\vec{q}|^2+k^2)\int dq_0\,\frac{1}{|\vec{q}|^2-q_0^{\,2}+s-i\epsilon}-\int\frac{d^3|\vec{q}|}{(2\pi)^4}\int dq_0\,\frac{q_0^{\,2}}{|\vec{q}|^2-q_0^{\,2}+s-i\epsilon}\;.$$

We can handle these integrals over dq_0 by the usual contour method. We obtain the result

$$I=\frac{i(k^2-s)}{(2\pi)^2}\int_0^\Lambda\frac{|\vec{q}|^2d|\vec{q}|}{(|\vec{q}|^2+s)^{1/2}}\;,$$

where we have introduced the (large) cutoff Λ since the integral is formally divergent. We can do this integral by elementary means, with the result

$$I=\int\frac{d^4q}{(2\pi)^4}\,\frac{(q+k)^2}{q^2+s-i\epsilon}=\frac{i(k^2-s)}{(2\pi)^2}\left[\tfrac{1}{2}\Lambda^2-\tfrac{1}{2}s\ln\left(\frac{2\Lambda}{\sqrt{s}}\right)\right].$$

Now, letting $q\to q-k$, $s\to s-k^2$, we find that

$$\int\frac{d^4q}{(2\pi)^4}\,\frac{q^2}{(q^2-2q\cdot k+s-i\epsilon)}=\frac{i(2k^2-s)}{2(2\pi)^2}\left\{\Lambda^2-(s-k^2)\ln\left[\frac{2\Lambda}{(s-k^2)^{1/2}}\right]\right\}.$$

Now differentiate both sides with respect to k:

$$\int\frac{d^4q}{(2\pi)^2}\,\frac{q_\alpha q^2}{(q^2-2q\cdot k+s-i\epsilon)^2}=\frac{ik_\alpha}{(2\pi)^2}\left\{(3k^2-2s)\ln\left[\frac{2\Lambda}{(s-k^2)^{1/2}}+k^2+\Lambda^2-\tfrac{1}{2}s\right]\right\}.$$

This is one of many integrals which we shall require to evaluate $\Sigma_{\rm I}$ and $\Sigma_{\rm II}$. The others are obtained in a similar manner, with the following results:

$$\Sigma_{\rm I}=\frac{iM_e}{128\pi^2}\left[30\ln\left(\frac{4\Lambda^2}{M_W^{\,2}}\right)+\frac{8\Lambda^2}{M_W^{\,2}}-11\right],$$

$$\Sigma_{\rm II}=\frac{M_e i\gamma\cdot p_2}{16\pi^2}\left[\frac{25}{12}\ln\left(\frac{4\Lambda^2}{M_W^{\,2}}\right)-\frac{233}{72}+\frac{2\Lambda^2}{M_W^{\,2}}\right]+\frac{1}{16\pi^2}\left[2\Lambda^2-M_W^{\,2}\ln\left(\frac{4\Lambda^2}{M_W^{\,2}}\right)\right]+O(M_e^{\,2}),$$

where we have made the approximation $k^2\ll M_W^{\,2}$ and neglected those terms containing $M_e^{\,2}$.

APPENDIX B: DIFFERENTIAL RATE

The amplitude corresponding to the interference of Fig. 1(b) and Fig. 2 is

$$\begin{aligned}A=\tfrac{1}{2}\alpha G^2\mathfrak{A}(\Lambda^2)[&\bar{\nu}(k_2)\gamma_\lambda(1+\gamma_5)u(p_2)\bar{\nu}(k_1)\gamma_\lambda(1+\gamma_5)\\&\times u(p_1)M_\mu\bar{u}(p_1)(1+\gamma_5)\nu(k_2)\\&\times\bar{u}(p_2)i\gamma\cdot p_1(1+\gamma_5)\nu(k_2)]\;.\end{aligned}$$

Summing over final spins and considering the incident muon to be polarized, we have

$$|A|^2 = \frac{\alpha G^2 \mathfrak{A}(\Lambda^2)}{2\pi} \sum_{i=1}^{10} A_{\alpha\beta}^{(i)} B_{\alpha\beta} ,$$

where

$$B_{\alpha\beta} = k_{1\alpha} k_{2\beta}$$

and

$$A_{\alpha\beta}^{(1)} = M_\mu{}^2 P_{2\alpha} P_{1\beta} ,$$
$$A_{\alpha\beta}^{(2)} = M_\mu{}^2 P_{1\alpha} P_{2\beta} \delta_{\alpha\beta} ,$$
$$A_{\alpha\beta}^{(3)} = -M_\mu{}^2 P_{1\alpha} P_{2\beta} ,$$
$$A_{\alpha\beta}^{(4)} = M_\mu (P_1 \cdot P_2) P_{1\beta} \hat{S}_\alpha ,$$
$$A_{\alpha\beta}^{(5)} = -2 M_\mu (P_1 \cdot P_2) P_{1\alpha} \hat{S}_\beta ,$$
$$A_{\alpha\beta}^{(6)} = M_\mu{}^3 P_{2\alpha} \hat{S}_{\alpha\beta} \delta_{\alpha\beta} ,$$
$$A_{\alpha\beta}^{(7)} = -M_\mu{}^3 P_{2\alpha} \hat{S}_\beta ,$$
$$A_{\alpha\beta}^{(8)} = M_\mu \epsilon_{\lambda\mu\nu\beta} P_{1\alpha} \hat{S}_\lambda P_{2\mu} P_{1\nu} ,$$
$$A_{\alpha\beta}^{(9)} = -M_\mu \epsilon_{\lambda\mu\nu\beta} P_{1\alpha} P_{2\lambda} P_{1\mu} \hat{S}_\nu ,$$
$$A_{\alpha\beta}^{(10)} = M_\mu (P_1 \cdot P_2) \epsilon_{\alpha\mu\nu\beta} P_{1\mu} \hat{S}_\nu ,$$

where $\hat{S}$ is the (unit) polarization four-vector of the muon.

Integrating over the two neutrino momenta, we obtain

$$\int B_{\alpha\beta} \frac{d^3k_1 d^3k_2 \delta^{(4)}(k_1 - k_2 - q)}{\omega_1 \omega_2} = \tfrac{1}{6}\pi (q^2 \delta_{\alpha\beta} + 2 q_\alpha q_\beta),$$

where

$$\omega_1 = |\vec{k}_1|, \quad \omega_2 = |\vec{k}_2|, \quad q = P_1 - P_2 .$$

We obtain

$$|A|^2 = \frac{\alpha G^2 \mathfrak{A}(\Lambda^2)}{2\pi} \{ \tfrac{1}{2}\pi M_\mu{}^2 (P_1 \cdot P_2) q^2 - (P_1 \cdot q) - \tfrac{1}{3}\pi M_\mu (P_1 \cdot P_2)(P_1 \cdot q)(\hat{S} \cdot q) + \tfrac{1}{3}\pi M_\mu{}^3 [(P_2 \cdot \hat{S}) q^2 - (P_2 \cdot q)(\hat{S} \cdot q)] \}$$

using

$$S_4 = \frac{\vec{P}_1 \cdot \hat{S}}{M_\mu} ,$$
$$\vec{s} = \vec{S} + \frac{(\vec{P}_1 \cdot \vec{S}) \vec{P}_1}{2 M_\mu{}^2} ,$$

where $\vec{S}$ is the unit polarization vector in the frame in which the muon is at rest, and evaluating the expression in the c.m. frame of the muon we have

$$|A|^2 = \tfrac{1}{2} G^2 \mathfrak{A}(\Lambda^2) M_\mu{}^3 \{ \tfrac{1}{2} E_e M_\mu (M_\mu - 2E_e) + \tfrac{1}{3} P_e \cos\theta [M_\mu (M_\mu - 2E_e) + E_e (E_e - 2M_\mu)] \} ,$$

where P_e is the electron three-momentum and E_e is its energy. The angle θ is the angle between the muon polarization vector and the electron momentum. Letting $W = \frac{1}{2} M_\mu$ be the electron energy, we can form the differential rate

$$dW = \frac{\alpha \mathfrak{A}(\Lambda^2) G^2 M_\mu P_e E_e \, dE_e \, d(\cos\theta)}{(2\pi)^3} \times \left\{ (W - E_e) - \frac{M_e{}^2}{M_\mu{}^2} (W - E_e) + \frac{1}{3} \frac{P_e}{E_e} \cos\theta \left[2(W - E_e) - \frac{E_e}{M_\mu} (4W - E_e) \right] \right\},$$

which is Eq. (2.7) of the text.

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[5]The procedure here is analogous to the magnetic moment model for spin-1 particles proposed by T. D. Lee [Phys. Rev. 140, B967 (1965)].

[6]Strictly speaking the Lorentz condition should be understood in the sense of an expectation value, viz., $\langle q_\mu A_\mu(q) \rangle = 0$. The choice of Lorentz gauge requires us to add gauge terms to the photon propagator, i.e.,

$$\Delta_{\mu\nu}(q^2) = -i\,\delta_{\mu\nu}/q^2 \rightarrow \frac{-i}{q^2}\left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right).$$

However, since all our currents are conserved (K_μ, J_μ, the nucleon current) even off the mass shell, these gauge terms do not contribute since $q_\mu J_\mu^\gamma = q_\mu K_\mu^\gamma = 0$.

[7]In the $q^2 \rightarrow 0$ limit, the longitudinal and timelike photons just cancel. Hence, the covariant treatment of A_μ for virtual photons is led back to just the usual transverse polarizations of a real photon.

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