Chiral Symmetry Breaking, Partial Conservation of Axial-Vector Currents, and the σ Commutator in the Kaon-Nucleon System*

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Chiral-symmetry-breaking effects are studied in the kaon-nucleon system, and discussed and compared with various symmetry-breaking schemes. We first derive a relation between low-energy parameters of the kaon-nucleon scattering amplitude and s- and p-wave scattering lengths, where contributions from $\bar{K}N$ unphysical regions and nearby singularities are calculated from field theory. This sum rule is nearly saturated. Together with consistency conditions of the isospin-even amplitude, the nucleon expectation value of the σ commutator is expressed in terms of s- and p-wave scattering lengths and a rather well-known integral over $K^{\pm}N$ cross sections. Although definitely not compatible with the recent estimate of Cheng and Dashen, our result for the magnitude of the σ term is in agreement with most of the calculations for the πN system and favors the (conventional) $(3,\bar{3})+(\bar{3},3)$ breaking scheme of SU(3) $\times$ SU(3) of Gell-Mann, Oakes, and Renner. The same conclusion has been reached by using a dispersive approach. Since these σ terms turn out to be slightly larger (by about a factor of 2) than conventional $(3,\bar{3})$ estimates, mechanisms and models are discussed in order to explain this (possible) enhancement. Nonsmooth higher-order effects due to the Λ' (1520) are found to be small, suggesting that even in the kaon-nucleon system second-order effects in chiral symmetry breaking, i.e., $O(m_K{}^4)$ can be neglected. Finally, PCAC (partial conservation of axial-vector current) for kaons has been directly compared with experiment using most recent phase-shift analyses, and its compatibility with the data has been confirmed, as suggested by generalized Goldberger-Treiman relations, contrary to previous calculations.

I. INTRODUCTION

For some time it has been apparent[1] that strong interactions are approximately SU(3)-symmetric. More recently, Gell-Mann suggested[2] that strong interactions are nearly symmetrical under the bigger group SU(3)$\times$SU(3), generated by the algebra of the vector and axial-vector currents of the hadrons. This larger symmetry does not, however, manifest itself only in multiplets of particles as does SU(3), but also through the appearance[3,4] of eight nearly zero-mass pseudoscalar mesons (Goldstone bosons in the exact symmetry limit). The idea that strong interactions are almost SU(3)$\times$SU(3)-symmetrical appears to be the only rational way in which one can understand the joint successes of current algebra and partially conserved axial-vector current (PCAC).[5] In addition, there is good evidence that the weak and electromagnetic currents of the hadrons generate the algebra of SU(3)$\times$SU(3); the hypothesis that the strong interactions are nearly invariant under this algebra clearly provides a beautiful connection between the symmetry of hadrons and their weak and electromagnetic interactions.

The combination of current algebra and PCAC leads to a large number of low-energy theorems[5] for processes involving soft pions and kaons. (Chiral symmetry does, in fact, have implications[3] other than soft-meson theorems.) These low-energy theorems, which relate the symmetry-breaking part of the total Hamiltonian to the scattering amplitude of zero-mass particles, are only approximate in the real world and would become exact in a limit where the pseudoscalar-meson masses vanish and the axial-vector currents are conserved. Thus, the soft-meson theorems may be thought of as consequences of approximate symmetry, which has been especially stressed by Weinberg.[6] Most important tests of theories of chiral symmetry breaking come therefore from low-energy theorems of, especially, meson-baryon scattering. (Whereas accepted current algebra predicts the low-energy values of the crossing-odd amplitudes, chiral symmetry breaking predicts the low-energy values of the crossing-even ones.) A detailed study of these low-energy theorems, namely the calculation of the meson-baryon σ terms, i.e., the nucleon expectation value of the equal-time commutator of the axial-vector current with its divergence, provides us crucial information about the chiral-symmetry-breaking mechanism, to what extent chiral symmetry must be broken and what symmetry-breaking models should be used. In addition, a reliable evaluation of the σ terms takes on further importance, as it may be useful in providing an understanding of the mechanism by which scale invariance is broken.[7]

Since low-energy theorems are valid outside the physical energy region and for zero-mass mesons, it is certainly not a trivial problem how to extrapolate off the mass shell and then to physical situations. In the case of pion-nucleon scattering, various techniques have been used in order to calculate the πN σ term, like off-mass-shell dispersion relations,[8] broad-area subtracted dispersion relations,[9] and threshold subtracted fixed-t dispersion relations[10] by making use of Adler zeros in the context of a systematic expansion in orders of chiral symmetry breaking, linear expansions[11] of πN amplitudes, sum rules[12] based on scattering lengths and Weinberg's smoothness hypothesis, and speculations originating in light-cone expansions.[13] In addition, the magnitude of the πN σ term has been extracted[14] from a study of π-nuclei interactions in π-mesonic atoms. Most of these calculations approximately agree with the $(3,\bar{3})+(\bar{3},3)$ chiral-symmetry-breaking scheme of Gell-Mann, Oakes, and Renner[15] (GMOR), and therefore give support to the notion that chiral SU(2)×SU(2) is in fact a better symmetry than SU(3). However, Cheng and Dashen,[9] and also the rather strongly model-dependent light-cone approach,[13] obtained a value for the pion-nucleon σ term roughly five times larger, indicating that the $(3,\bar{3})+(\bar{3},3)$ model might not be correct (at least with its conventional interpretation). Thus, the whole question appears to be still open.

However, very recently, several attempts have been made to calculate the σ term for the exceedingly more complicated reaction of kaon-nucleon scattering. Various independent methods have been used, like fixed-t dispersion relations using a first-order expansion in chiral symmetry breaking,[16] off-shell finite-energy sum rules,[17] and sum rules based on s- and p-wave scattering lengths;[18] all these estimates yield compatible results and strongly favor the GMOR model for chiral SU(3) ×SU(3) breaking. However, they are definitely inconsistent with the rather large values obtained in Refs. 9 and 13.

In a previous paper[18] we have considered the problem of how to extrapolate the kaon-nucleon scattering amplitude to the low-energy point by making full use of the smoothness hypothesis. Having calculated the discontinuities in the $\bar{K}N$ channel from field theory, we constructed an amplitude which can be expected to be a smooth function of all its energy and momentum variables. We assumed an expansion of this amplitude up to second order in kaon momenta; this expansion contains four unknown parameters. Together with the dispersive calculation of Ref. 16, one has then five independent conditions on these parameters, the others being (i) the Adler consistency condition (PCAC), (ii) the value of the amplitude at physical threshold, (iii) the value of the subtraction constant in forward dispersion relations, and (iv) a combination of s- and p-wave scattering lengths. Indeed, the resulting constraint for the low-energy expansion parameters is in remarkable agreement with experiment. Independently of Ref. 16 we also related the kaon-nucleon σ term to s- and p-wave scattering lengths and to a rather well-known integral over total $K^{\pm}N$ cross sections by keeping only second-order terms in kaon momenta, i.e., first-order terms in chiral symmetry breaking. These considerations are the starting point of the present paper.

Here, we pursue the idea of studying low-energy kaon-nucleon scattering and of determining the σ term by making full use of the smoothness hypothesis for the scattering amplitude. This possibility was already contained in our previous paper. However, in order to obtain an accurate determination of the σ term, it might be necessary to go beyond the simple second-order expansion for the scattering amplitude and to take into account possible sources of nonsmoothness arising from the presence of nearby singularities. In addition, these higher-order corrections could be particularly important in the kaon-nucleon system, contrary to the πN case, because of the rather large kaon mass. A reliable estimate of these effects can be given, and the only non-negligible contribution is due to the $\Lambda'(1520)$. Since this contribution is rather small in magnitude, about 5%, our results suggest that extrapolation techniques neglecting[16,18] terms proportional to m_K^4 are accurate enough to obtain reliable results – an approach which works very well for πN scattering[9,10,12] where contributions proportional to m_π^4 might be safely neglected *a priori*.

A basic assumption of all such calculations, for pion-nucleon as well as kaon-nucleon scattering, is the PCAC hypothesis. Some time ago the validity of PCAC for kaons was questioned[19] on the basis of rather incomplete experimental information. We recalculated Adler's consistency relation for kaons, using a subtracted fixed-t dispersion relation and most recent results of practically all existing kaon-nucleon phase-shift analyses as input, and found that kaon PCAC is certainly compatible with experiment – a result one might expect on the basis of generalized Goldberger-Treiman relations.[20]

For the sake of clarity we briefly summarize in Sec. II all relevant notations, partial-wave decompositions, and dispersion relations we use. In Sec. III, we discuss the low-energy theorem for kaon-nucleon scattering and give a brief outline of the $(3,\bar{3})+(\bar{3},3)$ chiral-symmetry-breaking model and the magnitudes for the various σ terms expected on purely theoretical grounds. In the first part of Sec. IV we describe the dispersive approach of how to calculate the kaon-nucleon σ term, whereas in the second part we study low-energy $K^{\pm}N$ scattering and calculate the σ term using the smoothness hypothesis; discontinuities in the $\bar{K}N$ channel are calculated from field theory and higher-order corrections from nearby singularities are studied. In the last part of Sec. IV our results are compared with other calculations and we discuss the implications for chiral-symmetry and scale breaking. Finally, in Sec. V, the PCAC hypothesis for kaons, which constitutes a basic assumption of our paper, is compared with experiment, and our conclusions are summarized in Sec. VI.

II. KINEMATICS, PARTIAL-WAVE DECOMPOSITIONS, AND DISPERSION RELATIONS

Consider the process $K^{\pm}(q)+N(p)\rightarrow K^{\pm}(q')+N(p')$ with four-momenta of the particles indicated in parentheses. The T matrix can be conventionally decomposed into

$$T_{fi}=A+\tfrac{1}{2}\gamma\cdot(q+q')B\,, \tag{1}$$

where the two invariant amplitudes A and B are chosen to be scalar functions of the kinematic invariants

$$\begin{aligned}\nu&=(p+p')\cdot(q+q')/4m_N\\&=\omega+t/4m_N\,,\\t&=(q-q')^2\end{aligned} \tag{2}$$

and ω denotes the total laboratory energy of the incoming kaon. To specify the various charge states, the amplitudes A and B are decomposed into

$$A(\nu,t)=A^{+}(\nu,t)+(\vec{\tau}_N\cdot\vec{\tau}_K)A^{-}(\nu,t) \tag{3}$$

and similarly for $B(\nu,t)$, where $\vec{\tau}_N$ and $\vec{\tau}_K$ are the isospin matrices for the nucleon and kaon, respectively. With respect to crossing ($\nu\rightarrow-\nu$, t fixed), A^{+} and B^{-} are even functions, whereas A^{-} and B^{+} are odd. In the s channel, these amplitudes are related to the amplitudes for a definite isospin state $I=0,1$ by

$$\begin{aligned}A^{+}&=\tfrac{1}{4}(A^0+3A^1)\,,\\A^{-}&=\tfrac{1}{4}(A^1-A^0)\,,\end{aligned} \tag{4}$$

and similarly for $B^{\pm}$. Therefore the amplitudes for $K^{\pm}N$ scattering are given by

$$\begin{aligned}A_{\pm}&=A^{+}\pm A^{-}\,,\\B_{\pm}&=B^{+}\pm B^{-}\,.\end{aligned} \tag{5}$$

The T-matrix normalization is chosen such that the differential cross section in the c.m. system is given by

$$\frac{d\sigma}{d\Omega}=\left(\frac{m_N}{4\pi W}\right)^2\sum|\bar{u}(p')T_{fi}u(p)|^2\,, \tag{6}$$

where $W=\sqrt{s}$, $s=(p+q)^2$, and $\sum$ denotes sum and/or average over nucleon spins, according to which differential cross section is being measured.

The invariant amplitudes may be decomposed into partial-wave amplitudes by

$$\begin{aligned}\frac{A(\nu,t)}{4\pi}&=\frac{W+m_N}{E+m_N}f_1-\frac{W-m_N}{E-m_N}f_2\,,\\\frac{B(\nu,t)}{4\pi}&=\frac{1}{E+m_N}f_1+\frac{1}{E-m_N}f_2\,,\end{aligned} \tag{7}$$

where E is the total c.m. energy of the nucleon and

$$\begin{aligned}f_1&=\frac{1}{k}\sum_l[f_{l+}P_{l+1}'(x)-f_{l-}P_{l-1}'(x)]\,,\\f_2&=\frac{1}{k}\sum_l(f_{l-}-f_{l+})P_l'(x)\,,\end{aligned} \tag{8}$$

with $P_l'=dP_l/dx$, $x=\cos\theta$, and k and θ being the c.m. momentum and scattering angle, respectively. The partial waves and phase shifts corresponding to total angular momentum $j=l\pm\frac{1}{2}$ are denoted by $f_{l\pm}$ and $\delta_{l\pm}$, respectively, where

$$f_{l\pm}=\frac{1}{2i}(e^{2i\delta_{l\pm}}-1)\,. \tag{9}$$

The subtracted fixed-t dispersion relations for $K^{\pm}N$ scattering may now be written[21] in the form:

$$\begin{aligned}\mathrm{Re}\,T_{\pm}(\nu,t)=&\,\mathrm{Re}\,T_{\pm}(\nu_0,t)\mp(\nu-\nu_0)\sum_{y=\Lambda,\Sigma}\frac{R_y}{(\nu_B+\Delta_y\pm\nu_0)(\nu_B+\Delta_y\pm\nu)}\\&\mp\frac{(\nu-\nu_0)}{\pi}P\int_{\bar{\nu}}^{\nu_0}d\nu'\,\frac{\mathrm{Im}\,T_{-}(\nu',t)}{(\nu'\pm\nu_0)(\nu'\pm\nu)}\mp\frac{(\nu-\nu_0)}{\pi}P\int_{\nu_0}^{\infty}d\nu'\,\frac{\mathrm{Im}\,T_{-}(\nu',t)}{(\nu'\pm\nu_0)(\nu'\pm\nu)}\\&\pm\frac{(\nu-\nu_0)}{\pi}P\int_{\nu_0}^{\infty}d\nu'\,\frac{\mathrm{Im}\,T_{+}(\nu',t)}{(\nu'\mp\nu_0)(\nu'\mp\nu)}\,,\end{aligned} \tag{10}$$

where we have defined the following combination of (on-shell) $K^{\pm}N$ scattering amplitudes:

$$T_{\pm}(\nu, t) = A_{\pm} + \nu B_{\pm} \ , \tag{11}$$

with $\nu_B = -q\cdot q'/2m_N$, $\nu_0 = m_K + t/4m_N$, and

$$R_y = \frac{g_y^2}{2m_N}(\nu_B + \Delta_y + m_N - m_y)\ ,$$
$$\overline{\nu} = \nu_B + [(m_\Lambda + m_\pi)^2 - m_N^2]/2m_N \ , \tag{12}$$

where $\Delta_y = (m_y^2 - m_N^2)/2m_N$ and g_y is the rationalized pseudoscalar coupling constant for the KyN vertex. The imaginary parts of the amplitudes in the dispersion relations of Eq. (10) are then uniquely determined by Eqs. (7), (8), and (9), once the phase shifts $\delta_{l\pm}$ are known.

From Eqs. (7) and (8) the amplitudes $T_{\pm}(\nu, t)$ can now be easily calculated at various energy points; at physical threshold we have

$$T_{\pm}(m_K, 0) = 4\pi\left(1 + \frac{m_K}{m_N}\right) a_{0+}^{\pm}\ , \tag{13}$$

where $a_{l\pm}^{\pm}$ is the scattering length of the lth $K^{\pm}N$ partial wave $f_{l\pm}^{\pm}$, defined by

$$a_{l\pm}^{\pm} = \lim_{k\to 0} f_{l\pm}^{\pm}/k^{2l+1}\ . \tag{14}$$

In addition we will need the value of $T_{\pm}$ at $\nu = m_K + m_K^2/4m_N$. Now for $t = 2m_K^2$,

$$\cos\theta = (m_K^2 + k^2)/k^2\ ,$$

and since at threshold

$$P_l'(x) \underset{x\to\infty}{\sim} \frac{l(2l)!}{2^l(l!)^2}x^{l-1}\ , \tag{15}$$

we obtain, using Eq. (15) in Eqs. (7) and (8),

$$T_{\pm}(m_K + m_K^2/4m_N, 2m_K^2) = 4\pi\left(1 + \frac{m_K}{m_N} + \frac{m_K^2}{4m_N^2}\right)\sum_{l=0}^{\infty} a_{l+}^{\pm}\frac{(l+1)[2(l+1)]!}{2^{l+1}[(l+1)!]^2}m_K^{2l}$$
$$+ 4\pi m_K^2\sum_{l=1}^{\infty}(a_{l-}^{\pm} - a_{l+}^{\pm})\frac{l(2l)!}{2^l(l!)^2}m_K^{2(l-1)}\ . \tag{16}$$

In practice, only s- and p-wave scattering lengths will be included: the only ones that are experimentally rather well known and that are the dominant contributions to Eq. (16).

III. LOW-ENERGY THEOREM AND CHIRAL SYMMETRY BREAKING

Before going into the details of the $(3,\overline{3})+(\overline{3},3)$ chiral-symmetry-breaking model, we briefly discuss the low-energy theorem for kaon-nucleon scattering.

A. Low-Energy Theorem

The basic relation for deriving low-energy theorems is given by

$$\int d^4x d^4y\, e^{iq'x}e^{-iqy}\langle p'|T\{\partial_\mu A_{K^+}^\mu(x)\partial_\nu A_{K^-}^\nu(y)\}|p\rangle = \int d^4x d^4y\, e^{iq'x}e^{-iqy}$$
$$\times\langle p'|(q'_\mu q_\nu T\{A_{K^+}^\mu(x)A_{K^-}^\nu(y)\} + iq'_\mu\delta(x^0 - y^0)[A_{K^+}^\mu(x), A_{K^-}^0(y)]$$
$$- \delta(x^0 - y^0)[A_{K^+}^0(x), \partial_\nu A_{K^-}^\nu(y)])|p\rangle\ , \tag{17}$$

where standard techniques[5] have been used to pull the derivatives through the time-ordered product; $A_a^\mu(x)$ is the axial-vector current, and the PCAC choice for the kaon field is

$$m_K^2 F_K \phi_{K^\pm}(x) = \partial_\mu A_{K^\pm}^\mu(x)\ , \tag{18}$$

with the semileptonic kaon decay constant F_K given by $F_K/F_\pi = 1.26$, corresponding to a Cabibbo angle $\theta_C = 0.26$, and $F_\pi = 96$ MeV. The first term on the right-hand side of Eq. (17) contributes both to the symmetric and to the antisymmetric part of the amplitude with respect to isospin indices, whereas the second term, the equal-time commutator known from current algebra, is antisymmetric in isospin indices and of first order in the kaon momentum. The last term corresponds to the so-called σ term. It is not a commutator which is given by current algebra, and derives its name from the σ model, where it simply reduces to the canonical σ field. In the soft-meson limit $q_\nu \to 0$, $q_\mu' \to 0$, the right-hand side of Eq. (17) reduces to

$$i(2\pi)^4\delta^4(q + p - q' - p')\langle p'|[F_{K^+}^5, [F_{K^-}^5, \epsilon\mathcal{H}']]|p\rangle$$
$$\equiv i(2\pi)^4\delta^4(q + p - q' - p')\sigma_{NN}^{KK}\ , \tag{19}$$

where the axial-vector charges are given by $F_a^5 = \int d^3x\, A_a^0(0,\vec{x})$ and $\mathcal{H}'$ is that part of the total Hamiltonian density which breaks chiral symmetry. As a formal device for keeping track of powers of symmetry breaking, we introduced the "small" scale parameter ϵ. (Of course, this decomposition of the total Hamiltonian into a symmetry-conserving and symmetry-breaking part means nothing until we add to it the assumption that an expansion about the limit $\epsilon \to 0$ makes sense, a hypothesis strongly suggested[4,22] by any Lagrangian field theory.) That σ_{NN}^{KK} is indeed symmetric in the SU(3) indices can easily be seen by writing a Jacobi identity for the double commutator in Eq. (19) and taking into account isospin and hypercharge conservation. Therefore, in the soft-meson limit, we obtain[5] the low-energy theorem

$$T^+(0,0,0,0) = -F_K^{-2}\sigma_{NN}^{KK}\ , \tag{20}$$

where $T^+(\nu, t, q^2, q'^2)$ denotes the isospin (crossing) even off-mass shell amplitude.

Unfortunately, objects like matrix elements of the σ commutator cannot be measured directly, but can be obtained by extrapolation from on-shell scattering amplitudes. Going off the mass shell through a power series expansion in q^2 and q'^2 (which has been used in Refs. 9, 10, and 16), then the Adler consistency conditions (PCAC) for kaons,

$$T^+(0, m_K^2, m_K^2, 0) = T^+(0, m_K^2, 0, m_K^2) = 0\,, \tag{21}$$

imply

$$\begin{aligned} T^+(0,0,0,0) &= -m_K^2(\partial/\partial q^2)T^+(0,0,0,0) + O(\epsilon^2) \\ &= -m_K^2(\partial/\partial q'^2)T^+(0,0,0,0) + O(\epsilon^2)\,, \end{aligned} \tag{22}$$

and it follows that

$$T^+(0, 2m_K^2, m_K^2, m_K^2) = -T^+(0,0,0,0) + O(\epsilon^2)\,, \tag{23}$$

where we have dropped terms of order m_K^4 since they are of order ϵ^2. Thus, provided these higher-order terms in chiral symmetry breaking can be neglected, Eq. (23) offers a unique relation between the off-mass-shell and on-mass-shell amplitudes independent of any (ambiguous) model-dependent off-mass-shell extrapolation procedure. In addition, the point $\nu = 0$, $t = 2m_K^2$ is clearly outside the physical region; it can, however, be reached for instance by a fixed-t dispersion relation. Still, two questions remain open: (i) Is the approximation in Eq. (23) acceptable, i.e., can terms like m_K^4 be safely neglected? (ii) To what extent is Eq. (21) compatible with experiment? We will come back to these points in Secs. IV and V.

B. The $(3,\bar{3})+(\bar{3},3)$ Model

Chiral symmetry is obviously broken by two physical effects: mass splittings within SU(3) multiplets and finite masses of pseudoscalar mesons. Assuming the simplest SU(3)×SU(3) transformation properties for the chiral-symmetry-breaking Hamiltonian density $\mathcal{H}'$, Eq. (19), namely that $\mathcal{H}'$ belongs to the $(3,\bar{3})+(\bar{3},3)$ representation of SU(3) ×SU(3), and if we require that $\mathcal{H}'$ conserves isospin, hypercharge, and parity, the most general form for $\epsilon\mathcal{H}'$ is

$$\epsilon\mathcal{H}' = u_0 + c u_8\,, \tag{24}$$

which has been suggested by Gell-Mann, Oakes, and Renner.[15] The basis of the $(3,\bar{3})+(\bar{3},3)$ representation is spanned by the set of scalar and pseudoscalar operators u_a and v_a $(a = 0, \ldots, 8)$, respectively. The parameter c fixes the relative scale between SU(3) breaking (through u_8) and SU(3)-invariant chiral symmetry breaking (through u_0). Fitting the pseudoscalar meson masses gives

$$\begin{aligned} c &\approx -2\sqrt{2}\, \frac{m_K^2 - m_\pi^2}{2m_K^2 + m_\pi^2} \\ &\approx -1.25\,. \end{aligned} \tag{25}$$

In the limit $m_\pi^2 \to 0$ we have $c = -\sqrt{2}$, i.e., exact SU(2)×SU(2) symmetry since $u_0 - \sqrt{2}\,u_8$ commutes with the vector and axial-vector charges F_a and F_a^5 for $a = 1, 2, 3$. Thus, deviations of c from $-\sqrt{2}$ are a direct measure of SU(2)×SU(2) breaking.

In this model the divergence of the axial-vector current is given by[15]

$$\partial_\mu A_a^\mu = -(\tfrac{1}{3})^{1/2}(\sqrt{2} - \tfrac{1}{2}c)v_a\,, \quad \text{for } a = 4, 5, 6, 7 \tag{26}$$

from which it follows, by calculating the commutator of Eq. (19),

$$\sigma_{NN}^{KK} = \tfrac{1}{3}(\sqrt{2} - \tfrac{1}{2}c)\langle N|\sqrt{2}\,u_0 + \tfrac{1}{2}\sqrt{3}\,u_3 - \tfrac{1}{2}u_8|N\rangle\ . \tag{27}$$

The matrix elements of u_i $(i = 1, \ldots, 8)$ between baryons are fixed by the measured SU(3) mass splittings:

$$\begin{aligned} \langle N|u_3|N\rangle &\approx 40\ \text{MeV}\,, \\ \langle N|u_8|N\rangle &\approx 170\ \text{MeV}\,. \end{aligned} \tag{28}$$

The matrix element $\langle N|u_0|N\rangle$ is not known, but the naive guess would be that its magnitude is similar to that of $\langle N|u_8|N\rangle$. The reason for this is that SU(3) mass splittings are always of the same order as the masses of the pseudoscalar-meson octet. This observation suggests that the strengths of the two symmetry-violating terms are comparable. Since u_0 breaks SU(3)×SU(3) and u_8 breaks SU(3) as well as SU(3)×SU(3), we cannot allow $\langle N|u_0|N\rangle$ to be different by as much as an order of magnitude, say, from $\langle N|u_8|N\rangle$ and still have the two symmetries broken by a comparable amount. (There could be a possible enhancement of $\langle N|u_0|N\rangle$ with respect to $\langle N|u_8|N\rangle$, if one assumes[23] u_0 to be coupled to the Goldstone boson of a further symmetry, namely scale invariance. Although this is an attractive possibility, there is no further hard experimental evidence for such a so-called "dilaton" and for the moment we have no *a priori* reason to consider such a situation.) Therefore, one obtains the following estimate for Eq. (27):

$$|\sigma_{NN}^{KK}| \approx 100 \text{ to } 200 \text{ MeV}. \qquad (29)$$

Since we will also compare our results with those for πN scattering, we briefly outline the prediction of the GMOR model for this case. The divergence of the axial-vector current for the πN system is given by

$$\partial_\mu A_a^\mu = -\frac{1}{\sqrt{3}}(\sqrt{2}+c)v_a \quad \text{for } a=1,2,3 \qquad (30)$$

and therefore the πN σ term takes the form

$$\sigma_{NN}^{\pi\pi} = \tfrac{1}{3}(\sqrt{2}+c)\langle N|\sqrt{2}\,u_0+u_8|N\rangle. \qquad (31)$$

Using the above estimates for the expectation values of u_0 and u_8, we obtain

$$|\sigma_{NN}^{\pi\pi}| \approx 10 \text{ to } 20 \text{ MeV}. \qquad (32)$$

It should be noted that the estimate of σ_{NN}^{KK}, Eq. (29), seems to be more reliable than in the case of πN scattering, Eq. (32), where $\sigma_{NN}^{\pi\pi}$ is proportional to $(\sqrt{2}+c)$, which is very sensitive to slight variations of the negative number c.

IV. CALCULATIONS OF THE σ TERM

We will consider two methods for calculating the magnitude of the σ commutator: One is a dispersive approach and the other one is an expansion of the scattering amplitude in terms of kinematic invariants, in which full use is made of Weinberg's smoothness hypothesis and where the $\bar{K}N$-channel discontinuities are calculated from field theory; in this way we also can study higher-order contributions, i.e., terms proportional to m_K^4, etc.

A. Dispersive Approach

For the time being we neglect higher-order terms in chiral symmetry breaking, which means that Eq. (23) is the correct relation between an on-shell and an off-shell amplitude. In order to reach the unphysical (but on-mass-shell) point $\nu=0$, $t=2m_K^2$, we will use a fixed-t dispersion relation. From Eq. (10), using Eq. (5), we get for $\nu=0$, $t=2m_K^2$,

$$T^+(0,2m_K^2) = \mathrm{Re}\,T^+(\nu_0,2m_K^2) - \nu_0^2\sum_{y=\Lambda,\Sigma}\frac{g_y^2}{4m_N^2}\,\frac{(m_y-m_N)^2}{\Delta_y(\Delta_y^2-\nu_0^2)} - \frac{2\nu_0^2}{\pi}P\int_{\nu_0}^{\infty}d\nu'\,\frac{\mathrm{Im}\,T^+(\nu',2m_K^2)}{\nu'(\nu'^2-\nu_0^2)}$$
$$-\frac{\nu_0^2}{\pi}P\int_{\bar{\nu}}^{\nu_0}d\nu'\,\frac{\mathrm{Im}\,T_-(\nu',2m_K^2)}{\nu'(\nu'^2-\nu_0^2)}, \qquad (33)$$

where, as in Sec. II, we no longer display the q^2, q'^2 dependence of an on-shell amplitude. The subtraction constant in Eq. (33) is given by Eq. (16); using recently determined[21,24] s- and p-wave scattering lengths, we obtain

$$\mathrm{Re}\,T^+(\nu_0,2m_K^2) = (8.9\pm3.9)m_K^{-1}. \qquad (34)$$

The KN coupling constants in the Born terms of Eq. (33) are taken to be

$$g_\Lambda^2/4\pi = 5.0\pm1.9, \qquad g_\Sigma^2/4\pi = 1.0\pm1.5. \qquad (35)$$

Using recent kaon-nucleon phase-shift analyses, feeding the various partial waves $f_{l\pm}$ into Eq. (8), and with the help of Eq. (7), the physical region integral in Eq. (33) can be easily calculated.[25] The s-wave part of the unphysical integral [last term in Eq. (33)] was evaluated using the K-matrix solution of Martin and Sakitt,[26] continued below the $\bar{K}N$ threshold. The p-wave unphysical region is assumed to be dominated by the $Y_1^*(1385)$ resonance, and here the narrow-width approximation was used:

$$\mathrm{Im}(f_{1+}/k)_{Y_1^*} = \tfrac{1}{2}\pi\alpha\,\frac{m_{Y_1^*}}{m_N}\,\delta(\nu-\nu_R), \qquad (36)$$

with

$$\alpha = \frac{g_{Y_1^*}^2}{4\pi}\,\frac{(m_N+m_{Y_1^*})^2-m_K^2}{3m_{Y_1^*}^2}\left(\frac{k_R}{m_N}\right).$$

The c.m. momentum at the Y_1^* resonance is denoted by k_R, and the $\bar{K}Y_1^*N$ coupling is taken to be

$$g_{Y_1^*}{}^2/4\pi = 1.2 \pm 0.6\,, \tag{37}$$

which is smaller by a factor 2–3 than its SU(3) value ($g_{Y_1^*}{}^2/4\pi \approx 2.4$), as suggested[21] by high-energy photoproduction of $Y_1^*(1385)$. With these input data, we obtain[16] from Eq. (33)

$$T^+(0, 2m_K{}^2) = (18.2 \pm 5.5)\, m_K{}^{-1}\,, \tag{38}$$

where practically all recent kaon-nucleon phase-shift analyses have been used. Equation (38) together with Eqs. (20) and (23) yields

$$\sigma_{NN}^{KK} = 540 \pm 160 \text{ MeV}\,. \tag{39}$$

For calculating the subtraction constant in Eq. (34), we have used a positive value for the real part of the $p_{3/2}$ K^-p scattering length in the isospin $I=1$ channel, which is the favored solution of most of the $\bar{K}N$ phase-shift analyses done up to now.[21] However, changing the sign of this scattering length only decreases the value of σ_{NN}^{KK} given in Eq. (39); we will come back to this point later.

In spite of the extrapolation from $t=0$ to $t=2m_K{}^2$, Eq. (39) shows that the errors in the partial waves are still kept within tolerable limits when extrapolated to the unphysical region. Because of this rather long-range extrapolation one might argue that our results strongly depend on the extrapolation procedure used. This is, however, not the case, as one can see from the partial-wave decomposition in Eq. (8): At low energies the main contributions are coming from s and p waves; the $s_{1/2}$ and $p_{1/2}$ contributions are independent of the extrapolation procedure, whereas the $p_{3/2}$ term depends only linearly on $\cos\theta$. Another possible enhancement in the t-channel extrapolation could come from $(K\bar{K})$ poles. However, we do not have any experimental evidence[27] that the $\epsilon(700)$, say, is coupled to the $K\bar{K}$ channel, giving rise to a significant contribution at $t=2m_K{}^2$.

The two basic assumptions we have used for estimating the σ commutator are (i) the PCAC condition [Eq. (21)] and (ii) the validity of Eq. (23), namely that higher-order terms in chiral symmetry breaking are negligible compared to first-order terms. That (i) is compatible with experiment will be shown in Sec. V, whereas higher-order terms are also calculated in Sec. IV B and are found to be rather unimportant.

B. Low-Energy $K^{\pm}N$ Scattering, Smoothness Hypothesis, and Higher-Order Corrections

Since we are interested in the (smooth) behavior of the scattering amplitude in the neighborhood of the Weinberg point $\nu = t = q^2 = q'^2 = 0$, we first have to construct (as far as possible) a smoothly varying amplitude in this region. In order to do this we must subtract from T^+ the Born terms and the various nonsmooth contributions of the unphysical regions in the $\bar{K}N$ channel. Taking this into account we define

$$F(\nu, t, q^2, q'^2) = \text{Re}\,T^+ - T_B^+ - T_{Y_0^*}^+ - T_{Y_1^*}^+\,, \tag{40}$$

where for our purpose we only consider the real parts of the amplitudes. [All quantities in Eq. (40) are assumed, where not explicitly stated, to represent the real parts of the appropriate amplitudes.] In Eq. (40) we did not include possible t-channel contributions since in our case, as mentioned above, there is no direct experimental evidence for such effects.

The Born terms in Eq. (40) are given by

$$T_B^+ = \sum_{y=\Lambda,\Sigma} \frac{g_y{}^2}{2m_N}\left[\frac{\nu_B^y(\nu_B^y + m_N - m_y)}{(\nu_B^y)^2 - \nu^2} + \frac{m_N - m_y}{m_N + m_y}\right], \tag{41}$$

where $\nu_B = (t - q^2 - q'^2)/4m_N$, $\nu_B^y = \nu_B + \Delta_y$, and $g_y{}^2$ is given by Eq. (35). The (nonsmooth) unphysical regions in the $\bar{K}N$ channel are dominated by the $I=0$ s-wave $Y_0^*(1405)$ and by the $I=1$ p-wave $Y_1^*(1385)$. Those two contributions are assumed to be described by effective Lagrangians where, in the gradient coupling theory, we have for the $Y_0^*(1405)$

$$\mathcal{L}_{Y_0^*} = \bar{g}_{Y_0^*}\bar{\psi}_{Y_0^*}\gamma^\mu\psi\partial_\mu\phi + \text{H.c.}\,, \tag{42}$$

with $\psi_{Y_0^*}$ representing the spin-$\frac{1}{2}$ Y_0^* field, and ψ and ϕ are the nucleon and kaon fields, respectively. This Lagrangian yields for the isospin-even Y_0^* amplitude in Eq. (40) the following expression:

$$T_{Y_0^*}^+ = \frac{g_{Y_0^*}{}^2}{m_N}\left[\frac{\nu_B^{Y_0^*}(\nu_B^{Y_0^*} + m_N + m_{Y_0^*})}{(\nu_B^{Y_0^*})^2 - \nu^2} + \frac{m_N + m_{Y_0^*}}{m_N - m_{Y_0^*}}\right], \tag{43}$$

with $g_{Y_0^*}{}^2 = \bar{g}_{Y_0^*}{}^2(m_{Y_0^*} - m_N)^2$. The $\bar{K}Y_0^*N$ coupling is taken to be[28,29]

$$g_{Y_0^*}{}^2/4\pi = 0.32 \pm 0.04\,. \tag{44}$$

The $\bar{K}Y_1^*N$ vertex is described by

$$\mathcal{L}_{Y_1^*} = (g_{Y_1^*}/m_N)\bar{\psi}_\mu\psi\partial^\mu\phi + \text{H.c.}\,, \tag{45}$$

where ψ_μ is the spin-$\frac{3}{2}$ Rarita-Schwinger field. We have used the following form for the $Y_1^*(J^P = \frac{3}{2}^+)$ propagator:

$$D_{\mu\nu}^{3/2^+}(m, P) = \frac{\not{P} + m}{P^2 - m^2}\left[g_{\mu\nu} + \frac{1}{3m}(P_\mu\gamma_\nu - \gamma_\mu P_\nu) - \frac{2}{3m^2}P_\mu P_\nu - \tfrac{1}{3}\gamma_\mu\gamma_\nu\right]. \tag{46}$$

It is well known that the spin-$\frac{3}{2}$ propagator is not unique. This nonuniqueness is immaterial as far as our analysis is concerned in that any other possible choice of $D_{\mu\nu}^{3/2^+}$ leads to a Born term which differs from ours by a polynomial in external momenta, which turns out to be entirely negligible and would only amount to a redefinition of the (smooth) amplitude F. The result for $T_{Y_1^*}^+$ is

$$T_{Y_1^*}^+ = \frac{1}{3}\frac{(g_{Y_1^*}/m_N)^2}{(\nu_B^{Y_1^*})^2-\nu^2}\left\{\frac{1}{4m}\left(1+\frac{m_N}{m}\right)(q^2+q'^2)\left[\left(\frac{1}{2}+\frac{m}{m_N}\right)\nu^2-\nu_B\nu_B^{Y_1^*}\right]+\frac{1}{4mm_N}q^2q'^2\left[\left(1+\frac{m_N}{2m}\right)\nu_B^{Y_1^*}+\frac{\nu^2}{2m}\right]\right.$$
$$\left.+\nu_B^{Y_1^*}\left[(m+m_N)\nu_B+\frac{m_N}{2m}\nu^2+\frac{m_N{}^2}{2m^2}(\nu^2+\nu_B{}^2)\right]+\frac{m_N}{2m^2}\nu^2\left[\nu^2-\nu_B{}^2+\nu_B\left(\frac{2m^2}{m_N}-m_N-m\right)\right]\right\}, \tag{47}$$

where m denotes the Y_1^* mass and $g_{Y_1^*}{}^2$ is given by Eq. (37).

Besides those two $\Lambda\pi$ and $\Sigma\pi$ discontinuities below the physical $\bar{K}N$ threshold, there are two additional three-particle channels open: the $\Lambda\pi\pi$ and $\Sigma\pi\pi$ just below and above threshold, respectively. However, close to threshold such final states are experimentally strongly suppressed and the measured branching ratios for those decays are small compared to the two-particle final states.[30] It is therefore reasonable to assume that such small three-body final-state contributions (if they are important at all) are included and well accounted for in our amplitude F defined in Eq. (40). Thus, the amplitude F is expected to be a smooth function in all its arguments. Finally, the amplitudes in Eqs. (41), (43), and (47) vanish at the Weinberg point and fulfill Adler's PCAC condition, Eq. (21), as a natural consequence of the gradient coupling theory.

Following Weinberg's original suggestion[31] for low-energy πN scattering, which has been recently applied by Altarelli, Cabibbo, and Maiani[12] in connection with the study of the πN σ commutator, and taking into account the symmetry properties of F, we can write:

$$F(\nu,t,q^2,q'^2)=A\,m_K{}^2+Bt+C(q^2+q'^2)+D\nu^2+R(\nu,t,q^2,q'^2), \tag{48}$$

where R measures the deviations of F from linearity and, of course, it has to vanish together with its first derivatives at the Weinberg point. These deviations can occur due to the presence of nearby singularities in each of the variables ν, t, q^2, and q'^2. It will turn out that the only non-negligible contribution in the $\bar{K}N$ channel comes from the $\Lambda'(1520)$ which lies closest to physical threshold. In the t channel, a possible contribution could come from the $\epsilon(700)$. However, as we have pointed out previously, no experimental evidence[27] exists for a coupling of the $\epsilon(700)$, if it exists at all, to the $(K\bar{K})$ channel giving a significant contribution in the region between $t=0$ and $2m_K{}^2$. Even in the case where the $\epsilon(700)$ was important, its contribution could well be included in the linear part of Eq. (48), since it is supposed to be a very broad, backgroundlike "resonance." It is difficult to say anything reliable about possible enhancements in the q^2 and q'^2 channels, but compared to the rather dominant effects of the $\Lambda'(1520)$, they should not be of substantial importance.

The next step is to find four equations (consistency conditions) for the four low-energy expansion parameters A, B, C, and D. If we expand F in powers of k^2 and $\cos\theta$ around the physical threshold and compare the coefficients of $k^2\cos\theta$, we obtain a relation between the low-energy parameters and a linear combination of kaon-nucleon s- and p-wave scattering lengths:

$$\frac{1}{4\pi}\left(2B+\frac{m_K}{m_N}D\right)+C_k+\frac{1}{4\pi}\frac{m_K}{m_N}\frac{\partial R}{\partial\nu^2}(m_K,0,m_K{}^2,m_K{}^2)+\frac{1}{2\pi}\frac{\partial R}{\partial t}(m_K,0,m_K{}^2,m_K{}^2)=X, \tag{49}$$

where

$$C_k=\frac{1}{4\pi}(T_{B,k}^+ + T_{Y_0^*,k}^+ + T_{Y_1^*,k}^+), \tag{50}$$

with $T_{y,k}^+=\partial T_y^+/\partial(k^2\cos\theta)\,|_{k^2\cos\theta=0}$, and

$$X=\frac{1}{16m_N{}^2}(2a_1+b_0+b_1)+\tfrac{1}{4}(2a_{11}+b_{01}+b_{11})+\left(\frac{1}{2}+\frac{3}{4}\frac{m_K}{m_N}\right)(2a_{13}+b_{03}+b_{13}). \tag{51}$$

The KN and $\bar{K}N$ s-wave scattering lengths for a definite isospin channel are given by a_I and $A_I=b_I+ic_I$, respectively, whereas the p-wave scattering lengths are denoted by $a_{I,2J}$ and $A_{I,2J}$

$=b_{I,2J}+ic_{I,2J}$, respectively. The constant C_k can be easily calculated using Eqs. (41), (43), and (47) which yields $C_k=(-0.95\pm0.23)m_K^{-3}$. Even if the effects of nearby singularities are neglected ($R\equiv 0$), the sum rule in Eq. (49) is nearly saturated.[18]

In addition to Eq. (49) we need three further equations for determining the low-energy parameters of F. These can be found from the knowledge of F at the following points: Adler's consistency condition (PCAC),

$$F(0, m_K^2, m_K^2, 0)=m_K^2(A+B+C)+R(0, m_K^2, m_K^2, 0) = 0, \tag{52}$$

and at physical threshold,

$$F(m_K, 0, m_K^2, m_K^2)=m_K^2(A+2C+D) + R(m_K, 0, m_K^2, m_K^2). \tag{53}$$

The left-hand side of Eq. (53) can be calculated using Eqs. (41), (43), and (47), in addition to

$$\mathrm{Re}\,T^+(m_K, 0, m_K^2, m_K^2)=\pi\left(1+\frac{m_K}{m_N}\right)(2a_1+b_0+b_1), \tag{54}$$

which follows from Eq. (13) with the appropriate isospin decomposition. Finally at $\nu=t=0$ we have

$$F(0,0,m_K^2,m_K^2)=m_K^2(A+2C) + R(0,0,m_K^2,m_K^2). \tag{55}$$

Again, it is straightforward to calculate $F(0,0)$ once we know $T^+(0,0)$, where we no longer display the q^2, q'^2 dependence of an on-mass-shell amplitude. This quantity appears as a subtraction constant in a subtracted forward dispersion relation at threshold; from Eq. (10) we get, for $\nu=m_K$, $t=0$,

$$\mathrm{Re}\,T^+(m_K,0)=T^+(0,0)+m_K^2\sum_{y=\Lambda,\Sigma}\frac{g_y^2}{4m_N^2}\,\frac{(m_y-m_N)^2-m_K^2}{\omega_y(\omega_y^2-m_K^2)}+I^+, \tag{56}$$

with

$$I^+=\frac{m_K^2}{\pi}P\int_{\bar\omega}^{m_K}d\omega'\,\frac{\mathrm{Im}\,T_-(\omega',0)}{\omega' k_L'^2}+\frac{m_K^2}{\pi}P\int_{m_K}^{\infty}d\omega'\,\frac{\sigma_++\sigma_-}{\omega' k_L'}, \tag{57}$$

where

$$\omega_y=\Delta_y-m_K^2/2m_N,\; k_L'^2=\omega'^2-m_K^2,$$

and

$$\bar\omega=[(m_\Lambda+m_\pi)^2-m_N^2-m_K^2]/2m_N,$$

and the total $K^\pm N$ cross sections are given by the optical theorem: $\mathrm{Im}\,T_\pm(\omega,0)=k_L\sigma_\pm$. Using the results of Perrin and Woolcock[32] for the forward dispersion integrals,

$$I^+=(-25.7\pm3.2)m_K^{-1}, \tag{58}$$

and for Eq. (54) taking the recent[21,24] s-wave scattering lengths, Eq. (56) yields

$$T^+(0,0)=(-34.9\pm4.6)m_K^{-1}. \tag{59}$$

Solving Eqs. (49), (52), (53), and (55) with respect to A, B, C, and D, one obtains

$$-m_K^2A=\left(1+\frac{m_K}{m_N}\right)F(0,0)-\frac{m_K}{m_N}F(m_K,0)+4\pi m_K^2(X-C_k)+R_\Delta, \tag{60}$$

$$m_K^2B=\tfrac12\left\{4\pi m_K^2(X-C_k)-\frac{m_K}{m_N}[F(m_K,0)-F(0,0)]+R(0,0)-2R(0,m_K^2,m_K^2,0)+R_\Delta\right\}, \tag{61}$$

$$m_K^2C=\left(1+\frac{m_K}{2m_N}\right)F(0,0)-\frac{m_K}{2m_N}F(m_K,0)+2\pi m_K^2(X-C_k)-\tfrac12 R(0,0)+\tfrac12 R_\Delta, \tag{62}$$

$$m_K^2D=F(m_K,0)-F(0,0)+R(0,0)-R(m_K,0), \tag{63}$$

where R_Δ is defined by

$$R_\Delta=\frac{m_K}{m_N}R(m_K,0)-\left(1+\frac{m_K}{m_N}\right)R(0,0)+2R(0,m_K^2,m_K^2,0) -\frac{m_K^3}{m_N}\frac{\partial R}{\partial\nu^2}(m_K,0,m_K^2,m_K^2)-2m_K^2\frac{\partial R}{\partial t}(m_K,0,m_K^2,m_K^2). \tag{64}$$

The amplitude F in Eq. (48) is now completely determined provided we can calculate R. As discussed above the only important contributions are expected to come from $\bar{K}N$ channel resonances. In this case we found that the only non-negligible contribution is due to the $\Lambda'(1520)$. Its pole term can be calculated using the effective Lagrangian

$$\mathcal{L}_{\Lambda'} = (g_{\Lambda'}/m_N)\bar{\psi}_\mu \gamma_5 \psi \partial^\mu \phi + \text{H.c.} \tag{65}$$

Therefore, the propagator of this $J^P = \frac{3}{2}^-$ resonance can easily be related to that of the $\frac{3}{2}^+$ resonance in Eq. (46):

$$\begin{aligned} D^{3/2^-}_{\mu\nu}(m,P) &= \bar{\gamma}_5 D^{3/2^+}_{\mu\nu}(m,P)\gamma_5 \\ &= D^{3/2^+}_{\mu\nu}(-m,P)\,, \end{aligned} \tag{66}$$

where the standard commutation relations for the γ matrices have been used. The $\Lambda'(1520)$ contribution can now be obtained from Eq. (47) by making the substitution $m \to -m$ and, in order to obtain R, we have to subtract from this expression its linear expansion around the Weinberg point, with the final result

$$\begin{aligned} R(\nu,t,q^2,q'^2) = \frac{1}{3}\,\frac{(g_{\Lambda'}/m_N)^2}{(\nu_B^{\Lambda'})^2-\nu^2}\Bigg\{ & \frac{1}{4M}\left(1-\frac{m_N}{M}\right)(q^2+q'^2)\left[\nu_B\nu_B^{\Lambda'}-\left(\frac{1}{2}-\frac{M}{m_N}\right)\nu^2\right]-\frac{1}{4Mm_N}\,q^2q'^2\left[\left(1-\frac{m_N}{2M}\right)\nu_B^{\Lambda'}-\frac{\nu^2}{2M}\right] \\ & +\nu_B^{\Lambda'}\left[\left(\frac{2m_N}{M+m_N}+\frac{m_N^2}{2M^2}\right)\nu_B^2-\frac{m_N}{2M}\left(1-\frac{m_N}{M}\right)\nu^2\right] \\ & +\frac{m_N}{2M^2}\,\nu^2\left[\nu^2-\nu_B^2-\frac{2m_N}{M+m_N}\left[\nu^2-(\nu_B^{\Lambda'})^2\right]+\nu_B\left(\frac{2M^2(M-m_N)}{m_N(M+m_N)}-m_N+M\right)\right]\Bigg\}\,, \end{aligned} \tag{67}$$

where M denotes the $\Lambda'(1520)$ mass. With a partial width for the $\Lambda'(1520)$ of $\Gamma_{\Lambda' \to \bar{K}N} = 7.2 \pm 1.1$ MeV, we have[28] $g_{\Lambda'}^2/4\pi = 0.55 \pm 0.1$ and find for R_Δ of Eq. (64) a value of: $R_\Delta = (-0.2 \pm 0.03)m_K^{-1}$. Using the recent determinations[21,24] of s- and p-wave scattering lengths to evaluate X in Eq. (51), we finally find for the low-energy parameters in Eqs. (60), (61), and (62)

$$-A = (16.4 \pm 3.7)m_K^{-3}\,, \tag{60a}$$

$$B = (9.3 \pm 1.4)m_K^{-3}\,, \tag{61a}$$

$$C = (7.1 \pm 2.3)m_K^{-3}\,. \tag{62a}$$

Equation (63), which does not depend on scattering lengths, yields

$$D = (53.1 \pm 3.0)m_K^{-3}\,. \tag{63a}$$

Calculating A, B, and C, we have used a positive value for b_{13}, the favored solution of most of the $\bar{K}N$ phase shift analyses done up to now.[21]

The low-energy theorem, Eq. (20), together with Eq. (48) tells us that the σ term is directly related to the coefficient A:

$$\sigma_{NN}^{KK} = -F_K^2 m_K^2 A\,, \tag{68}$$

which, by Eq. (60), relates the nucleon expectation value of the σ commutator to s- and p-wave scattering lengths and to a rather well-known integral over total $K^\pm N$ cross sections, where we have made full use of the smoothness hypothesis for F. Equation (68) together with Eq. (60a) gives us

$$\sigma_{NN}^{KK} = 480 \pm 110 \text{ MeV}\,. \tag{69}$$

Although the present method is entirely independent of the dispersive approach[16] of Sec. IV A, the above result agrees very well with the one of Eq. (39). However, like in the dispersive approach, it is very hard to make a reliable estimate of the lower limit in Eq. (69) due to, as mentioned earlier, the possibility[21] of a negative solution for b_{13}; in this case, Eq. (69) would read: $\sigma_{NN}^{KK} = 480^{+110}_{-600}$ MeV. A negative result is at least not favored by our analysis and we regard a negative σ_{NN}^{KK} as unlikely in this context. In this respect much work remains to be done, in that, especially for the p-wave scattering lengths, we are far from having universally accepted values.

In closing this section, we want to comment on the importance of higher-order corrections to chiral symmetry breaking. Comparing Eq. (69) with the calculation of Ref. 18, where we did not consider any nonsmooth contributions from nearby singularities, we find that higher-order corrections due to the $\Lambda'(1520)$ are less than 5%. Similarly, the higher-order corrections to Eq. (23), which is frequently used for dispersive approaches, turn out to be negligibly small. According to Eq. (23), taking into account the R term of Eq. (48), we obtain

$$-F(0,0,0,0) = F(0,2m_K^2,m_K^2,m_K^2) + \Delta\,, \tag{70}$$

with

$$\Delta = 2R(0, m_K^2, m_K^2, 0) - R(0, 2m_K^2, m_K^2, m_K^2).$$

Corrections to Eq. (23) turn out to be rather small: $\Delta \approx 10^{-2} m_K^{-1}$. It therefore appears that the non-smooth contribution due to the $\Lambda'(1520)$ is such that it does not appreciably modify the low-energy theorem if the amplitude is approximated by Eq. (23); higher-order terms (proportional to m_K^4) might be neglected and still obtain reliable results, in spite of the rather large kaon mass compared to m_π. In the case of πN scattering, one might expect such a result *a priori*, because of the exceedingly small factor m_π^4, and it has been explicitly shown by several authors.[12,33]

C. Discussion

On the basis of two entirely different approaches, we found a remarkable agreement of the two results, Eqs. (39) and (69), for the magnitude of the σ commutator. These results compare very well with the one obtained by Köpp, Walsh, and Zerwas[17] using off-mass-shell finite-energy sum rules. Within the quoted uncertainties, these estimates are compatible with Eq. (29), which one expects from a (conventional) $(3,\bar{3})+(\bar{3},3)$ breaking of chiral symmetry. Using Eq. (69) together with Eqs. (27) and (28), we can estimate the nucleon expectation value of u_0,

$$\langle N|u_0|N\rangle \approx 540 \pm 150 \text{ MeV}, \tag{71}$$

and find no significant enhancement with respect to $\langle N|u_8|N\rangle$. A strong enhancement comes about if one assumes[23] u_0 to be coupled to the Goldstone boson of a further symmetry, namely scale invariance: In this case we expect $\langle N|u_0|N\rangle \approx 1500$ MeV, in clear contradiction to Eq. (71).

Using the GMOR model, Sec. III B, we can predict the σ term for πN scattering: Equation (71) together with (28) and (31) yields

$$\sigma_{NN}^{\pi\pi} \approx 50 \pm 15 \text{ MeV}. \tag{72}$$

Although this result is definitely in serious disagreement with $\sigma_{NN}^{\pi\pi} \approx 110$ MeV (which implies $\langle N|u_0|N\rangle \approx 1350$ MeV) found by Cheng and Dashen,[9] it is in excellent agreement with most of the recent calculations[8,10-12,14] for the πN system and is not incompatible with the $(3,\bar{3})$ value in Eq. (32). Therefore, our results together with most of the πN calculations are in favor of the $(3,\bar{3})+(\bar{3},3)$ chiral-symmetry-breaking model, where SU(2) $\times$SU(2) is supposed to be a much better symmetry than SU(3), which suggests the following breaking chain:

$$SU(3)\times SU(3) \rightarrow SU(2)\times SU(2) \rightarrow SU(2), \tag{73}$$

contrary to the symmetry pattern

$$SU(3)\times SU(3) \rightarrow SU(3) \rightarrow SU(2) \tag{74}$$

suggested by the so-called weak pole dominance[13] and (possibly) required by a strongly enhanced $\langle N|u_0|N\rangle$ as the Cheng-Dashen[9] result indicates, provided no *ad hoc* "dilaton" of conformal invariance is assumed[23] to exist.

The relation between broken scale invariance and broken chiral symmetry is also intimately connected to $\langle N|u_0|N\rangle$. Some of these questions have been recently discussed by Renner.[34] Without going into details, such relatively small values of $\langle N|u_0|N\rangle$, as in Eq. (71) for example, indicate that the possibility of a c-number scale breaking, but SU(3)$\times$SU(3) conserving part of the hadronic energy density should be ruled out, provided one does not assume[35] an (*ad hoc*) scalar meson which dominates matrix elements of the trace of the energy-momentum tensor. In addition, Eq. (71) suggests the dimension d of $u_0 + cu_8$ to be $d<3$, and certainly rules out $d=3$ as obtained[23] by using the result of Ref. 9. However, within the framework of Ref. 15, the uniqueness of the dimension of the chiral-symmetry-breaking Hamiltonian is a delicate problem.[34]

Although our results favor, in agreement with several other recent calculations,[8,10-12,14,17] within quoted uncertainties the $(3,\bar{3})+(\bar{3},3)$ scheme for chiral symmetry breaking, they all yield values somewhat larger than the theoretical estimates of the GMOR model. Since *all* those entirely independent calculations produce slightly enhanced results (by about a factor of 2) with respect to the conventional $(3,\bar{3})+(\bar{3},3)$ estimates, it appears to us that this could be something more than just an accidental coincidence; especially if such slightly larger σ terms are unambiguously confirmed by more accurate future experiments (unique values for the various scattering lengths in KN as well as πN scattering). This, however, could mean that further admixtures in the symmetry-breaking Hamiltonian are required in addition to the $(3,\bar{3})$ $+(\bar{3},3)$ transforming part. An interesting attempt in this direction has been made by Sirlin and Weinstein[36] by studying a Hamiltonian belonging to a $(3,\bar{3})+(\bar{3},3)+(8,8)$ representation of SU(3)$\times$SU(3). In the limit where SU(2)$\times$SU(2) is nearly exact, and without requiring a large value for $\langle N|u_0|N\rangle$, they obtain an order of magnitude estimate for $\sigma_{NN}^{\pi\pi}$ in the neighborhood of 50 MeV.

A similar conclusion has been reached by Renner,[37] who estimated the meson-nucleon sigma terms based on the Li-Pagels[38] mechanism of calculating $(3,\bar{3})+(\bar{3},3)$ chiral symmetry breaking. In this model the octet enhancement is achieved by the threshold dominance of Goldstone-boson–pair states and one obtains[37] $\sigma_{NN}^{KK} \approx 400$ MeV. However, this close agreement with the above results could be accidental, since various uncertainties are contained in the estimate of Ref. 37, especially in evaluating dispersion relations for the form factors of the scalar operators u_a.

V. TEST OF KAON PCAC

It remains to be answered if the PCAC condition for kaons, Eq. (21), is indeed compatible with experiment. Confining ourselves to K^+N scattering, PCAC imposes[19] a nontrivial consistency condition on the A_+ amplitude [defined in Eq. (5)]

$$A_+(0, m_K^2, m_K^2, 0) = 0, \tag{75}$$

which, contrary to πN scattering, is a null condition. Some time ago, on the basis of rather incomplete experimental information, Martin[19] calculated the on-shell amplitude $A_+(0, 2m_K^2)$ to be about $-14.4 m_\pi^{-1}$. Giving convincing arguments, he therefore concluded that the off-shell condition in Eq. (75) is in violent disagreement with experiment.

In this section we are going to recalculate the on-shell amplitude A_+ using most recent results of practically all existing kaon-nucleon phase-shift analyses as input for a subtracted fixed-t dispersion relation. Similar to Eq. (10), the subtracted dispersion relation at $\nu = 0$, $t = 2m_K^2$ for A_+ reads

$$A_+(0, 2m_K^2) = A_+(\nu_0, 2m_K^2) - \frac{\nu_0}{2m_N} \sum_{y=\Lambda,\Sigma} \frac{g_y^2(m_y - m_N)}{\Delta_y(\nu_0 + \Delta_y)} - \frac{\nu_0}{\pi} P \int_{\nu_0}^{\infty} d\nu' \frac{\mathrm{Im} A_+(\nu', 2m_K^2)}{\nu'(\nu' - \nu_0)} + \frac{\nu_0}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\mathrm{Im} A_-(\nu', 2m_K^2)}{\nu'(\nu' + \nu_0)} + \frac{\nu_0}{\pi} \int_{\bar{\nu}}^{\nu_0} d\nu' \frac{\mathrm{Im} A_-(\nu', 2m_K^2)}{\nu'(\nu' + \nu_0)}, \tag{76}$$

where $\mathrm{Im} A_\pm(\nu', 2m_K^2)$ is given by the decomposition of Eqs. (7) and (8). Analogously to Eq. (16), the subtraction constant can be written, by keeping s- and p-waves, as

$$A_+(\nu_0, 2m_K^2) = \frac{2\pi}{m_N}(2m_N + m_K)(a_1 + 3m_K^2 a_{13}) - 8\pi m_K m_N (a_{11} - a_{13}), \tag{77}$$

where the p-wave scattering lengths are especially important, since they give a large contribution. With the recently determined[21,24] scattering lengths we obtain $A_+(\nu_0, 2m_K^2) = (7.0 \pm 1.6) m_\pi^{-1}$. Using the same sets of phase-shift analyses as in Ref. 16 for calculating the dispersion integrals in Eq. (76), and for the p-wave unphysical region assuming Eq. (36), we obtain an average value of

$$A_+(0, 2m_K^2) = (-3.4 \pm 1.1) m_\pi^{-1}, \tag{78}$$

compared to $-14.4 m_\pi^{-1}$ obtained by Martin.[19] This large difference in the two results is due to the fact that in Ref. 19 only the s-wave scattering length has been taken into account for calculating the subtraction constant $A_+(\nu_0, 2m_K^2)$, and furthermore, only s waves were used in order to calculate the integrals in Eq. (76), which have been truncated at rather low energies.

Up to this point we have been dealing with the on-shell amplitude. Unfortunately, in the present case, a simple reliable procedure for off-mass-shell extrapolations is not readily available and any attempt would yield rather (unrealistic) strongly model-dependent results. Previously, using a simple model, off-shell effects were explicitly calculated for pion-baryon scattering by several authors[39] and also for the strangeness-changing hyperon decays[40] requiring an extrapolation to zero kaon mass. Since these off-mass-shell effects turned out to be rather small, they could very well account for corrections to $A_+(0, 2m_K^2)$ in order to make Eq. (78) more consistent with (75), without changing our on-shell result in Eq. (78) drastically. [Note that the error estimate in Eq. (78) constitutes a purely statistical one not including any systematic errors.] In addition, the PCAC hypothesis for kaons receives further support by the fact that rather recent estimates[24,40] of the kaon Yukawa-coupling constants and of the semileptonic hyperon decay constants are compatible with generalized Goldberger-Treiman relations.[20]

VI. CONCLUSIONS

In studying chiral-symmetry-breaking effects we derived for low-energy kaon-nucleon scattering a relation between low-energy parameters of the scattering amplitude and s- and p-wave scattering lengths, taking into account nonlinear, higher-order effects due to singularities near threshold in the $\bar{K}N$ channel. Contributions from the unphysical $\bar{K}N$ regions and from resonances lying close to threshold are calculated using gradient-coupling effective Lagrangians. This sum rule based on scattering lengths turns out to be nearly saturated, using recently calculated low-energy parameters and experimentally measured scattering lengths.

Considering the sum rule as an independent equation in addition to the PCAC consistency condition and equations obtained at various energy points of the isospin-even amplitude, we expressed the nucleon expectation value of the σ commutator in terms of s- and p-wave scattering lengths and a rather well-known integral over total $K^{\pm}N$ cross sections, by making full use of Weinberg's smoothness hypothesis. Higher-order corrections, calculated from field theory, due to the $\Lambda'(1520)$ turn out to be rather small, contributing about 5%. The result obtained for the magnitude of the kaon-nucleon σ term is in agreement with most of the calculations (mainly for the πN system) done up to now and favors the $(3,\overline{3})+(\overline{3},3)$ scheme for chiral SU(3)×SU(3) breaking, giving support to the notion that chiral SU(2)×SU(2) is in fact a better symmetry than SU(3). In addition, we present an entirely different approach to calculate the σ term, by using subtracted fixed-t dispersion relations and working to first order in chiral symmetry breaking, and find essentially the same result. However, all these results are definitely not compatible with the rather large value for $\langle N|u_0|N\rangle$ obtained by Cheng and Dashen[9] studying πN scattering.

On rather general grounds, nonsmooth (nonlinear) contributions to the crossing-even amplitude between the Weinberg point and threshold are expected to come mainly from the presence of the $\Lambda'(1520)$ resonance close to threshold. On this basis we found that nonlinear corrections due to the $\Lambda'(1520)$ are such that they do not appreciably modify first-order expansions in kaon momenta squared of the off-shell scattering amplitude, an approximation frequently used in dispersive calculations of low-energy amplitudes (σ terms). It therefore appears that even for the kaon-nucleon system first-order calculations in chiral symmetry breaking (i.e., neglecting terms proportional to m_K^4) are accurate enough to yield reliable results.

Furthermore, the PCAC hypothesis for kaons (a basic assumption in the present approach) has been directly compared with experiment. Contrary to previous calculations, its compatibility with the data is very encouraging, as suggested by generalized Goldberger-Treiman relations.

Although, in agreement with several recent determinations of $\langle N|u_0|N\rangle$, the $(3,\overline{3})+(\overline{3},3)$ chiral-symmetry-breaking scheme is favored, it appears to be more than accidental that the magnitudes of the various σ terms obtained are slightly larger (by about a factor of 2) than the conventional $(3,\overline{3})$ estimates. This indicates that presumably either further admixtures in the symmetry-breaking Hamiltonian are required in addition to the $(3,\overline{3})+(\overline{3},3)$ transforming part, or other mechanisms, which generate, for example, octet enhancements, are necessary in order to calculate symmetry-breaking effects. Recent estimates using a Hamiltonian belonging to a $(3,\overline{3})+(\overline{3},3)+(8,8)$ representation of SU(3)×SU(3) or using the Li-Pagels mechanism for calculating symmetry-breaking effects have confirmed these enhanced σ terms with respect to the conventional $(3,\overline{3})+(\overline{3},3)$ values.

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Lepton Pair Production from Two-Photon Processes*

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An exact expression for the lepton-pair mass spectrum for an α^4 two-photon process in lepton-lepton, lepton-hadron, and hadron-hadron scattering processes is derived. This result is applied to muon pair production in proton-proton scattering to show that such a process is an important background to the α^2 one-photon process in certain energy ranges and can become physically significant by itself at very high energies. The general physical significance of such a two-photon process in hadron-hadron scattering is discussed, and comparison of our exact expression with some approximation schemes is made. The main differences between this work and earlier papers on the subject are that (1) exact calculations are given and (2) the inelastic contributions are included.

I. INTRODUCTION

Recently, lepton pair production in high-energy collisions has been the subject of various studies. The reactions under consideration are of the type

$$a_1(p_1)+a_2(p_2)\to l(l_1)+\bar{l}(l_2)+X, \qquad (1.1)$$

where a_i are the incident particles with momenta p_i, l and $\bar{l}$ are the produced lepton pair with momenta l_i and total invariant mass squared $Q^2=(l_1+l_2)^2$, and X may be either a definite exclusive state or anything inclusive. This type of reaction is important in studying the electromagnetic structure of hadrons and the purely electromagnetic interaction at high energies.

There are two important mechanisms contributing to the reactions (1.1), namely, the α^2 one-photon process and the α^4 two-photon process.