PHYSICAL REVIEW D VOLUME 7, NUMBER 11 1 JUNE 1973

Implications for the $\Delta N\pi$ Interaction and σ Term from Low-Energy πN Scattering*

M. G. Olsson and Leaf Turner
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

E. T. Osypowski
Department of Physics, University of Wisconsin, Marathon County Campus, Wausau, Wisconsin 54401
(Received 29 January 1973)

A model for low-energy πN scattering consisting of N, ρ, Δ, and σ exchanges together with current-algebra constraints is presented. We derive the Δ contributions to the scattering amplitude using a general Δ propagator and $\pi N\Delta$ interaction. We show that the data are consistent with the simplest structure for both the Δ propagator and the interaction. A σ term of 42 ± 10 MeV at the Cheng-Dashen point is found, and predictions of the σ term at $t = 0$, the πNN coupling constant, and the isospin-even s-wave scattering length are made.

I. INTRODUCTION

The current-algebra constraints on the on-mass-shell pion-nucleon scattering amplitude are well known.[1-4] The amplitude is given as a sum of equal-time commutator contributions and an axial-vector–nucleon amplitude contribution. The most appealing and common model for the axial-vector amplitude, that of dominance by baryon and meson exchanges, has in the past involved two difficulties. First, the Δ-exchange contribution is ambiguous because a general Δ propagator contains an unknown complex parameter.[5] Second, a realistic calculation of the σ-exchange amplitude has been hampered by a lack of information about the relevant vertices and propagator.

In this paper, we use the following approach to these difficulties:

(i) We allow a complex parameter Z entering the Δ contribution to be determined by comparison with experimental data.

(ii) We use a form for the σ-exchange contribution obtained recently by Schnitzer[2] from unitarization of $\pi\pi$ scattering.[6] As a result we are able to express the total σ contribution to the πN scattering amplitude in terms of the "σ term":

$$g_\sigma(t)=\tfrac{1}{3}\sum_a \langle p'|\sigma^{aa}(0)|p\rangle, \tag{1}$$

which is the nucleon matrix element of the "σ commutator"

$$\sigma^{ab}(x)=i[\,Q_5^a(x_0),\partial_\mu A_\mu^b(x)]. \tag{2}$$

In the above p and p' are the initial and final nucleon momenta,[7] $t=-(p'-p)^2$, A_μ^a is the axial-vector current with isospin index a, and

$$Q_5^a(x_0)=\int d^3x\,A_0^a(x). \tag{3}$$

Traditionally, predictions of models for low-energy πN scattering have been compared with s- and p-wave scattering-length data. Instead, we follow a suggestion of Höhler *et al.*[8] and make comparisons with the coefficients of an expansion of the invariant amplitudes in powers of $\nu=(s-u)/4M$ and t. The expansion can be written as follows:

$$\begin{Bmatrix} A^+ \\ A^-/\nu \end{Bmatrix} = (a_1^\pm + a_2^\pm t) + (a_3^\pm + a_4^\pm t)\nu^2 + a_5^\pm \nu^4 + \cdots ,$$
$$\begin{Bmatrix} \bar{B}^+/\nu \\ \bar{B}^- \end{Bmatrix} = (b_1^\pm + b_2^\pm t) + (b_3^\pm + b_4^\pm t)\nu^2 + b_5^\pm \nu^4 + \cdots , \tag{4}$$

where A and $\bar{B}$ are the usual invariant amplitudes minus the Born terms of pseudoscalar-coupling theory. The "experimental values" for the twenty coefficients $a_i^\pm$ and $b_i^\pm$ have been calculated from fixed-t dispersion relations by Höhler *et al.*[8]

Using our model we derive expressions for the coefficients in expansion (4) which involve four free parameters, the Δ coupling parameters g^* and Z (where Z is complex) and the σ term g_σ. We find that an excellent fit to the experimental coefficients is obtained provided

(i) the Δ exchange contributions chosen are consistent with those calculated with the simplest possible choice of Δ propagator and $\Delta N\pi$ interaction, a choice which corresponds to $Z=-\frac{1}{2}$;

(ii) the σ term is assigned the value

$$g_\sigma(2\mu^2)=42\pm 10\ \text{MeV},$$

where μ is the charged-pion mass.

Finally, using the information gained from the fit, we make predictions of $g_\sigma(0)$, for the πNN

coupling constant g^2, and for the isospin-even s-wave scattering length.

The current-algebra constraints on πN scattering are reviewed in Sec. II. In Sec. III our exchange model is described and theoretical expressions for the coefficients in expansion (4) are obtained. In Sec. IV a numerical comparison with the experimental values of the expansion coefficients is carried out, and the values of the Δ coupling parameters are discussed. In Sec. V the σ term is considered, and several predictions are made. Finally, a brief discussion and summary of our results is given in Sec. VI.

II. CURRENT-ALGEBRA AMPLITUDES

We begin with a brief review of the current-algebra constraints on[1-4] the scattering process[7]

$$\pi^a(q) + N(p) \to \pi^b(q') + N(p') .$$

We assume that the weak vector currents $V_\nu^a(x)$ and axial-vector currents $A_\nu^a(x)$ are related by the chiral $SU(2)\otimes SU(2)$ equal-time commutator

$$[A_0^a(x), A_\nu^b(y)]\delta(x_0 - y_0) = i\epsilon_{abc}\,\delta(x-y)V_\nu^c(y) . \tag{5}$$

We define the pion field ϕ^a by

$$\partial_\nu A_\nu^a(x) = F_\pi \mu^2 \phi^a(x) , \tag{6}$$

and the nonpionic part of the axial-vector current by

$$\hat{A}_\nu^a(x) = A_\nu^a(x) - F_\pi \partial_\nu \phi^a(x) , \tag{7}$$

where μ is the pion mass and F_π is the pion decay constant. We also assume that it is possible to define a local scalar field $\sigma^{ab}(x)$ by means of the equal-time commutator

$$[A_0^a(x), \partial_\mu A_\mu^b(y)]\delta(x_0 - y_0) = -i\delta(x-y)\sigma^{ab}(y) , \tag{8}$$

with $\sigma^{ab} = \sigma^{ba}$.

The definitions and commutation relations given above imply the well-known Ward identity[9]

$$\begin{aligned}\int dxdy\, e^{iq\cdot y} e^{-iq'\cdot x}(\Box_x{}^2 - \mu^2)(\Box_y{}^2 - \mu^2)\langle p'|T^*(\phi^b(x)\phi^a(y))|p\rangle \\ = \frac{1}{F_\pi{}^2} q'_\mu q_\nu \int dxdy\, e^{iq\cdot y}e^{-iq'\cdot x}\langle p'|T^*(\hat{A}_\mu^b(x)\hat{A}_\nu^a(y))|p\rangle + \frac{i}{F_\pi{}^2}\left(1 + \frac{q^2}{\mu^2} + \frac{q'^2}{\mu^2}\right)\int dy\, e^{i(q-q')\cdot y}\langle p'|\sigma^{ab}(y)|p\rangle \\ + \frac{1}{F_\pi{}^2}\epsilon_{abc}\tfrac{1}{2}(q+q')\mu \int dy\, e^{i(q-q')\cdot y}\langle p'|V_\mu^c(y)|p\rangle ,\end{aligned} \tag{9}$$

which involves the off-mass-shell πN scattering amplitude[10]

$$T^{ba}(p', q'; p, q) = -i\int dx\, e^{-iq'\cdot x}(q^2 + \mu^2)(q'^2 + \mu^2)\langle p'|T^*(\phi^b(x)\phi^a(0))|p\rangle . \tag{10}$$

The presence of only the nonpionic parts $\hat{A}_\mu^a$ in Eq. (9) indicates that all pion poles have been explicitly removed from the axial currents.

At this point we recast the identity (9) in a form more useful for our purposes. We define the amplitude R^{ba} by

$$R^{ba} = \frac{1}{F_\pi{}^2} q'_\mu q_\nu T_{\mu\nu}^{ba} , \tag{11}$$

where

$$T_{\mu\nu}^{ba}(p', q'; p, q) = -i\int dx\, e^{-iq'\cdot x}\langle p'|T^*(\hat{A}_\mu^b(x)\hat{A}_\nu^a(0))|p\rangle . \tag{12}$$

For the nucleon matrix element of the vector current, we use the standard form

$$\langle p'|V_\mu^a(0)|p\rangle = i\left[F_1^V(t)\gamma_\mu - F_2^V(t)\frac{\sigma_{\mu\nu}k_\nu}{2M}\right]\frac{\tau^a}{2} , \tag{13}$$

where $k = p' - p$, $t = -k^2$, and M is the nucleon mass. Also, we write

$$\langle p'|\sigma^{ab}(0)|p\rangle = \delta_{ab}\, g_\sigma(t) . \tag{14}$$

After inserting Eqs. (11) through (14) into Eq. (9) and performing a few elementary manipulations we arrive at

$$T^{ba} = R^{ba} + \frac{1}{F_\pi{}^2}\left(1 + \frac{q^2}{\mu^2} + \frac{q'^2}{\mu^2}\right)\delta_{ab}\, g_\sigma(t) + \frac{1}{2F_\pi{}^2}\{[F_1^V(t) + F_2^V(t)]\, i\gamma\cdot Q + \nu F_2^V(t)\}\tfrac{1}{2}[\tau^b, \tau^a] , \tag{15}$$

where $Q=\frac{1}{2}(q+q')$.

Finally, we go to the pion mass shell, $q^2=-\mu^2$, $q'^2=-\mu^2$, and write Eq. (15) in terms of the usual invariant amplitudes $A^\pm$ and $B^\pm$, which are defined by the decomposition

$$T^{ba}=(-A^+ + i\gamma\cdot QB^+)\delta_{ab}+(-A^- + i\gamma\cdot QB^-)\tfrac{1}{2}[\tau^b,\tau^a]. \tag{16}$$

The result, which contains the constraints of current algebra, is

$$A^+(\nu,t)=\bar{A}^+(\nu,t)+\frac{1}{F_\pi^{\,2}}g_\sigma(t), \tag{17a}$$

$$B^+(\nu,t)=\bar{B}^+(\nu,t), \tag{17b}$$

$$A^-(\nu,t)=\bar{A}^-(\nu,t)-\frac{\nu}{2F_\pi^{\,2}}F_2^V(t), \tag{17c}$$

$$B^-(\nu,t)=\bar{B}^-(\nu,t)+\frac{1}{2F_\pi^{\,2}}[F_1^V(t)+F_2^V(t)], \tag{17d}$$

where $\bar{A}^\pm$ and $\bar{B}^\pm$ are contributions coming from R^{ba}.

III. LOW-ENERGY EXPANSION

In this section we describe the model that we are using for the amplitude R^{ba}. Then we carry out the expansion (4) of the invariant amplitude in powers of ν and t.

We assume that R^{ba} can be represented by a sum of four terms

$$R^{ba}=R_N^{ba}+R_\rho^{ba}+R_\Delta^{ba}+R_\sigma^{ba} \tag{18}$$

corresponding to N, ρ, σ, and $\Delta(1236)$ exchanges. We do not consider higher baryon resonances because estimates[1,3] indicate that the contribution of all these resonances is not significant at points we are interested in.

For the nucleon-exchange contribution we use the pseudovector (pv) Born terms prescribed by current algebra:

$$\begin{aligned}&\bar{A}_N^+=\frac{g^2}{M},\quad \bar{A}_N^-=0,\\ &\bar{B}_N^+=\frac{g^2}{M}\,\frac{\nu}{\nu^2-\nu_B^{\,2}},\quad \bar{B}_N^-=\frac{g^2}{M}\,\frac{\nu_B}{\nu^2-\nu_B^{\,2}}-\frac{g^2}{2M^2},\end{aligned} \tag{19}$$

where $\nu_B=(2\mu^2-t)/2M$.

We treat the ρ-exchange contribution within the hard-pion approximation, keeping only the ρ one-particle reducible part of R^{ba} and using the $\rho\pi\pi$ vertex of Schnitzer and Weinberg.[11] It has been shown[4] that this procedure implies that the ρ-exchange contribution to the πN invariant amplitudes can be included by simply making the substitution

$$F_i^V(t)\to\left[1-\frac{(1+\delta)t}{4m_\rho^{\,2}}\right]F_i^V(t),\quad i=1,2 \tag{20}$$

in Eqs. (17). If one chooses $\delta=-\frac{1}{2}$ then one obtains agreement[11] with the experimental data for A_1 and ρ decay.

To calculate Δ-exchange contributions, we need to know both the form of the Δ propagator and the $\Delta N\pi$ interaction. For the Δ propagator we shall use:

$$\begin{aligned}P_{\mu\nu}(p)=\frac{1}{i\gamma\cdot p+M^*}&\left[\delta_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu+\frac{i}{3M^*}(\gamma_\mu p_\nu-\gamma_\nu p_\mu)+\frac{2}{3M^{*2}}p_\mu p_\nu\right]\\ &+\frac{i}{6M^{*2}}\left[\frac{2(A^*+1)}{2A^*+1}\gamma_\mu p_\nu+\frac{2(A+1)}{2A+1}\gamma_\nu p_\mu-\left|\frac{A+1}{2A+1}\right|^2\gamma_\mu(\gamma\cdot p)\gamma_\nu-iM^*\frac{(2AA^*+A+A^*)}{|2A+1|^2}\gamma_\mu\gamma_\nu\right],\end{aligned} \tag{21}$$

$M^*=\Delta_{\text{mass}}$,

which may be obtained from Eq. (109) of Aurilia and Umezawa,[5] where we have made the substitution[12] $a_1 = -\frac{1}{2}(1+3A^*)$.

The most general $\Delta N\pi$ interaction with gradient coupling is $\mathcal{L}_I = g^*\overline{\psi}_\mu \Theta_{\mu\nu}\psi\partial_\nu\phi + \text{H.c.}$, where $\Theta_{\mu\nu} = \delta_{\mu\nu} + C\gamma_\mu\gamma_\nu$ and where C is an arbitrary complex number. One might expect the Δ contribution to depend independently upon C and A. However, Nath *et al.*[13] have shown that for real C and A, the Δ contribution will depend on only one real parameter.

Their method can be trivially generalized to treat the case of complex parameters. Rewriting the interaction in the form

$$\mathcal{L}_I = g^*\overline{\psi}_\mu \Theta_{\mu\nu}\psi\,\partial_\nu\phi + \text{H.c.},$$
$$\Theta_{\mu\nu} = \delta_{\mu\nu} + [\tfrac{1}{2}(1+4Z)A + Z]\gamma_\mu\gamma_\nu, \tag{22}$$

where Z is an arbitrary complex parameter, we find that the total (free-field plus interaction) Lagrangian is invariant under the point transformation:

$$\psi'_\mu = \psi_\mu + a\gamma_\mu\gamma_\lambda\psi_\lambda,$$
$$A' = \frac{A-2a^*}{1+4a^*},$$
$$\psi' = \psi, \quad \phi' = \phi.$$

Because of this invariance, the Δ contributions to the S matrix cannot depend upon A.[14] However, they will be a function of Z.

The simplest form of propagator and interaction can be obtained from Eqs. (21) and (22):

$$P_{\mu\nu}(p) = \frac{1}{i\gamma\cdot p + M^*}\left[\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{i}{3M^*}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) + \frac{2}{3M^{*2}}p_\mu p_\nu\right],$$
$$\mathcal{L}_I = g^*\overline{\psi}_\mu\psi\partial_\mu\phi + \text{H.c.}, \tag{23}$$

with $Z=-\frac{1}{2}$, $A=-1$. This form has been used recently by many authors[3,4,15] to calculate Δ-exchange amplitudes. Nath *et al.*[13] used field-theoretic arguments to fix Z at the value $Z=\frac{1}{2}$. In this paper we leave Z in as a parameter to be determined by the data.

Using the Δ propagator (21) and the interaction (22), we obtain the following Δ-exchange contributions to the invariant amplitudes[16]:

$$\overline{A}_\Delta^+ = \frac{g^{*2}}{9M^*}(\alpha_1^* + \alpha_2^* t)\left(\frac{1}{\nu_\Delta - \nu} + \frac{1}{\nu_\Delta + \nu}\right) - \frac{4g^{*2}}{9M^*}(E^* + M)(2M^* - M)$$
$$- \frac{2g^{*2}}{9M^*}\left(4 + \frac{M}{M^*}\right)\mu^2 + \frac{4g^{*2}}{9M^*}(2\mu^2 - t)\left[|Z|^2\left(2 + \frac{M}{M^*}\right) + (\text{Re}Z)\left(1 + \frac{M}{M^*}\right)\right], \tag{24a}$$

$$\overline{B}_\Delta^+ = \frac{g^{*2}}{9M^*}(\beta_1^* + \beta_2^* t)\left(\frac{1}{\nu_\Delta - \nu} - \frac{1}{\nu_\Delta + \nu}\right) - \frac{16g^{*2}}{9M^*}\left(\frac{M}{M^*}\right)\nu|Z|^2, \tag{24b}$$

$$\overline{A}_\Delta^- = -\frac{g^{*2}}{18M^*}(\alpha_1^* + \alpha_2^* t)\left(\frac{1}{\nu_\Delta - \nu} - \frac{1}{\nu_\Delta + \nu}\right) - \frac{8g^{*2}}{9}\left(\frac{M}{M^*}\right)\nu\left[|Z|^2\left(2 + \frac{M}{M^*}\right) + (\text{Re}Z)\left(1 + \frac{M}{M^*}\right)\right], \tag{24c}$$

$$\overline{B}_\Delta^- = -\frac{g^{*2}}{18M^*}(\beta_1^* + \beta_2^* t)\left(\frac{1}{\nu_\Delta - \nu} + \frac{1}{\nu_\Delta + \nu}\right) + \frac{g^{*2}}{9}\left(1 + \frac{M}{M^*}\right)^2$$
$$+ \frac{4g^{*2}}{9M^{*2}}\{(\text{Re}Z)[2M(M+M^*) - \mu^2] + |Z|^2[2M(M+2M^*) + (\mu^2 - \tfrac{1}{2}t)]\}, \tag{24d}$$

where the relevant kinematic quantities are defined by

$$
\begin{aligned}
&E^* \pm M = \frac{1}{2M^*}[(M^* \pm M)^2 - \mu^2], && q^{*2} = E^{*2} - M^2, \\
&\nu_\Delta = \omega^* + \frac{t}{4M}, && \omega^* = \frac{M^{*2} - M^2 - \mu^2}{2M}, \\
&\alpha_1^* = 3(M+M^*)q^{*2} + (M^* - M)(E^* + M)^2, && \alpha_2^* = \tfrac{3}{2}(M+M^*), \\
&\beta_1^* = 3q^{*2} - (E^*+M)^2, && \beta_2^* = \tfrac{3}{2}.
\end{aligned}
\tag{25}
$$

To evaluate the σ-exchange contribution, we keep only the σ one-particle reducible part of R^{ba}, using an approximation recently developed by Schnitzer.[2] He assumed a form for the $\pi\pi\sigma$ vertex valid at small t that follows from unitarization of $\pi\pi$ scattering. The resulting approximate σ exchange contribution, when combined with the σ commutator contribution in Eq. (17a), leads to the total σ contribution

$$A_\sigma^+ = -\frac{1}{N(N+2)F_\pi^2}\left[4 - N(N+2) - \frac{2t}{\mu^2}\right] g_\sigma(t), \tag{26}$$

where N denotes the representation $(\frac{1}{2}N, \frac{1}{2}N)$ of chiral $SU(2)\otimes SU(2)$ to which $\partial\cdot A$ and the σ field belong.

Our complete model for the πN scattering amplitude can be summarized as follows:

$$A^+ = \bar{A}_\Delta^+ + \frac{g^2}{M} - \frac{1}{N(N+2)F_\pi^2}\left[4 - N(N+2) - \frac{2t}{\mu^2}\right] g_\sigma(t), \tag{27a}$$

$$\tilde{B}^+ = \bar{B}_\Delta^+, \tag{27b}$$

$$A^- = \bar{A}_\Delta^- - \frac{1}{2F_\pi^2}\left[1 - \frac{(1+\delta)t}{4m_\rho^2}\right] \nu F_2^V(t), \tag{27c}$$

$$\tilde{B}^- = \bar{B}_\Delta^- - \frac{g^2}{2M^2} + \frac{1}{2F_\pi^2}\left[1 - \frac{(1+\delta)t}{4m_\rho^2}\right][F_1^V(t) + F_2^V(t)], \tag{27d}$$

where the pseudoscalar-coupling Born terms have been separated out,

$$\tilde{B}^\pm = B^\pm - g^2\left(\frac{1}{M^2 - s} \mp \frac{1}{M^2 - u}\right), \tag{28}$$

and where the Δ contributions are given by Eqs. (24).

The expansion (4) of the amplitudes given by Eqs. (27) about the symmetry point ($\nu = 0$, $t = 0$) can be carried out easily. We present the nucleon, ρ, Δ, and σ contributions to the various expansion coefficients separately.

A. Nucleon Contributions

Since the Born terms of pseudoscalar coupling theory have been separated out, only the extra pieces of pv coupling theory remain. They yield the contributions

$$a_1^+ = \frac{g^2}{M}, \tag{29a}$$

$$b_1^- = -\frac{g^2}{2M^2}. \tag{29b}$$

B. ρ Contributions

The ρ contributions affect only the antisymmetric isospin amplitudes. The results are

$$a_1^- = -\frac{F_2^V(0)}{2F_\pi^2}, \tag{30a}$$

$$a_2^- = -\frac{F_2'^V(0)}{2F_\pi^2} + \frac{1+\delta}{8F_\pi^2 m_\rho^2} F_2^V(0), \tag{30b}$$

$$b_1^- = \frac{1}{2F_\pi^2}[F_2^V(0) + F_1^V(0)], \tag{30c}$$

$$b_2^- = \frac{1}{2F_\pi^2}[F_2'^V(0) + F_1'^V(0)] - \frac{1+\delta}{8F_\pi^2 m_\rho^2}[F_1^V(0) + F_2^V(0)]. \tag{30d}$$

C. Δ Contributions

1. Z-independent contributions:

$$\begin{bmatrix} a_1^+ \\ a_2^+ \\ a_3^+ \\ a_4^+ \\ a_5^+ \end{bmatrix} = \frac{2g^{*2}}{9M}\frac{\alpha_1^*}{\omega^*} \begin{bmatrix} 1 - \dfrac{M\omega^*}{\alpha_1^* M^*}\left[2(E^*+M)(2M^*-M) + \dfrac{4M^*+M}{M^*}\mu^2\right] \\ \alpha_2^*/\alpha_1^* - 1/4M\omega^* \\ 1/\omega^{*2} \\ (1/\omega^{*2})(\alpha_2^*/\alpha_1^* - 3/4M\omega^*) \\ 1/\omega^{*4} \end{bmatrix}, \tag{31a}$$

$$\begin{bmatrix} b_1^+ \\ b_2^+ \\ b_3^+ \\ b_4^+ \\ b_5^+ \end{bmatrix} = \frac{2g^{*2}}{9M}\frac{\beta_1^*}{\omega^{*2}} \begin{bmatrix} 1 \\ \beta_2^*/\beta_1^* - 1/2M\omega^* \\ 1/\omega^{*2} \\ (1/\omega^{*2})(\beta_2^*/\beta_1^* - 1/M\omega^*) \\ 1/\omega^{*4} \end{bmatrix}, \tag{31b}$$

$$\begin{bmatrix} a_1^- \\ a_2^- \\ a_3^- \\ a_4^- \\ a_5^- \end{bmatrix} = -\frac{g^{*2}\alpha_1^*}{9M\omega^{*2}} \begin{bmatrix} 1 \\ \alpha_2^*/\alpha_1^* - 1/2M\omega^* \\ 1/\omega^{*2} \\ (1/\omega^{*2})(\alpha_2^*/\alpha_1^* - 1/M\omega^*) \\ 1/\omega^{*4} \end{bmatrix}, \tag{31c}$$

$$\begin{bmatrix} b_1^- \\ b_2^- \\ b_3^- \\ b_4^- \\ b_5^- \end{bmatrix} = -\frac{g^{*2}\beta_1^*}{9M\omega^*} \begin{bmatrix} 1 - \dfrac{M\omega^*}{\beta_1^*}\left(\dfrac{M+M^*}{M^*}\right)^2 \\ \beta_2^*/\beta_1^* - 1/4M\omega^* \\ 1/\omega^{*2} \\ (1/\omega^{*2})(\beta_2^*/\beta_1^* - 3/4M\omega^*) \\ 1/\omega^{*4} \end{bmatrix}. \tag{31d}$$

2. Z-dependent contributions:

$$
\begin{aligned}
a_1^+ &= \frac{4g^{*2}\mu^2}{9M^*}y\,,\\
a_2^+ &= -\frac{4g^{*2}}{9M^*}y\,,\\
b_1^+ &= -\frac{16g^{*2}M}{9M^{*2}}|Z|^2\,,\\
a_1^- &= -\frac{8g^{*2}M}{9M^*}y\,,\\
b_1^- &= +\frac{8g^{*2}M}{9M^*}y+\frac{4g^{*2}\mu^2}{9M^{*2}}(|Z|^2-\mathrm{Re}Z)\,,\\
b_2^- &= -\frac{2}{9}\frac{g^{*2}}{M^{*2}}|Z|^2\,,
\end{aligned}
\tag{32}
$$

where

$$y=|Z|^2\left(2+\frac{M}{M^*}\right)+(\mathrm{Re}Z)\left(1+\frac{M}{M^*}\right).$$

D. σ Contributions

The σ contributions to the expansion parameters can be expressed in the form

$$a_1^+=-\left[\frac{4}{N(N+2)}-1\right]\frac{g_\sigma(0)}{F_\pi^{\,2}}\,,\tag{33a}$$

$$a_2^+=\frac{g_\sigma(2\mu^2)}{2F_\pi^{\,2}\mu^2}+\left[\frac{4}{N(N+2)}-1\right]\frac{g_\sigma(0)}{2F_\pi^{\,2}\mu^2}\,.\tag{33b}$$

IV. COMPARISON WITH THE EXPERIMENTAL CROSSING-SYMMETRIC EXPANSION

We have displayed in the preceding section the theoretical expressions for the coefficients in the power-series expansion (4). The twenty independent expansion coefficients will now be compared[17] with their experimental values. The experimental expansion coefficients have been evaluated directly in terms of derivatives of fixed-t dispersion relations by Höhler *et al.*[8,18] and are reproduced in Table I.

The theoretical contributions discussed in Sec. III are the (i) pv-nucleon pole, (ii) ρ-exchange, (iii) Δ-exchange, and (iv) σ-exchange contributions. Each of these will be considered in turn.

A. Nucleon and ρ Exchange

If we use the conventional πNN coupling constant[18]

$$\frac{g^2}{4\pi}=14.64\pm0.6\,,$$

the pv-nucleon contributions in Eq. (29) are

TABLE I. Experimental values for the coefficients of the crossing-symmetric expansion (4) as determined by Höhler *et al.* (Ref. 8). The pseudoscalar-nucleon pole contribution has been removed.

i	1	2	3	4	5
a_i^+	26.1 ± 0.3	1.15 ± 0.1	4.4	0	1.1
a_i^-	-8.4	-0.45	-1.15	0	-0.3
b_i^+	-3.3	0.2	-0.9	0.1	-0.3
b_i^-	8.0 ± 0.4	$0.3\ \pm0.2$	1.0	-0.05	0.25

$$a_1^+=\frac{g^2}{M}=27.4\pm1.1$$

and

$$b_1^-=-\frac{g^2}{2M^2}=-2.0\pm0.1\,.$$

The ρ-exchange contribution is given by Eqs. (30). If[18] $F_\pi=0.657$, $\delta=-\frac{1}{2}$, and the nucleon electromagnetic form factors[19] and slopes at $t=0$ are

$$F_1^V(0)=1.0\,,\quad F_1'^{\,V}(0)=0.046\,,$$

$$F_2^V(0)=3.7\,,\quad F_2'^{\,V}(0)=0.22\,,$$

then the ρ-exchange contributions are

$$a_1^-=-4.3\,,\quad a_2^-=-0.25\,,$$

$$b_1^-=5.4\,,\quad b_2^-=0.30\,.$$

Subtracting the pv-nucleon pole and ρ exchange contributions from the experimental expansion parameters of Table I results in a new set of parameters given in Table II. The entries in Table II should depend only on the Δ contribution and σ exchange.

B. Z-Independent Δ

The Δ contribution given by Eqs. (31) and (32) has been separated into two parts:
(i) Z-independent,
(ii) Z-dependent.

TABLE II. Expansion coefficients of Table I with the pseudovector-nucleon pole and the ρ-exchange contributions subtracted. The coefficients in this table depend on only Δ and σ contributions.

i	1	2	3	4	5
a_i^+	-1.3 ± 1.2	1.15 ± 0.1	4.4	0	1.1
a_i^-	-4.1	-0.2	-1.15	0	-0.3
b_i^+	-3.3	0.2	-0.9	0.1	-0.3
b_i^-	4.6 ± 0.4	0 ± 0.2	1.0	-0.05	0.25

The Z-independent part is nearly the same as the "pole" terms of all previous analyses, the main difference being in the coefficient a_1^+ where large "pole" and "nonpole" contributions nearly cancel in the Z-independent form.

The Z-independent coefficients depend on only one parameter, g^{*2}. This constant is often evaluated using the perturbation expression for the Δ width. However, since the pole term prediction for the Δ phase shift near the resonance position is quite poor,[18] the value of g^{*2} found by fitting the Δ width will not be reliable. Höhler *et al.*[8] compare a dispersive calculation of the real part of the resonance amplitude with the pole approximation and find a value of g^{*2} about 40% smaller than by the width method.

Since the expansion coefficients have been calculated dispersively, a direct experimental comparison involving those coefficients which depend on only the Z-independent Δ part can be used to evaluate g^{*2}. The twelve $i=3, 4, 5$ coefficients are dependent upon only the Z-independent terms. Of these the largest is

$$a_3^+ = 4.4 .$$

Using the Z-independent expression for a_3^+ from Eq. (31a)

$$a_3^+ = \frac{2g^{*2}\alpha_1^*}{9M\omega^{*3}} ,$$

and taking $M^* = 8.67$ (1211 MeV), which is the real part of the second sheet pole position,[20] we find that

$$g^{*2} = 2.90 .$$

The remaining Z-independent Δ contributions now can be calculated by the expression in Eqs. (31) with the above value of g^*. The Z-independent Δ parts of the expansion coefficients are given in Table III.

When the Z-independent Δ coefficients of Table III are subtracted from the entries of Table II, we are left with the coefficients of Table IV which depend on only the Z-dependent Δ and σ terms. One should note that all of the large coefficients in Table I with the exception of a_2^+ have thereby been reduced by an order of magnitude. Thus the Z-dependent contributions are expected to be small.

TABLE III. The Z-independent contribution to the coefficients of the crossing-symmetric expansion.

i	1	2	3	4	5
a_i^+	−1.3	0.70	4.4	0	0.95
a_i^-	−4.75	−0.1	−1.0	0	−0.2
b_i^+	−3.7	0.15	−0.8	0.05	−0.15
b_i^-	5.0	−0.1	0.85	−0.05	0.2

C. Z-Dependent Δ

The parameter Z which is a measure of the form of the $\Delta N\pi$ interaction is in general an arbitrary complex number. The Z-dependent expansion coefficients in Eqs. (32), neglecting terms of order $(1/M)^2$, can be written as follows[17]:

$$\begin{aligned} b_1^- &= \frac{8g^{*2}}{9}\left(\frac{M}{M^*}\right)y , \\ a_1^- &= -b_1^- , \\ a_1^+ &= \frac{1}{2M}b_1^- , \\ a_2^+ &= -a_1^+ , \\ b_1^+ &= -\frac{16}{9}g^{*2}\left(\frac{M}{M^{*2}}\right)|Z|^2 , \\ b_2^- &= \frac{1}{8M}b_1^+ , \end{aligned} \tag{34}$$

where we have defined as before

$$y = \left(2 + \frac{M}{M^*}\right)|Z|^2 + \left(1 + \frac{M}{M^*}\right)\mathrm{Re}Z .$$

In Fig. 1 the limits on y are plotted as a function of $|Z|$. The minimum value of y is −0.28, occurring when $Z=-0.32$. As can be seen from Eq. (34) only the coefficients $b_1^- = -a_1^- \simeq 2y$ are appreciable. By referring to Table IV, one sees that both a_1^- and b_1^- prefer a negative value of y. The total theoretical contribution to the coefficients a_1^-, a_2^+, and b_1^- are plotted in Fig. 2 as a function of y and compared with their experimental values. One may observe from Figs. 1 and 2 that the value $Z = \frac{1}{2}$ implies a value of y quite inconsistent with the data. However, the choice $Z=-\frac{1}{2}$, which is also

TABLE IV. Expansion coefficients of Table I with the N, ρ, and Z-independent exchanges subtracted. The entries of this table result from the difference between Table II and Table III. The remaining coefficients are explained by the Z-dependent and σ-exchange contributions in our model.

i	1	2	3	4	5
a_i^+	0 ± 1.2	0.45 ± 0.1	0	0	0.15
a_i^-	0.65	−0.1	−0.15	0	−0.1
b_i^+	0.4	0.05	−0.1	0.05	−0.15
b_i^-	−0.4 ± 0.4	0.1 ± 0.2	0.15	0	0.05

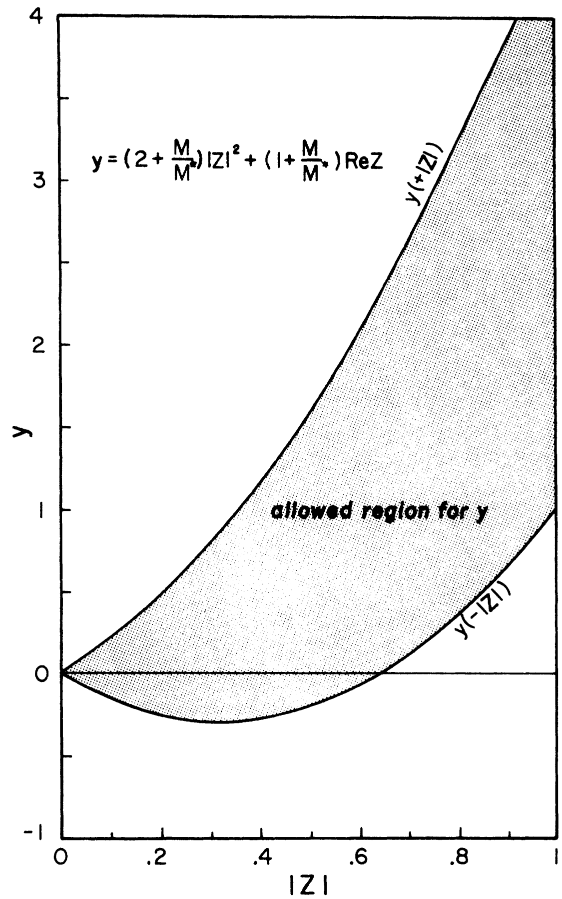

FIG. 1. The quantity y as a function of $|Z|$. Allowed values of y lie between the two curves. The bounding curves correspond to positive and negative real Z.

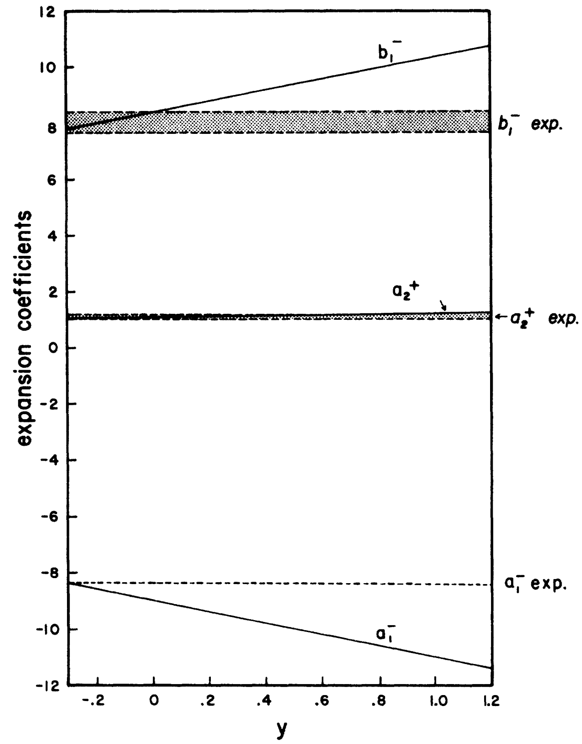

FIG. 2. Theoretical expressions for a_1^-, a_2^+, and b_1^- as a function of y compared to their experimental values. One should observe that negative values of y are preferred by a_1^- and b_1^- and that a_2^+ is not strongly y-dependent.

attractive from a theoretical point of view, is consistent with the data.

If a slightly higher value of g^{*2} were chosen, then the $i=3, 4, 5$ coefficients would exhibit slightly better (and a_3^+ slightly worse) agreement with the data. The magnitudes of a_1^- and b_1^- in Table IV would be larger, making it even more likely that y is near its minimum value.[21]

Finally, the σ term contributing to a_1^+ and a_2^+ will be considered in the following section.

V. σ TERM

The matrix element between nucleon states of the σ commutator, $g_\sigma(t)$, introduced in Eqs. (1)–(3) has been the center of considerable interest and controversy. This term will be discussed in detail in this section.

Using Eq. (27a), one obtains the A^+ amplitude at the unphysical point $\nu=0$, $t=2$ [henceforth referred to as the Cheng-Dashen (CD) point][22]:

$$A^+(0,2)=a_1^+ + 2a_2^+ = \bar{A}_\Delta^+(0,2) + \frac{g^2}{M} + \frac{g_\sigma(2)}{F_\pi{}^2} . \quad (35)$$

From Table II we see that $\bar{A}_\Delta^+(0,2)$ is negligible.[23] The above equation provides the basis for a direct determination of $g_\sigma(2)$ by extrapolation. Unfortunately the quantity $A^+(0,2)-g^2/M$ is experimentally uncertain and the resulting $g_\sigma(2)$ is poorly determined.

From Table IV we note that the coefficient $(a_2^+)_\sigma$ is relatively well determined and has the value

$$(a_2^+)_\sigma = 0.45 \pm 0.10 . \quad (36)$$

In our model this coefficient represents the σ-term contribution as given by Eq. (33b),

$$2F_\pi{}^2(a_2^+)_\sigma = g_\sigma(2) + \left[\frac{4}{N(N+2)} - 1\right] g_\sigma(0) . \quad (37)$$

The difference between the σ term at the CD point and at $t=0$ can be defined as

$$g_\sigma(2) - g_\sigma(0) \equiv \lambda . \quad (38)$$

We expect that λ is small but may not be negligible. Most authors have neglected the difference between $g_\sigma(2)$ and $g_\sigma(0)$. However, Pagels and

Pardee[24] have noted that the two-pion-state contribution to $g_\sigma(t)$ gives

$$\lambda = 0.1\,. \tag{39}$$

Combining Eqs. (37) and (38) we obtain

$$g_\sigma(2) = \tfrac{1}{2}N(N+2)F_\pi{}^2(a_2^+)_\sigma + \lambda F_\pi{}^2[1 - \tfrac{1}{4}N(N+2)]\,. \tag{40}$$

With the common assumption that the chiral symmetry breaking transforms as a $(\frac{1}{2}, \frac{1}{2})$ representation of $SU(2)\otimes SU(2)$ (i.e., $N=1$), we have

$$g_\sigma(2) = \tfrac{3}{2}F_\pi{}^2(a_2^+)_\sigma + \tfrac{1}{4}\lambda F_\pi{}^2\,. \tag{41}$$

Using the Pagels-Pardee value of $\lambda = 0.1$, the λ correction, Eq. (41), is less than 4% and yields

$$g_\sigma(2) = 42 \pm 10 \text{ MeV}\,. \tag{42}$$

The corresponding value for $g_\sigma(0)$ is

$$g_\sigma(0) = 28 \pm 10 \text{ MeV}\,. \tag{43}$$

The above value of $g_\sigma(2)$ can be used to predict the magnitude of $g^2/4\pi$ by use of Eq. (35):

$$\begin{aligned}\frac{g^2}{4\pi} &= \frac{M}{4\pi}\left[a_1^+ + 2a_2^+ - \frac{1}{F_\pi{}^2}g_\sigma(2)\right]\\ &= \frac{M}{4\pi}(a_1^+ + \tfrac{1}{2}a_2^+ + 1.02)\\ &= 14.9 \pm 0.2\,,\end{aligned} \tag{44}$$

where, from Table III, $(a_2^+)_\sigma = a_2^+ - 0.70$. This prediction can be compared to the usual[18] value of

$$\frac{g^2}{4\pi} = 14.64^{+0.54}_{-0.72}\,,$$

and to the analysis of Lichard and Presnajder,[25] who find

$$\frac{g^2}{4\pi} = 14.52 \pm 0.18$$

by use of the technique of analytic extrapolation.

The symmetric scattering length a_{0+}^+ can be written in terms of the expansion parameters as[26]

$$\begin{aligned}4\pi\frac{M+1}{M}a_{0+}^+ &= a_1^+ - \frac{g^2}{M} + 1.31\\ &= g_\sigma(2)/F_\pi{}^2 - 2a_2^+ + 1.31\,.\end{aligned} \tag{45}$$

For our value of the σ term in Eq. (42) the value of a_{0+}^+ is predicted to be

$$a_{0+}^+ = -0.022 \pm 0.004\,. \tag{46}$$

A specific model for calculation of the σ term is provided by the generalized σ model discussed by Turner and Olsson.[27] The σ term in this model is given by

$$g_\sigma(2) = \frac{N(N+2)}{3}\frac{M\alpha}{m_\sigma{}^2}\,, \tag{47}$$

where α is the fraction (any positive or negative number) of the pion mass contributed by the chirally symmetric nonderivative portion of the Lagrangian and m_σ is the σ mass. If $N=1$ and $\alpha=1$ we recover the original Gell-Mann–Lévy σ model.[28] If m_σ is taken to be the ρ mass and $N=1$, we find numerically

$$g_\sigma(2) = 31\alpha \text{ MeV}\,. \tag{48}$$

The primary attempts to find $g_\sigma(2)$ by direct extrapolation using dispersion relations have been

$g_\sigma(2) \sim 110$ MeV (Cheng-Dashen, Ref. 22),

$g_\sigma(2) \sim 40$ MeV (Höhler *et al.*, Ref. 26).

TABLE V. The σ term as found by various authors.

Authors	Reference	σ term (MeV)
F. von Hippel and J. Kim	31	~26
T. P. Cheng and R. Dashen	22	~110
E. Osypowski	4	~60
G. Höhler *et al.*	26	~40
G. Altarelli *et al.*	32	80 ± 30
M. Ericson and M. Rho	33	~34
S. J. Hakim	34	51 ± 9
B. Renner	35	33 or 43
C. C. Shih and H. K. Shepard	30	-46 ± 140
H. Jakob	29	43^{+12}_{-6}
This work		42 ± 10

The analysis of Cheng and Dashen[22] emphasized the low-energy data (below the Δ resonance). This analysis was criticized by Höhler *et al.*,[26] who obtain a value[29] in the vicinity of 40 MeV. Recently Shih and Shepard[30] have used the technique of analytic extrapolation to obtain

$$g_\sigma(0) = -46 \pm 140 \text{ MeV}.$$

A number of other authors[31-35] have calculated the σ term under a variety of theoretical assumptions. Table V contains a collection of these results.

Finally, we present in Table VI the expansion coefficient residue after all the contributions of our model have been removed. The entries in this table differ from those in Table IV by the subtraction of

(i) Z-dependent terms with $Z=-\frac{1}{2}$,

(ii) a_1^+ and a_2^+ calculated from Eqs. (33), with $g_\sigma(2)=42$ MeV and $g_\sigma(0)=28$ MeV.

The success of the model can be evaluated by comparing Table VI to the experimental coefficients in Table I.

TABLE VI. Residues of the experimental expansion coefficients left after all of the model contributions have been removed. The entries of this table differ from those of Table IV by the subtraction of (i) Z-dependent terms with $Z=-\frac{1}{2}$ and (ii) values of a_1^+ and a_2^+ calculated with $g_\sigma(2)=42$ MeV and $g_\sigma(0)=28$ MeV.

i	1	2	3	4	5
a_i^+	0.15 ± 1.2	0 ± 0.1	0	0	0.15
a_i^-	0.35	0.1	−0.15	0	−0.1
b_i^+	0.50	0.05	0.1	−0.05	−0.15
b_i^-	-0.1 ± 0.4	0.1 ± 0.2	0.15	0	0.05

VI. CONCLUSIONS

We have shown that a pole model with current-algebra constraints can adequately account for the experimentally determined πN scattering amplitude at low energy. The experimental data[8,18] used are the twenty coefficients of the power series in ν^2 and t given in Eq. (4). This set of coefficients serves to fix the free parameters of our model much better than the conventional s- and p-wave scattering lengths.[8]

The free parameters of our model are determined separately. Only the Z-independent Δ exchange amplitude contributes to the twelve $i=3, 4, 5$ coefficients, thus fixing g^{*2}. Information on the parameter Z is obtained by examining the residue left after the ρ exchange and Z-independent Δ exchange contributions are subtracted from the experimental values of the coefficients a_1^- and b_1^-. Since this residue is small, a small value of Z is preferred as is shown by Figs. 1 and 2. A value of Z as large as $Z=\frac{1}{2}$ is clearly ruled out, and if Z is chosen real it should fall in the range $-0.8 \leq Z \leq 0$. Once the Δ contribution is removed from the coefficient a_2^+ the remainder must be due to the σ-commutator contribution. Using the work of Schnitzer[2] we can immediately relate the a_2^+ coefficient to the σ term.

We noted earlier that when the Δ contributions to the scattering amplitudes are calculated with the propagator and interaction given in Eqs. (21) and (22), the amplitudes will be a function of Z and independent of A. Without loss of generality, we can choose $A=-1$ which yields the simplest form of propagator [Eq. (23)]. The interaction in Eq. (22) then becomes $\Theta_{\mu\nu} = \delta_{\mu\nu} - (Z+\frac{1}{2})\gamma_\mu\gamma_\nu$. The choice $Z=-\frac{1}{2}$ used by many authors[3,4,15] leads to the simplest interaction [Eq. (23)]. We have shown that this choice is consistent with the experimental data.[36]

Finally, we have noted that the σ-term contribution to a_2^+ is easily extracted from the experimental data. Under the assumption that the chiral symmetry breaking is characterized by $N=1$, the σ term at the CD point is[29]

$$g_\sigma(2) = 42 \pm 10 \text{ MeV}.$$

Using this value of $g_\sigma(2)$, the πNN coupling constant and the isospin symmetric s-wave scattering length are predicted to be

$$\frac{g^2}{4\pi} = 14.9 \pm 0.2$$

and

$$a_{0+}^+ = -0.022 \pm 0.004,$$

respectively.

---

*Supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alummni Research Foundation, and in part by the U. S. Atomic Energy Commission under contract AT(11-1)-881, COO-881-350.

[1]L. S. Brown, W. J. Pardee, and R. D. Peccei, Phys. Rev. D 4, 2801 (1971).

[2]H. J. Schnitzer, Phys. Rev. D 5, 1482 (1972); Phys.

Rev. D 6, 1801 (1972).

[3]H. J. Schnitzer, Phys. Rev. 158, 1471 (1967); K. Raman, Phys. Rev. 164, 1736 (1967).

[4]E. T. Osypowski, Nucl. Phys. B21, 615 (1970).

[5]A. Aurilia and H. Umezawa, Phys. Rev. 182, 1478 (1966). One should note that two errors in phase were made in obtaining their Eq. (109).

[6]H. J. Schnitzer, Phys. Rev. Lett. 24, 1384 (1970); Phys. Rev. D 2, 1621 (1970).

[7]In the metric we are using $x_\mu x_\mu = \vec{x}^2 - x_0^2$; $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\delta_{\mu\nu}$; $\{\gamma_\mu\}$ to γ_5 are all Hermitian.

[8]G. Höhler, H. P. Jakob, and R. Strauss, Nucl. Phys. B39, 232 (1972).

[9]S. Weinberg, in *Lectures on Elementary Particles and Quantum Field Theory*, 1970 Brandeis University Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1971), Vol. 1.

[10]In Eq. (10) $p' + q' = p + q$ is understood. In Eqs. (10) and (12)–(14), nucleon spinors are suppressed.

[11]H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).

[12]This substitution yields the same Lagrangian discussed by C. Fronsdal, Nuovo Cimento Suppl. 9, 416 (1958).

[13]L. M. Nath, B. Etemadi, and J. D. Kimel, Phys. Rev. D 3, 2153 (1971).

[14]S. Kamefuchi, L. O'Raifeartaigh, and A. Salam, Nucl. Phys. 28, 529 (1961); J. S. R. Chisholm, Nucl. Phys. 26, 469 (1961).

[15]D. Amati and S. Fubini, Ann. Rev. Nucl. Sci. 12, 419 (1962); L. N. Chang, Phys. Rev. 162, 1497 (1967); B. Petersson, Lecture notes of the International Summer School at Karlsruhe, 1968 (unpublished).

[16]The Δ amplitudes used in Ref. 8 can be obtained from Eqs. (24) by the substitution $Z = -(C/2)/(1+2C)$. The authors of Ref. 8 used a general propagator [with C replacing A in Eq. (21)], but with the simplest interaction of Eq. (23) giving the above relation between Z and C.

[17]Henceforth for all numerical calculations we will set the charged pion mass $\mu = 1$ unless explicitly stated otherwise.

[18]G. Ebel *et al.*, Nucl. Phys. B33, 317 (1971).

[19]E. Lohrmann, *Proceedings of the Fifth International Conference on Elementary Particles, Lund, 1969*, edited by G. von Dardel (Berlingska Boktryckeriet, Lund, Sweden, 1970), p. 11.

[20]J. S. Ball *et al.*, Phys. Rev. Lett. 28, 1143 (1972); Particle Data Group, Phys. Lett. 39B, 1 (1972).

[21]The increase in the Z-dependent contributions to a_1^- and b_1^- due to the increase in g^{*2} is negligible, since the Z-dependent contribution is much smaller than the Z-independent contribution.

[22]T. P. Cheng and R. Dashen, Phys. Rev. Lett. 26, 594 (1971).

[23]By direct evaluation of Eq. (24a) at the CD point we find independent of Z

$$\bar{A}_\Delta^+(0,2) = \frac{2g^{*2}}{9M^{*2}} \frac{2M^* + M}{M^{*2} - M^2} = 0.0069 .$$

[24]H. Pagels and W. J. Pardee, Phys. Rev. D 4, 3335 (1971).

[25]P. Lichard and P. Presnajder, Nucl. Phys. B33, 605 (1971). Conversely, if this value of g^2 is used, Eq. (35) implies that $g_\sigma(2) = 75 \pm 30$ MeV by direct extrapolation. The large error arises from combining the errors in g^2 and a_1^+.

[26]G. Höhler, H. Jakob, and R. Strauss, Phys. Lett. 35B, 445 (1971).

[27]Leaf Turner and M. G. Olsson, Phys. Rev. D 6, 3522 (1972).

[28]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

[29]A recent analysis, using the method of Ref. (24), by H. Jakob, CERN Report TH 1446, has found $g_\sigma(2) = 43^{+12}_{-6}$ MeV. If the quoted errors are accepted, this result when combined with ours implies $N = 1$ dominance for the chiral-symmetry-breaking term.

[30]C. C. Shih and H. K. Shepard, Phys. Lett. 41B, 321 (1972).

[31]F. von Hippel and J. Kim, Phys. Rev. D 1, 151 (1970).

[32]G. Altarelli, N. Cabibbo, and L. Maiani, Nucl. Phys. B34, 621 (1971).

[33]M. Ericson and M. Rho, Phys. Lett. 36B, 93 (1971).

[34]S. J. Hakim, Nuovo Cimento Lett. 5, 377 (1972).

[35]B. Renner, Phys. Lett. 40B, 473 (1972).

[36]The choice $Z = +\frac{1}{2}$ advocated by Nath *et al.*[13] can be interpreted[8] as the Rarita-Schwinger propagator [W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941)] and the simplest type of interaction.

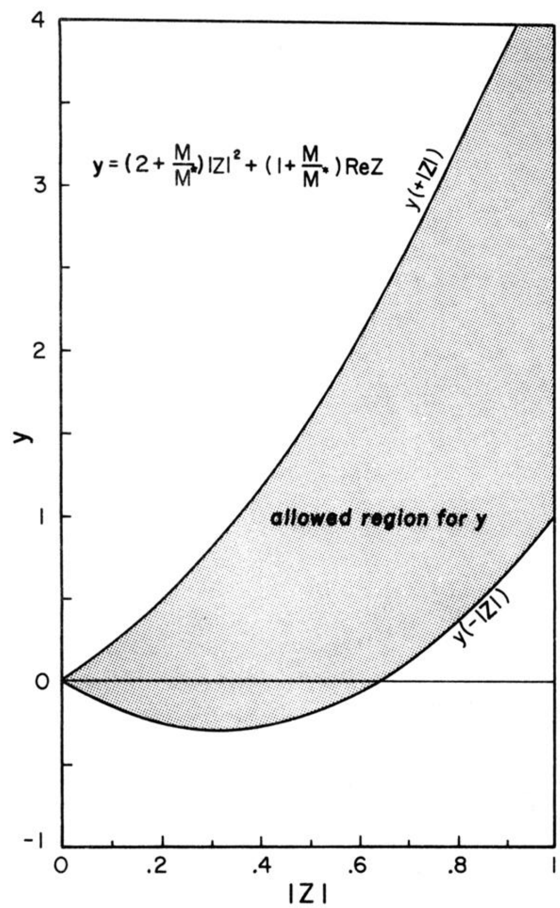

FIG. 1. The quantity y as a function of $|Z|$. Allowed values of y lie between the two curves. The bounding curves correspond to positive and negative real Z.

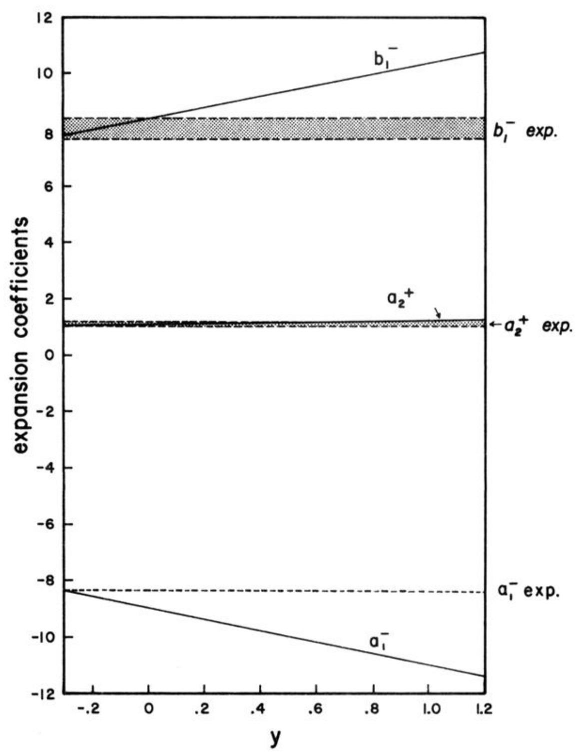

FIG. 2. Theoretical expressions for a_1^-, a_2^+, and b_1^- as a function of y compared to their experimental values. One should observe that negative values of y are preferred by a_1^- and b_1^- and that a_2^+ is not strongly y-dependent.