Calculation of $K_2^0 \to \pi^+\pi^-\gamma$ Decay as a Consistency Test of a Current-Current Quark Model*

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The CP-conserving decay $K_2^0 \to \pi^+\pi^-\gamma$ is studied in a modified fermion-loop model as a consistency test of a current-current quark model. The weak Hamiltonian is phenomenologically constructed from one-baryon octet matrix elements. The predicted branching ratio $r = R(K_2^0 \to \pi^+\pi^-\gamma)/R(K_2^0 \to \text{all modes}) = 3.0 \times 10^{-4}$ is below the present experimental upper limits of Thatcher *et al.* ($r < 4 \times 10^{-4}$) and may be compared with the tree-graph estimate $2.6 \times 10^{-4} < r < 4 \times 10^{-4}$ of Moshe and Singer; however, the present model has no adjustable parameters.

Recently it was shown[1] that when the baryon-loop model introduced by Steinberger[2] to explain the decay $\pi^0 \to \gamma\gamma$ is suitably modified for weak interactions,[1] it unexpectedly provides a qualitative explanation for $K_2^0 \to \gamma\gamma$ decay as well.[3,4] As we remarked in that work, this is a very attractive alternative to the usual tree-graph description[5] of this $(B=0)$ process, since (a) it utilizes input[6] derived from an analysis of a group of $B=1$ processes, the nonleptonic hyperon decays, thus pointing to a connection between weak, strangeness-changing processes with differing baryon numbers (such a connection *cannot* be made in the framework of the tree-graph model), and (b) it is a model for $K_2^0 \to \gamma\gamma$ with *no adjustable parameters*. The question of whether others of the weak, strangeness-changing $(B=0)$ processes can be successfully treated by this modified fermion-loop approach would appear to be of great interest. Not only would such applications test the consistency of this alternative description, but they would be expected to yield predictions of branching ratios similarly free of adjustable parameters. As a first step, then, in such an exploration, we study the (CP-conserving) decay mode $K_2^0 \to \pi^+\pi^-\gamma$ in the extended baryon-loop model.

One recalls that the modified baryon-loop approach presumes the replacement of the weak nonleptonic Hamiltonian density relevant for hyperon decays,[1,6]

$$\mathcal{H}_G = \sqrt{2}\, G \cos\theta \sin\theta\, \tfrac{1}{2}\{\mathcal{J}_\mu^{(1-i2)}, \mathcal{J}_\mu^{(4+i5)}\}, \qquad (1)$$

by an equivalent weak Hamiltonian,

$$\mathcal{H}_W = -\sqrt{2} F \,\mathrm{Tr}([\bar{B}, B]\lambda_6) + \sqrt{2}\, D\,\mathrm{Tr}(\{\bar{B}, B\}\lambda_6), \quad (2)$$

expressed in terms of physical baryon fields,[7] and constructed from the parity-conserving one-baryon octet matrix elements of $\mathcal{H}_G$, so that

$$\langle B_j | \mathcal{H}_W | B_i \rangle = \langle B_j | \mathcal{H}_G | B_i \rangle = 2\sqrt{2}\, \bar{u}_j (-i f_{6ji} F + d_{6ji} D) u_i \qquad (3)$$

in Gronau's[6] parametrization. His application of the symmetric quark model to a (semiphenomenological) current-algebraic treatment of nonleptonic hyperon decays finds a remarkable fit to the experimental amplitudes obtained for the values $F = 4.7\times10^{-5}$ MeV, $D/F = -0.85$. (This D/F ratio is to be compared with that required by the symmetric quark model[6] for the octet-baryon matrix elements of $\mathcal{H}_G$, $D/F = -1$.) As in our earlier[1] two-photon-decay calculation, and following Gronau,[6] we take $d/f = 1.8$, where d and f $(d+f=1)$ are the symmetric and antisymmetric MBB couplings.[8] The two possible mechanisms[9] for the decay $K_2^0 \to \pi^+\pi^-\gamma$ via baryon loop are graphically illustrated in Figs. 1 and 2. The effective interaction Lagrangian sufficient for the calculation of the four sets of graphs of Fig. 1 (these are the graphs involving the emission of "uncorrelated" pions) is[1]

$$\begin{aligned}\mathcal{L}_{\text{int}} = &-\sqrt{2}\, gf\, \mathrm{Tr}([\bar{B} i \gamma_5, B] M) \\ &+ \sqrt{2}\, gd\, \mathrm{Tr}(\{\bar{B} i \gamma_5, B\} M) \\ &- \tfrac{1}{2} e A_\mu\, \mathrm{Tr}([\bar{B}\gamma_\mu, B] Q) + \mathcal{L}_W,\end{aligned} \qquad (4)$$

with $\mathcal{L}_W = -\mathcal{H}_W$ and $Q = \lambda_3 + \lambda_8/\sqrt{3}$. Although it is characteristic of our *modus operandi*[1,6] to have SU(3) conserved at vertices but broken in hadron masses, we have, for simplicity, neglected such breaking in the baryon octet and replaced the differing intermediate baryon masses by a "mean" baryon mass (m) of 1 GeV.[1] Furthermore, in the case of "uncorrelated" pionic emission, we discuss the decay amplitude,[10]

$$\mathfrak{M}(K_2^0\to\pi^+\pi^-\gamma)=(16K_0p_0^+p_0^-q_0)^{1/2}\langle\gamma(q)\pi^+(p^+)\pi^-(p^-)\text{ out }|\mathcal{H}_W(0)|K_2^0(K)\rangle$$

$$=\epsilon(p_+p_-K\epsilon(q,\lambda))\,A(p_+^2,p_-^2,K^2,p_+\cdot p_-,p_+\cdot K,p_-\cdot K),\qquad(5)$$

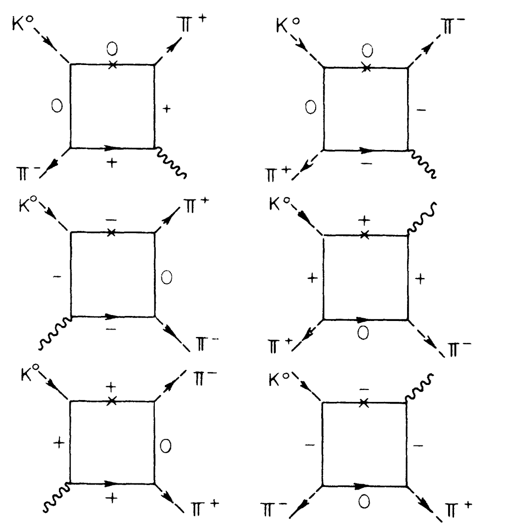
in the limit of vanishing invariant arguments, $p_+^2,\ldots,p_-\cdot K$,[11] this being the zeroth approximation in an expansion of A^{uncorr} in "external invariants." It is no minor undertaking to produce the result

$$A^{\text{uncorr}}=\frac{\sqrt{2}\,eg^3}{(4\pi)^2m^4}\,4\{fF(5f^2-\tfrac{23}{3}d^2)+dD(3f^2+\tfrac{37}{9}d^2)\}.\qquad(6)$$

This must be further corrected for the emission of "correlated" pions which could arise in the virtual decay of ρ^0 emitted from the fermion loop as shown in Fig. 2.[12] The enlargement of the effective Lagrangian of Ref. 1 [Eq. (4) above] to accommodate the additional ρ couplings which now enter in is

$$\mathcal{L}_{\text{int}}^{(\rho)}=-\sqrt{2}\,\varphi\,\text{Tr}([\bar{B}\gamma_\mu,B]V_\mu^{(3)})+\sqrt{2}\,\delta\,\text{Tr}(\{\bar{B}\gamma_\mu,B\}V_\mu^{(3)})-i(g_\rho/\sqrt{2})\,\text{Tr}([M,\partial_\mu M]V_\mu^{(3)}),\qquad(7)$$

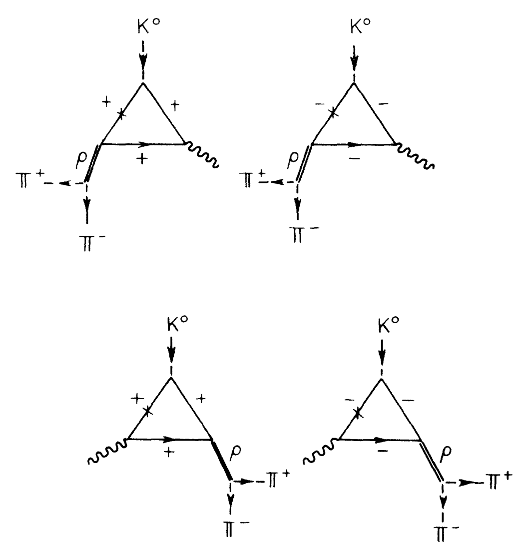
with $V_\mu^{(3)}=\rho_\mu^0\lambda_3/\sqrt{2}$, and where δ and φ are "average" couplings that simulate and average over the effects due to the γ_μ and $\sigma_{\mu\nu}$ couplings of vector mesons to baryons.[13] Following Gronau,[6] we take $\delta/\varphi=-0.5$, which, it has been noted,[6] is not inconsistent with similar values obtained from Regge-type analysis of meson-baryon scattering[14] and K^+ photoproduction.[15] Since Gronau's calculation[6] of the "half-parameter" $c=3.2\times10^{-9}$ MeV^{-1}, which fixes the contribution of the K^* pole in his fit, apparently uses the Barger-Olsson[14] values,



+ (three sets of similar diagrams with the x on other sides)

FIG. 1. Baryon-loop graphs for emission of "uncorrelated" pions in CP-conserving $K_2^0\to\pi^+\pi^-\gamma$ decay. The contribution to the invariant amplitude A from the first graph of the figure is

$$A^{(1)}=4\sqrt{2}\,eg^3[2df(-fF+\tfrac{1}{3}dD)-(f+d)^2(fF+\tfrac{1}{3}dD)-d^2(fF+\tfrac{1}{9}dD)-f^2(fF+dD)]\,(32\pi^2m^4)^{-1}.$$



+(two sets of similar diagrams with the x on other sides)

FIG. 2. Baryon-loop graphs for emission of "correlated" pions (from virtual ρ decay) in CP-conserving $K_2^0\to\pi^+\pi^-\gamma$ decay.

$g_\rho{}^2/4\pi \simeq 2.5$ and $2\varphi = (1.25/1.03)g_\rho = 6.7$,

these values are adopted in this calculation as well. We find

$$A^{(\rho)} = \frac{\sqrt{2}egg_\rho\phi}{(4\pi)^2 m^2} 16\left[3(fF + dD) - \frac{\delta}{\phi}(fD + dF)\right]\frac{1}{m_K{}^2 - m_\rho{}^2 - 2m_K q}. \tag{8}$$

Note that in the approximate evaluation of this *destructively interfering*[16] contribution to the invariant amplitude A, we do *not* neglect the (important) variation of the ρ-meson propagator with photon energy q. The CP-invariant rate is given by[17]

$$R(K_2^0 \to \pi^+\pi^-\gamma) = \int_0^{(m_K{}^2 - 4m_\pi{}^2)/2m_K} dq\left(\frac{dR}{dq}\right), \tag{9}$$

with

$$\frac{dR}{dq} = \frac{m_K}{384\pi^3}\,|A^{\text{uncorr}} + A^{(\rho)}|^2 q^3(m_K{}^2 - 4m_\pi{}^2 - 2m_K q)\left(1 - \frac{4m_\pi{}^2}{m_K{}^2 - 2m_K q}\right)^{1/2}; \tag{10}$$

the photon energy distributions dR/dq with and without the effect of virtual ρ decay are shown in Fig. 3. We would like to argue that the theoretical branching ratio

$$\left[\frac{R(K_2^0 \to \pi^+\pi^-\gamma)}{R(K_2^0 \to \gamma\gamma)}\right]_{\text{theor}}$$

is more reliable[3,4] than the result given by expression (9) [with $R(K_2^0 \to \pi^+\pi^-\gamma) = 1.45 \times 10^{-3}\ \text{sec}^{-1}$], and on this basis we predict a branching ratio for this process, with *no adjustable parameters*,

$$\begin{aligned} r &= \frac{R(K_2^0 \to \pi^+\pi^-\gamma)}{R(K_2^0 \to \text{all})} \\ &= \left[\frac{R(K_2^0 \to \pi^+\pi^-\gamma)}{R(K_2^0 \to \gamma\gamma)}\right]_{\text{theor}} \left[\frac{R(K_2^0 \to \gamma\gamma)}{R(K_2^0 \to \text{all})}\right]_{\text{exp}} \\ &= 3.0 \times 10^{-4}. \end{aligned} \tag{11}$$

It is interesting to note that in the tree-graph model of Moshe and Singer,[17] which we regard as the most successful of such models in describing the weak radiative decays of K mesons as well as the two-photon decay of the K_2^0,[5] this branching ratio is predicted[16] to occur at the relative rate $2.6 \times 10^{-4} < r < 4 \times 10^{-4}$.

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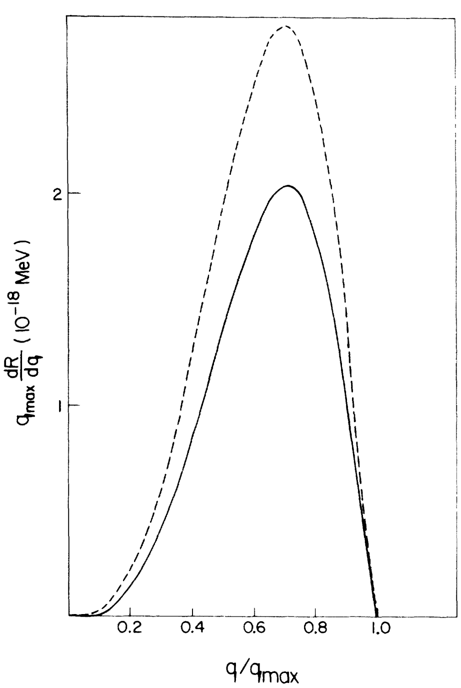

FIG. 3. Photon-energy distributions for the CP-invariant $K_2^0 \to \pi^+\pi^-\gamma$ decay vs photon energy q in units of the maximum photon energy q_{max}. The dashed curve is the distribution for "uncorrelated" pionic emission only; the solid curve is the distribution including virtual ρ^0 decay ("correlated" pionic emission).

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[1]R. Rockmore and T. F. Wong, Phys. Rev. Lett. 28, 1736 (1972).

[2]J. Steinberger, Phys. Rev. 76, 1180 (1949).

[3]In their recent comment, R. Amado and J. Noble [Phys. Rev. D 6, 2696 (1972)] point out that our "explanation" (Ref. 1) of the $K_2^0 \to \gamma\gamma$ rate would give $R(K_2^0 \to \mu^+\mu^-)/R(K_2^0 \to \gamma\gamma) \sim 5 \times 10^{-5}$, "some four times the unitarity bound" for it. However, this is quite compatible with the recent result of the Columbia-CERN-NYU group reported at the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972 (unpublished), $R(K_2^0 \to \mu^+\mu^-)/R(K_2^0 \to \text{all}) = (1 \pm 0.4) \times 10^{-8}$, which implies $[R(K_2^0 \to \mu^+\mu^-)/R(K_2^0 \to \gamma\gamma)]_{\text{exp}} = (2.0 \pm 0.5) \times 10^{-5}$.

[4]Note that the qualitative agreement with experiment found in the loop model for $K_2^0 \to \gamma\gamma$ decay may be radically improved in the Duffin-Kemmer-Petiau (DKP) description of mesons lately put forward by E. Fischbach *et al.* [Phys. Rev. Lett. 29, 1046 (1972)] to resolve the disagreement between experiment and the simple SU(3) prediction for the decay-width ratio $R(\eta \to \gamma\gamma)/R(\pi^0 \to \gamma\gamma)$. If one ignores the perturbation on the usual f and d mixing parameters expected in the DKP theory, our earlier result is to be multiplied by the factor $(m_K/m_\pi) = 3.7$. The resulting range of values, $R(K_2^0 \to \gamma\gamma)/R(K_2^0 \to \text{all}) = (2.9 - 5) \times 10^{-4}$, then compares more favorably with the present branching ratio of 4.9×10^{-4}. One of us (R.R.) wishes to thank Dr. M. M. Nieto for a useful discussion on this point.

[5]R. Rockmore, Phys. Rev. 182, 1512 (1969); M. Moshe and P. Singer, Phys. Rev. Lett. 27, 1685 (1971).

[6]M. Gronau, Phys. Rev. Lett. 28, 188 (1972); Phys. Rev. D 5, 118 (1972).

[7]$B = \lambda_i \psi_i / \sqrt{2}$ is the traceless baryon matrix; $M = \lambda_i \phi_i / \sqrt{5}$ is the traceless Hermitian meson matrix as in S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966), Chap. 18.

[8]We also adopt the value of the pion-nucleon coupling $g^2/4\pi = 14.6$, used by Gronau (Ref. 6) in his fit.

[9]Note that the contribution from the virtual emission of a scalar meson (say, ϵ) vanishes under the trace operation.

[10]We use the notation $\epsilon(abcd) = \epsilon_{\mu\nu\rho\sigma} a_\mu b_\nu c_\rho d_\sigma$.

[11]We estimate the error involved in this last approximation at less than 10% of our result.

[12]Thus, the diagrams of Figs. 1 and 2 correspond in a sense to those of the tree-graph theory (Ref. 5) in which there occur (virtual) vector mesons in the t (or u) and s channels.

[13]J. J. Sakurai, Phys. Rev. 132, 434 (1963).

[14]V. Barger and M. Olsson, Phys. Rev. 146, 1080 (1966).

[15]C. Michael and R. Odorico, Phys. Lett. 34B, 422 (1971).

[16]The ρ contribution is found to reduce the rate for "uncorrelated" emission by approximately 30%.

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Theory of Boson Resonances*

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We propose a bootstrap theory of boson resonances which consists of new self-consistency conditions for boson resonances. We derive these conditions by saturating in terms of resonances the once-subtracted dispersion relations for their two-body decay amplitudes where subtractions are introduced by choosing appropriately an arbitrary parameter that appears in the usual continuations of these amplitudes. We then study the simplest solution to all these conditions in the sense that the resonances are minimal in their numbers, with the input that pseudoscalar and vector mesons form the observed nonets. The boson spectrum predicted by this solution is not very different from the observed one, some crucial tests of which are also given.

I. INTRODUCTION AND SUMMARY

It is the purpose of this paper to discuss the new self-consistency conditions for boson resonances, which are the once-subtracted dispersion relations for their two-body decay amplitudes (or more generally three-point functions), saturated in terms of resonances. We expect these conditions not only to be well satisfied, but also to be able to explain the entire spectrum of boson resonances[1] when some of them are known. In other words, we propose that these conditions constitute a new bootstrap theory of boson resonances. In this theory we bypass such questions as their internal structure in terms of quarks or singularities in the angular momentum plane. We can incorporate unitary symmetry partially into this theory by requiring as an input that the least massive me-