Regge Pole and Cut Model of Pion-Nucleon Charge Exchange

J. W. Coleman* and R. C. Johnson
Mathematics Department, Durham University, United Kingdom
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The reaction $\pi^- p \to \pi^0 n$ is described quantitatively in a model involving a nonsense-choosing ρ Regge pole modified by a mainly imaginary and spin-nonflip $\rho \otimes P$-type Regge cut. The details are determined with the aid of continuous-moment finite-energy sum rules, as well as scattering data. The Regge cut is found to have peaked t dependence and approximately evasive behavior at $t = 0$. This feature (in addition to its phase and spin coupling) disagrees strongly with predictions of conventional Regge-cut models. Some implications are mentioned.

I. INTRODUCTION

It now seems clear that Regge cuts play an important part in high-energy hadron scattering, and in nondiffractive meson-baryon processes a tentative picture has emerged for the natural-parity mesonic exchanges.[1] The evidence suggests a scheme of exchange-degenerate Regge poles modified by cut terms that are strongly and destructively interfering in certain spin-nonflip amplitudes, and relatively weak (perhaps negligible) in certain spin-flip amplitudes. This paper presents a quantitative analysis in these terms of $\pi^- p \to \pi^0 n$ at high energies and small angles.

We discuss $\pi^- p$ charge exchange because of its cleanness (only ρ quantum numbers exchanged) and because of the quantity of data available.[2] The data include low-energy phase-shift analyses[3] which allow exploitation of analyticity through sum rules.[4] Also near 6 GeV/c there are enough πN measurements to permit a complete amplitude decomposition,[5] which gives further information. As a result, with reasonable assumptions, it is possible to make a separation of Regge-pole and Regge-cut effects.

In Sec. II we recall the evidence for the proposed Regge-pole and -cut description of $\pi^- p \to \pi^0 n$, and go on to formulate a specific model. Sec. III examines the agreement between the model and the data. A fit is made to high-energy measurements and continuous-moment sum rules[6] (CMSR) together. The CMSR constraints lead to a better understanding of the process than was achieved in our previous work.[7]

It is found that indeed in the spin-nonflip amplitude a large cut contribution is required, but its phase and t dependence are quite different from the predictions of either absorptive[8] or eikonal[9] models.

To a very good first approximation the spin-flip amplitude is described by the pole term alone, but both CMSR and polarization data[10] are somewhat better accounted for with a small secondary contribution.

The conclusions are summarized in Sec. IV.

II. MODEL

A. General Features

We use amplitudes A' and B, convenient for CMSR because of their crossing properties.[11] The following features of these amplitudes in $\pi^- p \to \pi^0 n$ at high energy and small angles are known:

(i) the forward dip of $d\sigma/dt$ (Ref. 12) shows that B dominates;

(ii) the power-law energy dependence of both $d\sigma/dt$ (Ref. 13) and the integrated cross section[14] is what is expected from ρ-Regge-pole exchange, with canonical trajectory

$$\alpha(t) = 0.55 + t \ (\text{GeV}/c)^2; \tag{1}$$

(iii) from (i) and (ii) together, the natural explanation of the dip of $d\sigma/dt$ near $t = -0.55$ (Ref. 12) is the presence of the ρ-pole nonsense factor in B at $\alpha = 0$;

(iv) the mirror symmetry and approximate double zero near $t = -0.55$ in the polarization $\mathcal{P}$ in the elastic $\pi^\pm p$ processes[15] is thus consistently accounted for[16] by the double zero in ReB;

(v) the crossover effect[17] in the elastic $\pi^\pm p$ angular distributions[18] indicates a zero in ImA' near $t = -0.2$;

(vi) detailed fits,[19] amplitude analyses,[5,20] and sum-rule integrals[4,19] all lend support to the preceding deductions, and in addition suggest that ReA', if it vanishes at all, has a zero near $t = -0.5$ or -0.6.

The general structure of the spin-flip amplitude B therefore may be described by the ρ Regge pole alone. The most economical explanation of the behavior of the nonflip amplitude A' is that the nonsense-choosing ρ-pole exchange is modified by a destructive and mainly imaginary contribution. The crossover effect is thus explained without factorization difficulties,[21] and interference between the ρ pole in B and the secondary term in A' provides a mechanism for the nonzero polarization $\mathcal{P}$.[10]

This viewpoint is reasonably consistent with the over-all systematics of similar processes involving the natural-parity exchanges related by SU(3) and duality.[1]

The qualitative features of the secondary contributions (destructive sign, larger coupling to the spin-nonflip amplitude) agree with general expectations based on the physical mechanism of absorption.[8,22] It is therefore natural to identify the corrections to the ρ Regge pole with the absorptively generated $\rho\otimes P$ Regge cut. However, the details of both the strength and phase of the $\rho\otimes P$ cut, as conventionally calculated in either the absorptive or eikonal framework, are known to be in clear disagreement with the data.[23,24]

Various modifications of the usual models have been proposed[23,25] with differing degrees of justification and of success. Here we introduce as the correction to the ρ Regge pole a phenomenological $\rho\otimes P$ Regge-cut parametrization with some generally accepted features,[26] but with its strength and phase described by parameters free to be fixed by the data. In particular as a first approximation its coupling to B is taken to be zero. We assume, consistent with exchange degeneracy etc., that Regge-Regge cuts tend to cancel[27] and so perhaps can be safely ignored at this stage.

B. Amplitude

The ρ-meson contributions to A' and νB (crossing-odd) are written

$$R_r(\nu, t) = i\gamma_r \alpha(t) e^{a_r t}(-i\nu)^{\alpha(t)}, \tag{2}$$

where r (= A, B) labels the amplitude, $\nu=(s-u)/4m$, and the trajectory $\alpha(t)$ is given by (1).

The constants a_r in the exponential residue factors are expected to be of order 2–6 $(\text{GeV}/c)^{-2}$, as is typical of many high-energy forward peaks,[28] and the remaining over-all couplings γ_r are assumed constant.

The contributions (to A' only) of the $\rho\otimes P$ Regge cut is represented by the expression[7,29]

$$C(\nu, t) = i\lambda(t)(-i\nu)^{\alpha_c(t)}[c+\ln(-i\nu)]^{\beta}, \tag{3}$$

where $\alpha_c(t)$ is the branch-point trajectory.

We assume that the cut originates mainly from simultaneous exchange of the ρ pole and an effective Pomeranchukon pole with trajectory $1+\frac{1}{2}t$.[30] Thus we have

$$\alpha_c(t) = 0.55 + \tfrac{1}{3}t. \tag{4}$$

The presence of the logarithmic factor is typical of a Regge cut.[26] The parameter β is linked to the behavior of the discontinuity near $J=\alpha_c(t)$, and in the t-channel physical region where nonlinear unitarity applies is constrained by $\beta < -1$.[31]

Most models in fact suggest the constant value $\beta=-1$. We may suspect that perhaps $\beta+1$ is small. The fits shown here have $\beta=-1.4$, constant in the region of interest.[8,9,26]

The parameter c controls the energy-scale of the logarithmic factor, and therefore has an important influence on the phase of the cut. Defining a "phase-effective trajectory" $\bar{\alpha}(\nu, t)$ for the Regge cut by

$$\text{Re}C/\text{Im}C = \tan\left(\frac{\pi}{2}\bar{\alpha}(\nu, t)\right), \tag{5}$$

we have for the parametrization (3) that

$$\bar{\alpha}(\nu, t) = \alpha_c(t) + \frac{2\beta}{\pi}\arctan\left(\frac{\pi}{2(c+\ln\nu)}\right). \tag{6}$$

The larger c is, the slower the energy-variation of the log term, and the closer are both energy-dependence and phase to that of an effective pole with trajectory $\alpha_c(t)$. Since $\beta<0$, this can be interpreted as a bunching of the J-plane discontinuity near the branch point, and conversely a smaller value of c may be viewed as an effective smearing out of the discontinuity.[29]

In the absorptive and eikonal models[8,9] (AE models) – irrespective of dip mechanism – the numerical value of c is fixed by the two exchanged poles. We find [see, e.g., Ref. 26]

$$c = \frac{a_1 + a_2}{\alpha_1' + \alpha_2'}, \tag{7}$$

where $a_{1,2}$ are exponential residue factors, and $\alpha_{1,2}'$ are trajectory slopes. Therefore, for the $\rho\otimes P$ cut, the AE models predict typically $c \simeq 4$.

In the Gribov-Reggeon calculus[32] the value of c is similarly related to the exchanged trajectories and to the slopes of the two contributing fixed-pole residues. For these there are no convincing *a priori* estimates.

To investigate a variety of phase possibilities we take in this analysis c as a (t-independent) free parameter, to be determined by the data. Similarly the coupling function $\lambda(t)$ appearing in (3) is left to be fixed phenomenologically, although of course its over-all sign is expected to make the cut a destructive correction to the pole.

III. DATA AND FITS

A. Data

Our amplitudes A' and B are identical to the A'^-, and B^- of Barger and Phillips.[19] The formulas connecting them to the high-energy observables

$$\Delta\sigma \equiv \sigma_T(\pi^- p) - \sigma_T(\pi^+ p),$$

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n),$$

$$\mathcal{P}(\pi^- p \to \pi^0 n)$$

are standard.[33]

The following subset of the available high-energy data was used to fix the model parameters by least-squares fitting:

$$\Delta\sigma, \quad p_{\rm lab} = 8\text{–}22 \text{ GeV}/c \qquad \text{(Ref. 34)},$$

$$\frac{d\sigma}{dt}, \quad p_{\rm lab} = 10.0, 13.3, 18.2 \text{ GeV}/c, \quad |t| \le 1.4 \ (\text{GeV}/c)^2 \qquad \text{(Ref. 12)},$$

$$\mathcal{P}, \quad p_{\rm lab} = 5, 8 \text{ GeV}/c, \quad |t| \le 1.4 \ (\text{GeV}/c)^2 \qquad \text{(Ref. 10)}.$$

These measurements, taken with the CMSR, are more than sufficient to determine the model and so give a good description of intermediate data.

The CMSR evaluations were made using the 1971 CERN phase shifts,[35] with cutoff $N = 2.075$ GeV (at $t = 0$) and covering the range $0 \le |t| \le 1$ (GeV/c)^2.

We used the integrals

$$\frac{1}{N}\int^N \mathrm{Im}[(\nu_0^{\,2} - \nu^2)^{-(\epsilon+1)/2} F(\nu)]\, d\nu, \tag{8}$$

where F is crossing-odd ($= A'$ or νB) and the physical threshold is $\nu = \nu_0$. Nucleon pole terms are understood to be included. The range $-3 \le \epsilon \le 0$ was used, so that the sum rules sampled both real and imaginary parts of the amplitudes in more than one fashion, without undue emphasis on any particular region of energy.

Some idea of the uncertainties in the CMSR integrals can be gained by comparison with evaluations made from other phase-shift analyses.[3] As expected, it is found that at small $|t|$ higher-moment sum rules are the less reliable since the integrals are weighted towards the cutoff where small non-Regge amplitude fluctuations are still evident. At larger t values the lower-moment sum rules become the more suspect, because they emphasize amplitudes which are extrapolated outside their physical region, and indeed for $t < -0.52$ the s- and u-channel cuts overlap.

The utility of the sum rules is that they constrain individual amplitudes directly. Here the CMSR for A' are particularly important, because for $t \ne 0$ the best-measured high-energy bilinear $(d\sigma/dt)$ does not determine well this relatively small amplitude.

The zero-moment ($\epsilon = -1$) contributions of the individual pole and cut terms to the CMSR (8) are as follows: For the pole term R of (2) we have[4]

$$\gamma_r \alpha(t) e^{a_r t} \frac{N^{\alpha(t)}}{\alpha(t)+1} \cos\frac{\pi\alpha(t)}{2}. \tag{9}$$

For the Regge cut (3) we find similarly the contribution[29]

$$\lambda(t)\frac{N^{\alpha_c(t)}}{\alpha_c(t)+1} q(N)\, \mathrm{Re}[e^{-i\pi\bar\alpha(N,t)/2} g(\beta, x)], \tag{10}$$

where $\bar\alpha(\nu, t)$ is given by (6), and

$$q(\nu) = [(c + \ln\nu)^2 + \tfrac{1}{4}\pi^2]^{\beta/2},$$

$$x = [\alpha_c(t) + 1][c + \ln(-iN)].$$

The phase function $g(\beta, x)$ is given by

$$g(\beta, x) = e^{-x} x^{-\beta}\int^x (x')^\beta e^{x'} dx', \tag{11}$$

with $g(0, x) = g(\beta, \infty) = 1$.

The cut expression (10) is written to emphasize its similarity to the pole term (9). As discussed in detail elsewhere,[29] the Regge cut appears in the sum rules like an effective pole displaced in the J plane from $\alpha = \alpha_c$ to $\alpha \approx \bar\alpha(N)$, since $g \approx 1$ to zeroth order in $1/x$.

For nonzero moments ($\epsilon \ne -1$) we replace as appropriate

$$\alpha \to \alpha - \epsilon - 1,$$

$$\alpha_c \to \alpha_c - \epsilon - 1$$

in these expressions. The errors thus committed are of order $(\nu_0/N)^2$, which are negligible.

B. Fits

The $d\sigma/dt$ data determine the parameters of the pole amplitude describing B, up to an over-all sign. This is fixed by the CMSR (consistent with the $\pi^\pm p$ elastic polarizations). At $t = 0$, $\mathrm{Im}A'$ is determined by $\Delta\sigma$, $\mathrm{Re}A'$ is fixed by the forward $d\sigma/dt$, and both are constrained at lower energies by the CMSR. For $t \ne 0$, the main information on A' comes from $\mathcal{P}$ and the CMSR together.

Sample fits to data are shown in Figs. 1–6. Figure 1 shows the CMSR for A' at fixed moment as functions of t. The sum rules with $\epsilon = -1, -3$ involve only the imaginary part, and show a clear zero of $\mathrm{Im}A'$ near $t = -0.1$. This is the crossover zero.[17] The sum rules involving only $\mathrm{Re}A'$ show no such zero; there is a hint of a possible minimum in the region $t = -0.4$ to -0.6.

A nonsense-choosing ρ alone would show a simple zero of ImA' near $t=0.55$, and a double zero of ReA' at the same place [with trajectory given by (1)]. Figure 1 therefore suggests (as already outlined above) that the cut correction is destructive, and mainly imaginary. This is the case if the phase-effective cut trajectory $\overline{\alpha}(\nu, t)$ defined by (5) is close to zero. With the present parametrization [$\overline{\alpha}(\nu, t)$ given by (6), $\alpha_c(t) \simeq 0.55+\frac{1}{3}t$, and $\beta \simeq -1$], $\overline{\alpha}$ is small at intermediate energies if $c \simeq 0-1$.

This value is to be contrasted with the AE prediction $c \simeq 4$, which leads to a Regge cut with a much larger real part. Therefore a typical AE prediction for A' has destructive pole (and/or cut) interference in both the real and imaginary parts, so that also Re$A'=0$ at small $|t|$. This is a feature which besides disagreeing with the CMSR leads to a direct conflict between the AE-model polarization predictions and the data.

In the forward direction the phase of A' over a range of energy can be determined directly from the data [$\Delta\sigma$, $d\sigma/dt$ $(t=0)$] and has been calculated

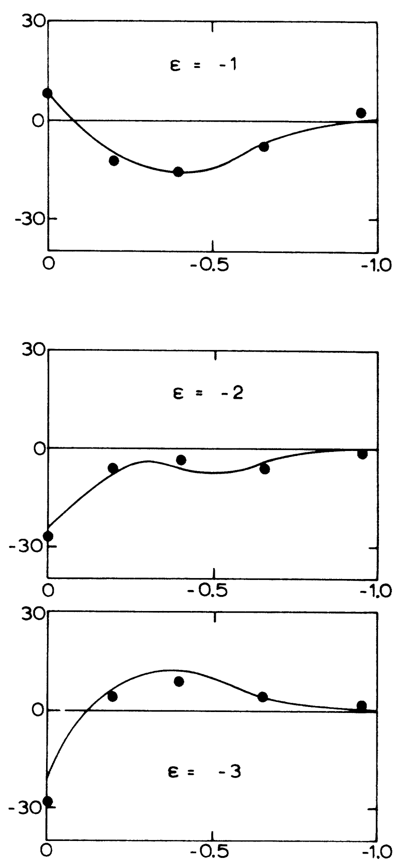

FIG. 1. Sum rules for A' (in GeV^{-1}) at fixed moment as functions of t [in $(\text{GeV}/c)^2$]. Values $\epsilon=-1,-3$ involve only ImA', and $\epsilon=-2$ involves only ReA'. Lines are pole + cut model fit.

in good agreement from forward dispersion relations.[36] The phase so determined (Fig. 2) is completely consistent with pure ρ Regge-pole exchange with $\alpha(0)\simeq 0.55$ [as required by $d\sigma/dt$, Eq. (1)]. Several detailed calculations have shown in particular that at $t=0$ possible secondary terms with phase-effective trajectory near zero must be relatively very small.[19,29,37] Therefore we conclude that in the energy range under consideration the mainly imaginary Regge-cut term in A' approximately decouples at $t=0$.

Consequently, as $-t$ increases, the strength of the cut increases rapidly at first to cancel the pole at the crossover zero. This is consistent with the observed large positive polarization $\mathcal{P}$ for $-t \simeq 0.2-0.4$ (Ref. 10), which arises mainly from interference between the spin-flip pole and the nonflip cut. The smaller values of $\mathcal{P}$ for $|t| \gtrsim 0.7$ (Ref. 10) suggest that the cut dies away at larger momentum transfers.

So the coupling function $\lambda(t)$ appearing in the cut amplitude (3) is deduced to be large near $-t=0.3$, falling rapidly both as $t\to 0$ and as $|t|\to\infty$. Various successful parametrizations were found, differing in detail but with these general features. The fits quoted in the figures have

$$\lambda(t) = -0.72\gamma_A \exp[-8.7(t+0.33)^2], \qquad (12)$$

where $\gamma_A = 21.8\ (\text{GeV}/c)^{-1}$. The other parameters are $\gamma_B = 165.0\ (\text{GeV}/c)^{-2}$, $a_A = 5.0\ (\text{GeV}/c)^{-2}$, $a_B = 1.04\ (\text{GeV}/c)^{-2}$, and $c=1.64$.

We contrast (12) with the form of $\lambda(t)$ predicted by the usual absorptive and eikonal models.[8,9,26] For two Regge poles with residue dependence

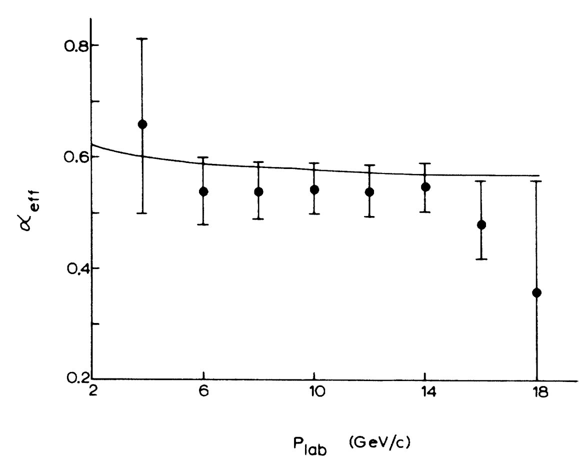

FIG. 2. Plot of $\alpha_{\text{eff}} = (2/\pi)\arctan(\text{Re}A'/\text{Im}A')$, where the real and imaginary parts of A' are calculated direct from $\Delta\sigma$ and $d\sigma/dt\,(t=0)$. The line is the fit described in the text, with ρ intercept $\alpha(0)=0.55$. The effect on the phase of a relatively small amount of destructive cut is clearly noticeable.

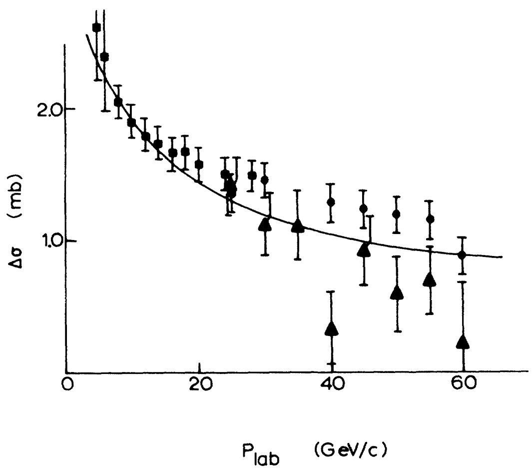

FIG. 3. Fit to Brookhaven measurements of Δσ (■) (Ref. 34) extrapolated and compared with Serpukhov data on $\sigma_T(\pi^- p) - \sigma_T(\pi^+ p)$ (●) and $\sigma_T(\pi^+ n) - \sigma_T(\pi^+ p)$ (▲) (Refs. 14, 38).

$\exp(a_{1,2}t)$, the Regge cut arising from their simultaneous exchange has

$$\lambda(t) \propto \exp\left(\frac{a_1 a_2}{a_1 + a_2} t\right). \tag{13}$$

This falls smoothly from $t=0$ more slowly than either pole, reflecting its relatively large content of low partial waves. The form (13) (used previously in Ref. 7) conflicts seriously with the A' CMSR at small t, as well as distorting the fit to $\Delta\sigma$ and $d\sigma/dt$ $(t=0)$ at higher energies.

The fit to the Brookhaven $\Delta\sigma$ data is shown in Fig. 3, with predictions up to 60 GeV/c compared with Serpukhov measurements of both $\sigma_T(\pi^- p) - \sigma_T(\pi^+ p)$ (Ref. 14) and $\sigma_T(\pi^+ n) - \sigma_T(\pi^+ p)$ (Ref. 38). The former lie consistently above the curve; the latter are less regular but fall mainly below. If

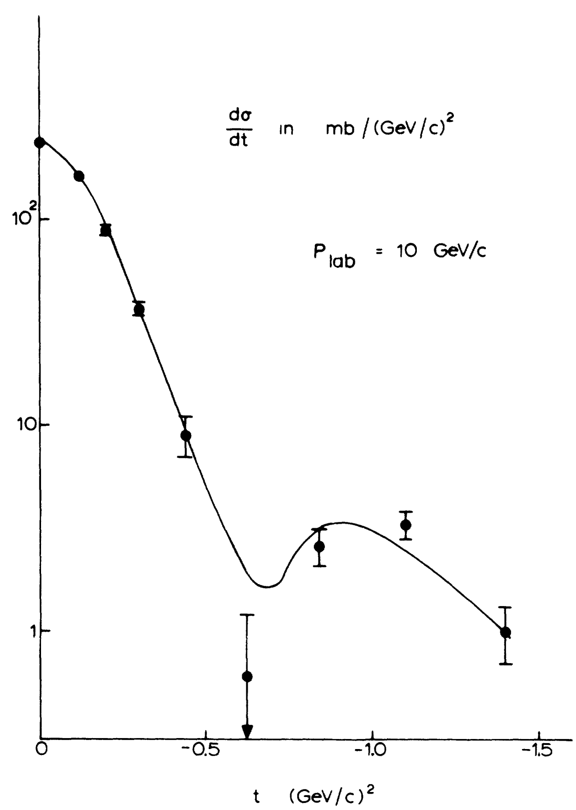

FIG. 4. Sample fit to some 10-GeV/c $d\sigma/dt$ measurements (Ref. 12).

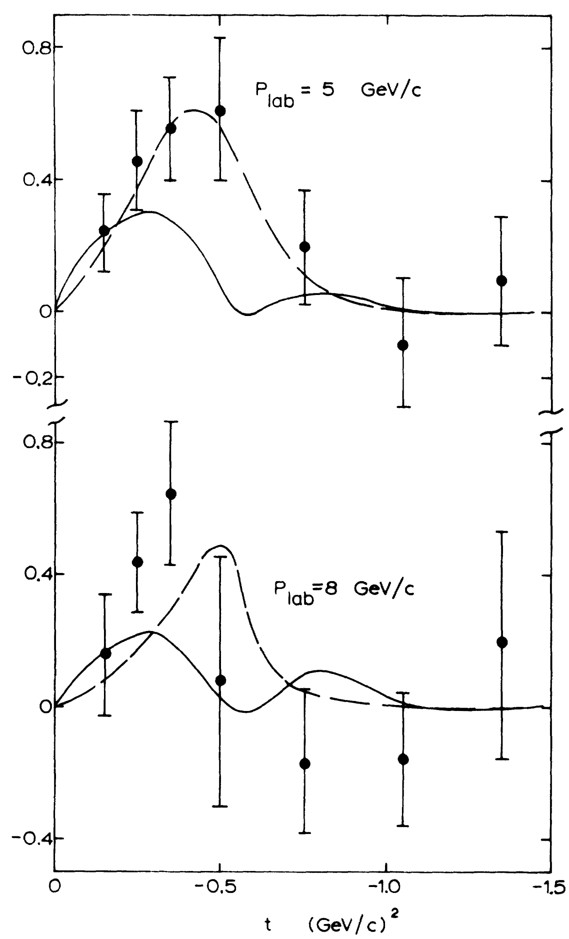

FIG. 5. Fits to latest polarization data at 5 and 8 GeV/c (Ref. 10). Full line is the model with pure ρ-Regge-pole B amplitude, dashed line with $\rho + \rho'$ B amplitude (Ref. 19).

the proton data are taken the more seriously, then their over-all energy dependence may indicate the growing strength of destructive cuts in A' above about 20 GeV/c. However analyticity would reflect such contributions into ReA' at lower energies and a large effect may conflict with measured $d\sigma/dt$ at $t=0$. In view of the large systematic uncertainties in the total cross sections and the smallness of the differences involved, the issue cannot be resolved conclusively.

The fit to $d\sigma/dt$ is illustrated in Fig. 4. There is no evidence here for secondary flip contributions, and the ρ-pole parameters in the B amplitude are well determined.

Figure 5 shows the polarization $\mathcal{P}$ which because of the cut phase is generally correct in sign but is however rather small in magnitude. (This was found before Ref. 7.) Moreover, the mechanism of interference between flip pole and nonflip cut alone predicts $\mathcal{P}=0$ at $t=-0.55$. This is not well supported by the data[10] especially at 5 GeV/c.

However the CMSR for B (Fig. 6) hint at the possible presence of a small secondary term in this amplitude, since they show a sign-change in ImB nearer to $t\simeq -0.4$ than to the dip in $d\sigma/dt$ at $t \simeq -0.55$. The CMSR do not determine a secondary term well (and $d\sigma/dt$ not at all), although a $\rho\otimes P$ cut contribution may be naturally expected. By way of illustration Fig. 5 includes predictions for $\mathcal{P}$ using Barger and Phillips $\rho+\rho'$ B parametrization,[19] along with our $(\rho+\rho\otimes P)$ A' amplitude. The

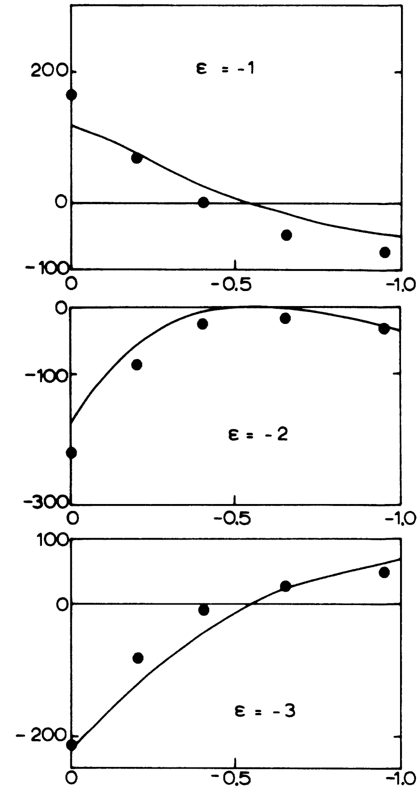

FIG. 6. Sum rules for νB (in GeV^{-1}) at fixed moment as functions of t [in (GeV/c)^2]. The lines are predictions of the ρ-pole model for B, determined by fitting $d\sigma/dt$.

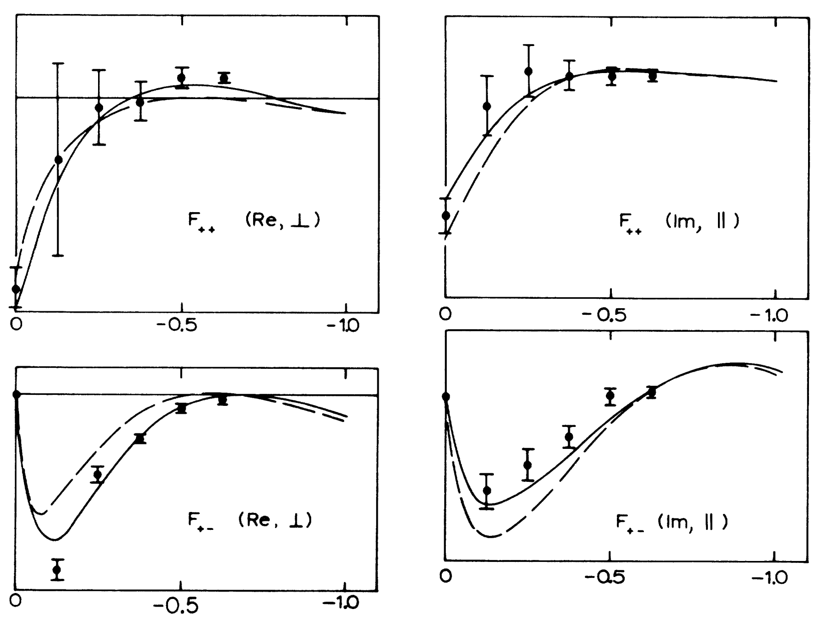

FIG. 7. Predicted s-channel helicity amplitudes $F_{+\pm}$ (in arbitrary units on a linear scale) for $\pi^- p \to \pi^0 n$ at 6 GeV/c as functions of t [in (GeV/c)^2] compared with the results of the amplitude analysis, Ref. 5. Full lines are components parallel (‖) and perpendicular (⊥) to the imaginary part of the nonflip t-channel isoscalar amplitude of Ref. 19; dashed lines are real and imaginary parts.

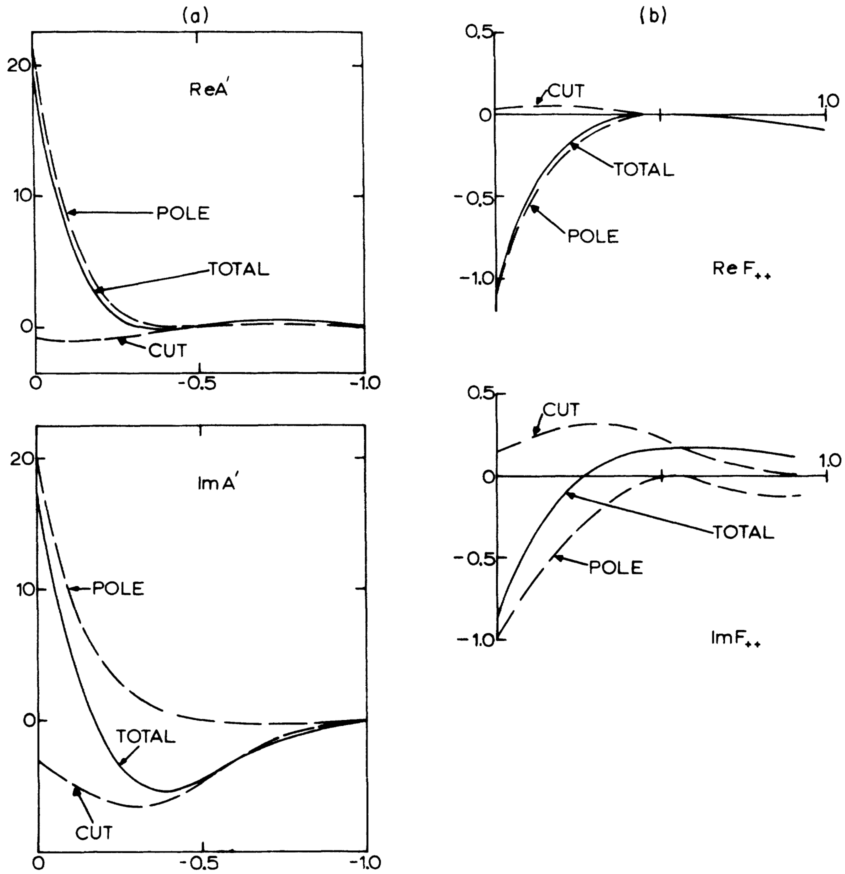

FIG. 8. Relative pole and cut contributions (arbitrary units on a linear scale) to (a) A' and (b) F_{++} at 6 GeV/c against t in (GeV/c)^2.

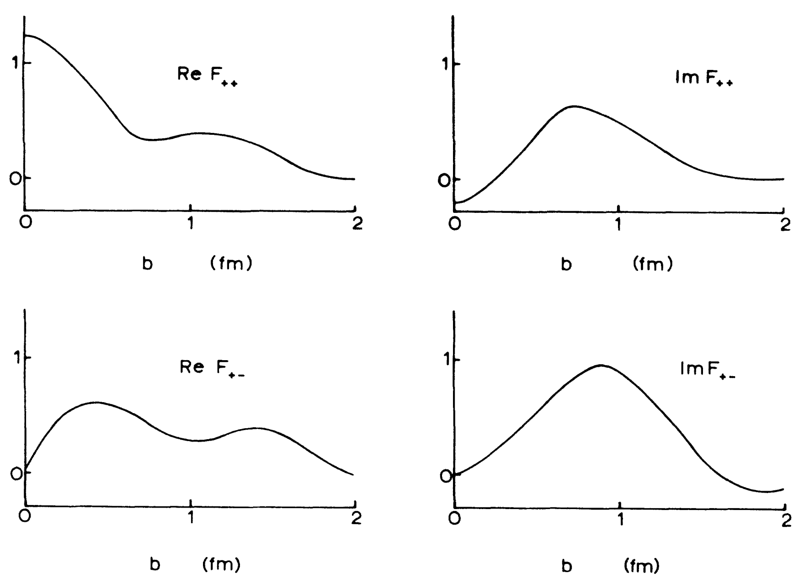

FIG. 9. Impact-parameter (b) distribution of s-channel helicity amplitudes according to Eq. (14) of the text, in arbitrary units on a linear scale.

agreement with the data is now excellent.

The 6-GeV/c s-channel helicity amplitudes of the model with the ρ pole alone in B are compared in Fig. 7 with the results of the amplitude analysis of Halzen and Michael.[5] Predicted components parallel and perpendicular to the helicity-nonflip t-channel isoscalar amplitudes of Ref. 19 are shown. The agreement here is good, emphasizing the smallness of any extra contributions to the helicity-flip amplitude.

Figure 8 shows corresponding individual pole and cut contributions to A' and to F_{++} at 6 GeV/c. The mainly imaginary cut term, with peaked t dependence, is quite evident in both amplitudes.

Figure 9 is a plot of impact-parameter distributions of the helicity amplitudes, given by

$$F_{+\pm}(b) = \frac{1}{s}\int_{-\infty}^{0} F_{+\pm}(t) J_{0,1}(b\sqrt{-t})dt, \qquad (14)$$

where the order 0 (1) of the Bessel function corresponds to nonflip (flip). The integral is truncated at $t = -1.5$ $(\text{GeV}/c)^2$, beyond which the amplitudes are negligible.

The peripherality of the imaginary parts is clearly shown. ReF_{+-} is less peripheral, and ReF_{++} is dominantly central but contains a substantial peripheral component. The agreement with the results of other similar analyses[20, 39] is very good.

The reason for making this decomposition is that $F_{+\pm}(b)$ appear natural for discussion of physical absorptive effects. The kind of absorptive ("elastic") amplitude which can strongly affect ImF_{++} while leaving its real part essentially unaffected has been discussed elsewhere.[40] In order to make the cut contribution to the spin-nonflip amplitude tend to vanish at $t = 0$ as well, the "elastic" amplitude needs not only the usual central and mainly imaginary ("Pomeranchukon") piece, but also a peripheral component of opposite phase.

IV. CONCLUSIONS

We summarize several points.

The proposed Regge pole and cut model of $\pi^- p \to \pi^0 n$ is quite successful as a representation of the data. Up to small corrections (affecting mainly the magnitude of the polarization) B is described by the nonsense-choosing ρ pole alone, whereas A' requires an additional strongly destructive and mostly imaginary cut term. A $\rho \otimes P$ cut ansatz accommodates the data and sum rules, although presumably effectively including possible ρ' and/or residual uncanceled[27] Regge-Regge cut ($\rho \otimes f^0$, $A_2 \otimes \omega$) contributions.

We reemphasize that not only are the phase and the spin-coupling characteristics of the phenomenological $\rho \otimes P$ cut in conflict with conventional prescriptions, but so also are its peaked t-dependence and near-evasive forward behavior. These features need theoretical understanding.

However we recall[40] that if attention is confined just to nonflip scattering, and if the absorptive framework is appropriate, then the kind of "elastic" amplitude that generates the Regge cut here gives also some limited understanding of other related exchanges (ω,[41] f^0, A_2, K^*-K^{**}) in meson-baryon processes.

Furthermore,[40] the necessary peripheral component in the "elastic" amplitude, if present in the physical diffractive amplitude, may be the origin of the observed small-angle structure in pp scattering at high energies. Detailed suggestions on these lines have been made recently.[42]

We would like to thank Geoffrey Bennett for calculating the data points in Fig. 2.

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*Science Research Council Research Student.

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Odorico's Bootstrap of Pseudoscalar Mesons*

Richard H. Capps
Physics Department, Purdue University, West Lafayette, Indiana 47906
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Odorico has derived bootstrap conditions for P (pseudoscalar) meson interactions from the hypothesis that the zeros in PP scattering amplitudes are linear in the Mandelstam plane. We explain the relation between this hypothesis and an earlier bootstrap hypothesis based on duality. If the solution proposed by Odorico for the PP scattering bootstrap is correct, other bootstrap conditions involving virtual P mesons suggest that a tenth member should be associated with the P nonet. If the 1422-MeV E meson is pseudoscalar, it is a possible candidate for the tenth member. It is shown that in some, but not all, hadron-hadron scattering amplitudes, a simple condition based on duality and isotopic-spin invariance is almost sufficient to predict linear zeros.

I. INTRODUCTION

Odorico has proposed a strong bootstrap condition for VPP and TPP interactions, where P denotes a pseudoscalar meson, and V and T Reggeized vector and tensor mesons.[1] His basic hypothesis is that the zeros in $PP\to PP$ amplitudes are straight lines in the Mandelstam plane. The bootstrap condition requires not only an internal symmetry group, but, if the group is SU(3), it also requires the η-X mixing angle to be $\tan^{-1}(1/\sqrt{2})$.[1]

Recently, the author published a paper (to be referred to as C1) extending Odorico's condition to $VP\to PP$ and $\gamma P\to PP$ amplitudes, where γ denotes a photon.[2] Another result of C1 is that the quark-model values of VPP and TPP interactions lead to a solution of the $PP\to PP$ conditions overlooked in Ref. 1.