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Impulse Approximation and Multiple-Scattering Series for $\pi^+ d \leftrightarrow pp$ at High Energy

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A new model for $\pi^+ d \leftrightarrow pp$ at high energy, based on the impulse approximation, is formulated. It is observed that the shape of the differential cross section for this process is qualitatively consistent with the deuteron momentum-space density, implying that nuclear structure plays a dominant role in determining the momentum-transfer distribution. Despite the drastically nondiffractive nature of the reaction, a multiple-scattering-like series can be formulated from the Lippmann-Schwinger equation and evaluated by the linearized-propagator method. The resulting amplitude yields a good four-parameter fit to $pp \to \pi^+ d$ data at 3.74, 10.0, 14.2, 21.1, and 24.0 GeV/c. Arguments against some aspects of the Reggeistic interpretation of this process are also presented.

I. INTRODUCTION

The reaction $pp \to \pi^+ d$ at high energy has recently been studied rather extensively in two experiments,[1] but corresponding theoretical progress has been lacking. Barger and Michael[2] analyzed the process using Regge exchange of the N_α and N_γ trajectories, the latter being included (via broken exchange degeneracy) to account for the absence of a wrong-signature nonsense zero indicated in the N_α residue by backward $\pi^+ p$ data. Little other work has been published since that of Yao[3] in 1964, which used a triangle-diagram approach in which an exchanged pion scattered backwards at its second vertex.

In other high-energy deuteron reactions, both coherent and incoherent, it is well accepted that the impulse approximation, with or without multiple-scattering corrections, should be used. The essential content of this approximation is that a high-energy particle (momentum $\vec{k}$) incident upon a deuteron interacts with only one of its nucleons, leaving the other as a spectator. The momentum distribution of the latter should then simply be representative of the deuteron's momentum-space density. The corresponding view of the process $\pi^+ d \to pp$ is that one of the nucleons absorbs the pion and flies away, leaving the other to emerge unscathed, as shown in Fig. 1(a). Its momentum (in the deuteron rest frame) is kinematically related to the invariant momentum transfer squared t by

$$p^2 = [(M+m)^2 - t][(M-m)^2 - t]/4M^2 , \qquad (1)$$

where M and m are the deuteron and nucleon masses, respectively. Carrying out the impulse approximation, one expects to find

$$\frac{d\sigma}{d\Omega}(\pi^+ d \to pp) \approx g^2(t) P_d(p^2) , \qquad (2)$$

where $g(t)$ is the coupling constant for the vertex in Fig. 1(a), and $P_d(p^2)$ is the deuteron momentum-space density distribution.

We show in Fig. 2 a comparison of experimental $pp \to \pi^+ d$ data at five different proton lab momenta with $P_d(p^2) = [|\varphi_S(\vec{p})|^2 + |\varphi_D(\vec{p})|^2]/4\pi$, where φ_S and φ_D are the s- and d-state deuteron wave functions obtained from the Reid soft-core potential.[4] It is notable that $P_d(p^2)$ (which has been normalized arbitrarily in this graph) decreases at a rate which is quite comparable to the near-forward differential cross section. While the agreement is not exact, it seems significant and suggestive that the decrease of the differential cross section over two orders of magnitude is predominantly a result of the nuclear structure of the deuteron.

This viewpoint is antipodal to the Reggeistic approach used by Barger and Michael. Instead of treating the deuteron as an elementary particle, we emphasize its composite nature. Since there is no nucleon exchange in the traditional sense, we lose information on the energy dependence provided by Regge theory, but in return we gain a new insight into the momentum-transfer dependence as a result of the nuclear structure.

In this paper we wish to present a derivation yielding the impulse approximation (2) as its leading term, but including the corrections to it as well. The impulse approximation has previously been applied only to processes in which nothing drastic happens to the deuteron, whereas in high-energy disintegration by pions it is violently broken apart. As we shall show, however, a model of this type can indeed be formulated, although it differs somewhat in interpretation from the usual situation. The results lead to a multiple-scattering series very much like Glauber theory, even though the process is not itself diffractive in the usual sense. To our knowledge, this approach has not been tried before.

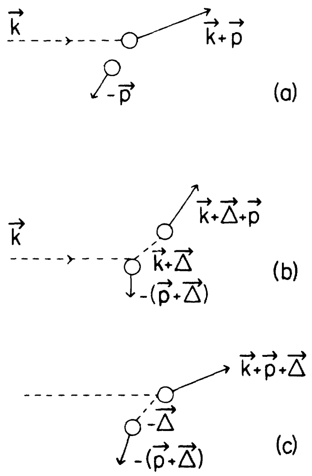

FIG. 1. Diagrams describing the process $\pi^+ d \to pp$: (a) impulse approximation, in which the pion interacts with only one nucleon; (b) double-scattering term, in which the pion scatters elastically forward before being absorbed; (c) double-scattering term analyzed by Yao (Ref. 3), involving backward pion-nucleon scattering prior to absorption.

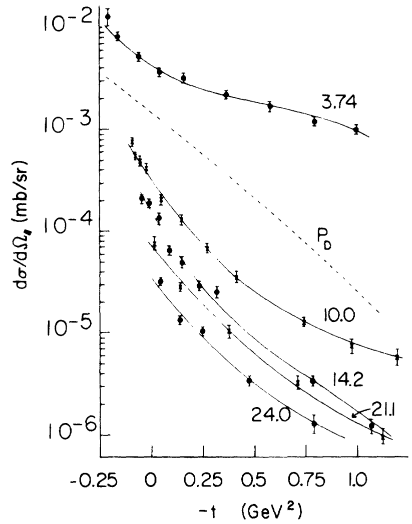

FIG. 2. Differential cross sections for $pp \to \pi^+ d$ at incident momenta 3.74, 10.0, 14.2, 21.1, and 24.0 GeV/c. The dashed line, arbitrarily normalized, shows the t variation expected from the deuteron momentum-space wave function. The solid curves are our fits.

II. DERIVATION OF IMPULSE APPROXIMATION

We consider the interactions of a three-body system including a pion (mass μ) and two nucleons bound in a deuteron, including the possibility that the pion can be absorbed (or created) by either of the nucleons. The Hamiltonian of the system, assuming only two-body interactions, is written

$$H = K_p + K_n + K_\pi + V_{pn} + V_{\pi n} + V_{\pi p} + A_p + A_n\,, \tag{3}$$

where K_i is the kinetic energy of particle i, V_{ij} is the elastic scattering potential between i and j, and A_N stands for the pion creation-plus-annihilation operators on nucleon N. The Lippmann-Schwinger equation will be used to obtain a cluster expansion for the total transition matrix in terms of the individual interaction terms. To show the process clearly, we consider first the scattering from a single free nucleon, omitting K_n, V_{pn}, $V_{\pi n}$, and A_n from H to write

$$H_{\rm free} = K_p + K_\pi + V_{\pi p} + A_p\,. \tag{4}$$

Choosing the kinetic energies as the unperturbed Hamiltonian, we obtain a Lippmann-Schwinger equation

$$T_{\rm free} = (V_{\pi p} + A_p) + (V_{\pi p} + A_p) G_K T_{\rm free}\,, \tag{5}$$

with $G_K = (E_K - K_\pi - K_p + i\epsilon)^{-1}$, and an iterative solution (omitting subscripts for neatness)

$$\begin{aligned} T_{\rm free} &= (V+A) + (V+A)G_K(V+A) + \cdots \\ &= (V+A)[1 - G_K(V+A)]^{-1}\,. \end{aligned} \tag{6}$$

By making use of the operator identity

$$R(F+G) = R(F) + R(G) + R(F)R(G) + R(G)R(F) + \cdots\,, \tag{7}$$

where $R(F) = F[1-F]^{-1}$ and the sum contains all combinations in which $R(F)$ and $R(G)$ alternate, this result can be rearranged as a power series in A,

$$\begin{aligned} T_{\rm free} &= V(1 - G_K V)^{-1} \\ &\quad + (1 - VG_K)^{-1} A (1 - G_K V)^{-1} + \cdots\,, \end{aligned} \tag{8}$$

where the omitted higher-order terms correspond to multipion intermediate states. To the extent that these may be neglected, we can identify

$$T_{\rm el} = V(1 - G_K V)^{-1} \tag{9}$$

as the elastic scattering part of $T_{\rm free}$, and

$$A_r = (1 - VG_K)^{-1} A (1 - G_K V)^{-1} \tag{10}$$

as the pion creation-annihilation part, *renormalized* by the elastic scattering.

Now let us proceed similarly in the three-body case. We choose $H_0 = K_p + K_n + K_\pi + V_{pn}$ as the unperturbed Hamiltonian, and $V = V_{\pi p} + V_{\pi n} + A_p + A_n$ as the perturbation, with $G_0 = (E - H_0 + i\epsilon)^{-1}$ the propagator for the πd system. The T matrix is now the solution of

$$T=(V_{\pi p}+V_{\pi n}+A_p+A_n)$$
$$+(V_{\pi p}+V_{\pi n}+A_p+A_n)G_0T\,. \tag{11}$$

Proceeding similarly, we find

$$T=T_0+\tilde{A}+\cdots\,, \tag{12}$$

where

$$T_0=(V_{\pi p}+V_{\pi n})[1-G_0(V_{\pi p}+V_{\pi n})]^{-1} \tag{13}$$

generates the usual elastic multiple-scattering series, while

$$\tilde{A}=[1-(V_{\pi p}+V_{\pi n})G_0]^{-1}(A_p+A_n)$$
$$\times[1-G_0(V_{\pi p}+V_{\pi n})]^{-1} \tag{14}$$

describes the creation-annihilation process renormalized by the nuclear interactions, and the higher-order terms again are multipion intermediate states. A further clarification of $\tilde{A}$ is now possible, however, by using the identity (7) again. After some manipulation we find that

$$\tilde{A}=\lambda_p\tilde{A}_p\rho_p+\lambda_n\tilde{A}_n\rho_n\,, \tag{15}$$

where

$$\tilde{A}_N=(1-V_NG_0)^{-1}A_N(1-G_0V_N)^{-1}$$

describes the creation-annihilation process on nucleon N, renormalized by the elastic scattering *and* the deuteron binding potential. The rescattering operators λ_p and ρ_p are given by

$$\lambda_p=[1+R(V_{\pi n}G_0)][1-R(V_{\pi p}G_0)R(V_{\pi n}G_0)]^{-1}\,,$$
$$\rho_p=[1-R(G_0V_{\pi n})R(G_0V_{\pi p})]^{-1}[1+R(G_0V_{\pi n})]\,, \tag{16}$$

with λ_n and ρ_n obtained by simply interchanging $V_{\pi p}$ and $V_{\pi n}$. Explicitly, we have

$$\lambda_p\tilde{A}_p\rho_p=[1+R(V_{\pi n}G_0)+R(V_{\pi p}G_0)R(V_{\pi n}G_0)+\cdots]\tilde{A}_p$$
$$\times[1+R(G_0V_{\pi n})+R(G_0V_{\pi n})R(G_0V_{\pi p})+\cdots]$$
$$=\tilde{A}_p+R(V_{\pi n}G_0)\tilde{A}_p+\tilde{A}_pR(G_0V_{\pi n})+\cdots\,. \tag{17}$$

The series is easily pictured graphically with each R corresponding to an "elastic scattering."

The impulse approximation for this process clearly corresponds to neglecting all of the rescattering terms in (17), so that

$$\tilde{A}\approx\tilde{A}_p+\tilde{A}_n\,. \tag{18}$$

In the usual procedure, one would also equate the single-scattering operators $\tilde{A}_N$ with the free-scattering operators corresponding to A_r in Eq. (10). Here, however, we have

$$\tilde{A}_N=A_r+V_N(G_0-G_K)A_r+A_r(G_0-G_K)V_N+\cdots\,, \tag{19}$$

with the difference between bound and free propagators given by

$$G_0-G_K=G_KV_{pn}G_K(1-V_{pn}G_K)^{-1}\,.$$

Since the kinetic energies of the bound nucleons are fairly large (i.e., they are a considerable distance off the mass shell), G_0-G_K will not produce negligible corrections, and $\tilde{A}_N$ cannot be identified with A_r. In Regge language, this means one would *not* expect the coupling to correspond to the nucleon trajectory's residue as determined, e.g., by Chiu and Stack,[5] in fitting π^+p backward scattering. It should also be noted that the dependence of $\tilde{A}_N$ on the deuteron binding means that the coupling obtained from it may be a function of the πd center-of-mass energy s as well as of the momentum transfer t.

We shall therefore not assume any connection between $\tilde{A}_N$ and the pion-nucleon coupling constant, but simply write

$$\tilde{A}_N=g(t)\vec{k}\cdot\vec{\sigma}\,\vec{\pi}(\vec{k})\cdot\vec{\tau}+\text{c.c.}\,, \tag{20}$$

where $\vec{\sigma}$ and $\vec{\tau}$ are the nucleon spin and isospin operators, and $\vec{\pi}(\vec{k})$ is the (isovector) annihilation operator for a pion of momentum $\vec{k}$. The matrix element of $\tilde{A}$ between the initial and final states can then be taken straightforwardly, using the formalism for deuteron breakup given in an earlier paper.[6] We write the initial state $|i\rangle=|\vec{k}\rangle|\varphi_d\rangle|\chi_d\rangle$, where $|\vec{k}\rangle$ is the pion state, and

$$|\varphi_d\rangle|\chi_d\rangle=2^{-1/2}\int d^3p[\varphi_S(\vec{p})+(1/\sqrt{8})\varphi_D(\vec{p})S_{12}(\vec{p})]$$
$$\times|\vec{p}\rangle|-\vec{p}\rangle|\chi_d\rangle \tag{21}$$

is the usual deuteron spatial wave function with d state included, $|\chi_d\rangle$ being the appropriate SU(4) spin-isospin state. (Both φ_S and φ_D are functions of p^2 only, but we shall find it convenient to indicate the vector $\vec{p}$ explicitly.) The final two-nucleon state is written analogously as $|f\rangle=|0\rangle|\varphi_{f\pm}\rangle|\chi_{f\pm}\rangle$, where $|0\rangle$ is the pion's vacuum state,

$$|\varphi_{f\pm}\rangle=2^{-1/2}[|\vec{p}_1\rangle|\vec{p}_2\rangle\pm|\vec{p}_2\rangle|\vec{p}_1\rangle]\,, \tag{22}$$

and $|\chi_{f\pm}\rangle$ is the approximately symmetrized spin-isospin state. Carrying out the calculation with the matrix elements of Eq. (20) leads without difficulty to the result (2),

$$\frac{d\sigma}{d\Omega}(\pi^+d\to pp)=g^2(t)P_d(p^2)\,,$$

where

$$P_d(p^2)=4\pi[|\varphi_S(\vec{p})|^2+|\varphi_D(\vec{p})|^2] \tag{23}$$

is the deuteron momentum-space density.

III. MULTIPLE-SCATTERING CORRECTIONS

In order to interpret the correspondence between the differential cross section and the deuteron density shown in Fig. 2 as a manifestation of the first-order result derived in Sec. II, we must establish that the correction terms in (15) do not alter the good agreement of the two. In this regard we may note that, unless an appropriately increasing t dependence is ascribed *ad hoc* to g, the differential cross section is initially somewhat steeper than $P_d(p^2)$, but then becomes less steep than the impulse approximation alone.

This behavior is analogous to that expected from the usual analyses of rescattering effects. We therefore shall attempt an evaluation of the corrections to $\tilde{A}$ arising from succeeding terms of (17), such as

$$A^{(1)} = \tilde{A}_p R(G_0 V_{\pi n}) + \tilde{A}_n R(G_0 V_{\pi p}) . \qquad (24)$$

We shall relabel the nucleons 1 and 2, since they are isosymmetrically indistinguishable. Either term of (24) may then be expanded in the form

$$\tilde{A}_1 R(G_0 V_2) = \tilde{A}_1 (G_0 V_2 + G_0 V_2 G_0 V_2 + \cdots) . \qquad (25)$$

This series is easily pictured graphically, the first term corresponding to Fig. 1(b).

It is also illuminating to expand it in terms of the off-shell scattering matrix $t_2 = V_2(1 - G_K V_2)^{-1}$, rather than V_2. Then one obtains

$$\tilde{A}_1 R(G_0 V_2) = \tilde{A}_1 G_0 t_2 [1 + (G_K - G_0) t_2]^{-1} . \qquad (26)$$

Since, as mentioned above, an expansion involving $(G_K - G_0)$ is not expected to converge well for this process, we see that the corrections are more appropriately related to the pion-nucleon potential V_2 than to the scattering operator obtained from it.

We shall evaluate these corrections by making use of the linearized propagator technique, which Osborn[7] has shown is the essential gambit of the Glauber[8] multiple-scattering theory. (The usual Glauber theory cannot be assumed here because it concerns diffractive scattering, with small, transverse momentum transfers.) To simplify our treatment of these terms we shall temporarily omit the d-state part of the deuteron wave function, denoting it simply as $\varphi(\vec{p})$. The matrix element of the first term of Eq. (25), for example, is then given by

$$\begin{aligned} \langle f | \tilde{A}_1 G_0 V_2 | i \rangle &= \delta(\vec{k} - \vec{p}_1 - \vec{p}_2) A_{12}(\vec{k}, \vec{p}_2) , \\ A_{12}(\vec{k}, \vec{p}_2) &= \int d^3p\, \varphi(\vec{p}) g(\vec{p}_1 - \vec{p}) G_0(\vec{p}, \vec{p}_2, \vec{p}_1 - \vec{p}) \\ &\quad \times v_2(\vec{k}, -\vec{p}; \vec{p}_1 - \vec{p}, \vec{p}_2) , \end{aligned} \qquad (27)$$

where

$$\langle \vec{K} | \langle \vec{P} \,|\, V_2 | \vec{k} \rangle | \vec{p} \rangle = \delta(\vec{K} + \vec{P} - \vec{k} - \vec{p}) v_2(\vec{K}, \vec{P}; \vec{k}, \vec{p}) \qquad (28)$$

describes the pion-nucleon elastic interaction in Fig. 1(b) and $G_0(\vec{p}, \vec{p}_2, \vec{p}_1 - \vec{p})$ is the propagator for the three-particle intermediate state with the indicated momenta. For notational ease, we have also now written $g(t)$ as a function of the momentum $\vec{p}_1 - \vec{p}$ transferred to the struck nucleon.

The crucial step in simplifying this integral is the assumption that the amplitude Eq. (28) is appreciable only for certain values of the momentum transfer. Diffractive elastic scattering at high energy implies that the matrix element of t_2 is small unless $\vec{p}_2 \approx -\vec{p}$, and we shall assume the same is true for v_2. The propagator

$$\begin{aligned} G_0(\vec{p}, \vec{p}_2, \vec{p}_1 - \vec{p}) = [&E + i\epsilon - p^2/2m - p_2{}^2/2m \\ &- (\vec{p}_1 - \vec{p}^{\,2}/2\mu]^{-1} \end{aligned}$$

is then well approximated[9] by

$$\begin{aligned} G_0(\vec{p}, \vec{p}_2, \vec{p}_1 - \vec{p}) &\approx 2\mu[k^2 + i\epsilon - (\vec{p}_1 - \vec{p})^2]^{-1} \\ &\approx 2\mu[2\vec{k}\cdot(\vec{p}_2 + \vec{p}) + i\epsilon]^{-1} . \end{aligned} \qquad (29)$$

The integration over the component of $\vec{p}$ parallel to $\vec{k}$ is thus converted into a simple pole term. Neglecting the principal part of the integral in favor of the δ function yields

$$G_0(\vec{p}, \vec{p}_2, \vec{p}_1 - \vec{p}) \approx \frac{\mu i \pi}{k} \delta(p_{\parallel} + p_{2\parallel}) , \qquad (30)$$

where $\parallel$ denotes the component parallel to $\vec{k}$, leading to

$$A_{12}(\vec{k}, \vec{p}_2) \approx \frac{\mu i \pi}{k} \int d^2p_\perp\, \varphi(\vec{p}) g(\vec{p}_1 - \vec{p}) v_2(\vec{k}, -\vec{p}; \vec{p}_1 - \vec{p}, \vec{p}_2) , \qquad (31)$$

with the constraint $p_{\parallel} = -p_{2\parallel}$.

This integral can be evaluated by approximating the functions involved diffractively. We write

$$\begin{aligned} v_2(\vec{k}, -\vec{p}; \vec{p}_1 - \vec{p}, \vec{p}_2) &= F e^{-\alpha(\vec{p}_2 + \vec{p})_\perp{}^2} , \\ g(\vec{p}_1 - \vec{p}) &= g_0 e^{-\gamma(\vec{p}_1 - \vec{p})^2} , \\ \varphi(\vec{p}) &= \varphi_0 e^{-\lambda p^2} , \end{aligned} \qquad (32)$$

and it follows that

$$A_{12} = \frac{i\pi\mu}{k} g(\vec{k}) \varphi(\vec{p}_2) \frac{\pi F}{\alpha + \gamma + \lambda} e^{\lambda^2 p_\perp{}^2/(\alpha + \gamma + \lambda)} , \qquad (33)$$

where $p_\perp$ is the component of $\vec{p}_1$ normal to $\vec{k}$. We expect this formula to be a good approximation to A_1 even if $g(\vec{k})$ and $\varphi(\vec{p})$ are not precisely of the form (32), provided their slopes are satisfactorily represented by γ and λ. The latter is true for the Reid soft-core wave functions we have used, for which $P_d(p^2) \propto \exp(-5.75p^2)$ quite accurately over the range of p involved here, corresponding to $\lambda = 2.875$ GeV^{-2}.

The succeeding terms of the multiple-scattering series can also be evaluated in this way. With the above parametrization of v_2, it follows that

$$\langle \vec{K} | \langle \vec{P} | V_2 G_0 V_2 | \vec{k} \rangle | \vec{p} \rangle = \delta(\vec{K} + \vec{P} - \vec{k} - \vec{p}) v_2{}^2(\vec{K}, \vec{P}; \vec{k}, \vec{p}),$$

with

$$v_2{}^2(\vec{K}, \vec{P}; \vec{k}, \vec{p}) = \frac{\mu i \pi^2}{k} \frac{F^2}{2\alpha} e^{-\alpha K_\perp{}^2/2}, \tag{34}$$

where $K_\perp$ is the component of $\vec{K}$ normal to $\vec{k}$. The generalization to

$$\langle \vec{K} | \langle \vec{P} | V_2 (G_0 V_2)^n | \vec{k} \rangle | \vec{p} \rangle = \delta(\vec{K} + \vec{P} - \vec{k} - \vec{p}) \times v_2{}^n(\vec{K}, \vec{P}; \vec{k}, \vec{p})$$

can be shown inductively to yield

$$v_2{}^n(\vec{K}, \vec{P}; \vec{k}, \vec{p}) = \frac{F}{n+1} \left(\frac{i\pi^2 \mu F}{k\alpha} \right)^n e^{-\alpha K_\perp{}^2/(n+1)}. \tag{35}$$

In fact, the summation over all terms of (35) can be carried out using the integral relation

$$\int_0^\infty b\, db\, e^{-nb^2/\alpha} J_0(bK_\perp) = \frac{2\alpha}{n} e^{-\alpha K_\perp{}^2/n}$$

to obtain an impact-parameter representation

$$\langle \vec{K} | \langle \vec{P} | V_2 R(G_0 V_2) | \vec{k} \rangle | \vec{p} \rangle = \delta(\vec{K} + \vec{P} - \vec{k} - \vec{p}) \frac{i\pi^2 \mu F^2}{2k\alpha^2} \int_0^\infty b\, db \frac{J_0(bK_\perp)}{e^{b^2/\alpha} - i\pi^2 \mu F^2/k\alpha}. \tag{36}$$

The final integration involving $\tilde{A}_1$, however, cannot be carried out analytically using this integral form. The series obtained for

$$\langle f | \tilde{A}_1 R(G_0 V_2) | i \rangle = \delta(\vec{k} - \vec{p}_1 - \vec{p}_2) \mathcal{A}_1(\vec{k}, \vec{p}_2)$$

is given by

$$\mathcal{A}_1(\vec{k}, \vec{p}_2) = g(\vec{k}) \varphi(\vec{p}_2) \sum_{n=1}^\infty \frac{\alpha}{\alpha + n(\gamma + \lambda)} \left(\frac{i\pi^2 \mu F}{k\alpha} \right)^n e^{\lambda^2 p_\perp{}^2/(\gamma + \lambda + \alpha/n)}. \tag{37}$$

If all terms in Eq. (37) are to be included in the amplitude, however, it is also necessary to keep the higher-order terms in Eq. (17) describing elastic scattering on nucleon 1 prior to annihilation, e.g., $\langle f | \tilde{A}_1 G_0 V_2 G_0 V_1 | i \rangle$. If we neglect internal quantum numbers, it is reasonable to assume that $G_0 V_1$ commutes with $G_0 V_2$. In that case, all $G_0 V_1$ terms may be moved to the left, and the relevant part of Eq. (17) becomes

$$\tilde{A}_1 \rho_1 = \sum_{m=0}^\infty \sum_{n=1}^\infty \tilde{A}_1 \frac{(m+n-1)!}{m!(n-1)!} (G_0 V_1)^m (G_0 V_2)^n, \tag{38}$$

the binomial coefficient arising from the number of possible orderings of terms. The matrix elements of (38) for $m > 0$ can also be evaluated as above [assuming the same parametrization (32) for v_1 as for v_2] with the result

$$\langle f | \tilde{A}_1 (G_0 V_1)^m (G_0 V_2)^n | i \rangle = \delta(\vec{p}_1 + \vec{p}_2 - \vec{k}) \mathcal{A}_{mn}(\vec{k}, \vec{p}_2),$$

where

$$\mathcal{A}_{mn}(\vec{k}, \vec{p}_2) = \int d^2 p_\perp \int d^2 P_\perp\, \varphi(\vec{p}) g(\vec{p}_1 - \vec{P}) v_1{}^m(\vec{P} - \vec{p}) v_2{}^n(\vec{p}_2 + \vec{p})$$
$$= \frac{\alpha^2 g(\vec{k}) \varphi(\vec{p}_2)}{(m\gamma + \alpha)(n\lambda + \alpha) + n\gamma\alpha} \left(\frac{i\pi^2 \mu F}{k\alpha} \right)^{n+m} \exp\left[\lambda^2 p_\perp{}^2 \Big/ \left(\lambda + \frac{\alpha}{n} + \frac{\gamma\alpha}{m\gamma + \alpha} \right) \right]. \tag{39}$$

This form reduces correctly for $m = 0$ as well, so we may write

$$\langle f | \tilde{A}_1 \rho_1 | i \rangle = \delta(\vec{k} - \vec{p}_1 - \vec{p}_2) g(k) \varphi(p_2) [1 + C(p_\perp{}^2)], \tag{40}$$

with

$$C(p_\perp{}^2) = \sum_{m=0}^\infty \sum_{n=1}^\infty \frac{(m+n-1)!}{m!(n-1)!} \frac{\alpha^2}{(m\gamma + \alpha)(n\lambda + \alpha) + n\gamma\alpha} \left(\frac{i\pi^2 \mu F}{k\alpha} \right)^{n+m} \exp\left[\lambda^2 p_\perp{}^2 \Big/ \left(\lambda + \frac{\alpha}{n} + \frac{\gamma\alpha}{m\gamma + \alpha} \right) \right]. \tag{41}$$

This final result gives, in the diffractive approximation employed, the amplitude for the process $\pi^+ d \to pp$ with all orders of multiple scattering correctly included.

IV. EVALUATION AND FITS TO DATA

The series (41) describes the multiple-scattering corrections to the impulse approximation result (20). It may be noted at the outset that the envelope of these corrections, as a function of $p_\perp^2$, is simply a multiple of $\exp(\lambda p_\perp^2)$. If $|C(0)| \ll 1$, we obtain the usual situation in which single scattering predominates at small momentum transfers, but multiple scattering takes over for larger $p_\perp^2$. [In the latter situation, the t dependence is simply $\varphi(\vec{p}_2)e^{\lambda p_\perp^2} = \varphi(\vec{p}_{2\parallel})$.] This behavior is perfectly consistent with the data.

Let us now estimate the magnitude of the corrections. The relevant parameter in the expansion (41) is the power term $\pi^2\mu|F|/2k\alpha$, which depends on the magnitude and slope of the matrix element of the pion-nucleon interaction V. Although we have pointed out that this quantity may differ measurably from the off-shell scattering matrix, and *a fortiori* from the on-shell one, it seems nonetheless reasonable to estimate it from the available data. In the normalization used for the expansion (8), the total cross section will be $\sigma_T = -16\pi^3\mu \,\mathrm{Im}\langle T\rangle_0/k$, where $\langle T\rangle_0$ is the forward matrix element of T. Taking $|F| = -\mathrm{Im}\langle T\rangle_0$ and a total cross section of 25 mb yields

$$\pi^2\mu|F|/2k\alpha \approx (1.3\ \mathrm{GeV}^{-2})/2\alpha\,.$$

If the slope α is taken from typical diffraction peak values, say, $2\alpha \sim 10\ \mathrm{GeV}^{-2}$, then each correction term is about 15% of its predecessor, a numerical relation common in high-energy multiple-scattering phenomenology. This implies that our corrections are indeed corrections, in the sense that they represent a decreasing series of terms.

To test these results more quantitatively, we have fitted them to the experimental data. Terms up to $n=m=4$ in the sum (41) were included explicitly. In the higher-order terms we assume that $\alpha/4\lambda$ and $\alpha/4\gamma$ are negligible, so that the contribution of terms with $n \geq 4$ and $m \geq 4$ can be approximated according to

$$C'(p_\perp^2) = \frac{\alpha^2}{\gamma\lambda} e^{\lambda p_\perp^2} \sum_{m=4}^{\infty}\sum_{n=4}^{\infty} \frac{(n+m-1)!}{mm!n!}\left(\frac{i\pi^2\mu F}{k\alpha}\right)^{n+m} \tag{42}$$

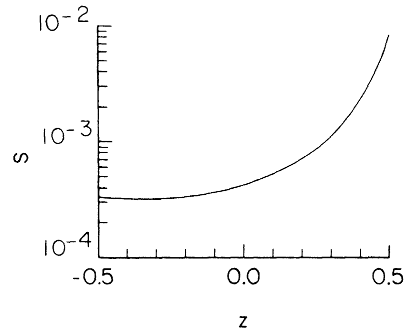

FIG. 3. Evaluation of the series

$$S = \sum_{m=4}^{\infty}\sum_{n=4}^{\infty}(m+n-1)!z^{m+n-8}/mm!n!$$

appearing in the multiple-scattering series, for real values of z.

The double summation in $C'(p_\perp^2)$ can be transformed into

$$S(z) = \sum_{k=8}^{\infty}\left[\sum_{m=4}^{k-4}\frac{1}{mm!(k-m)}\right](k-1)!z^{k-8}, \tag{43}$$

which can be evaluated numerically. It does not converge particularly quickly, requiring terms up to $k \approx 50$ to achieve 0.1% accuracy for $|z| \approx 0.5$. The values obtained for real z are shown in Fig. 3. The remaining terms in Eq. (41) are those with $m<4$ and $n \geq 4$, or vice versa; these can be summed explicitly.

The amplitude obtained in this way was fitted to experimental $pp+\pi^+d$ data[1,10] at proton lab momenta 3.74, 10.0, 14.2, 21.1, and 24.0 GeV/c, taking g_0, γ, and z as free parameters. The best-fit values of these parameters are summarized in Table I, and the fits are shown graphically in Fig. 2; the agreement is good at all five energies.

The values obtained for g_0 decrease proportionately to $k^{-1.25}$ to a good approximation. As pointed out earlier, this quantity is not directly interpretable as the usual pion-nucleon coupling constant due to the presence of the deuteron binding potential. Its simple energy dependence implies a corresponding relationship between $\tilde{A}_N$ and A_r via (19). The values of γ show somewhat more

TABLE I. Parameters of best fit to $pp \to \pi^+d$.

P_{lab} (GeV/c)	g_0 ($\mathrm{mb}^{1/2}\times 10^{-2}$)	γ (GeV^{-2})	Rez	Imz	χ^2/point
3.74	4.23 ± 0.06	5.08 ± 0.09	0.396 ± 0.003	0.313 ± 0.002	0.98
10.0	1.30 ± 0.02	12.8 ± 0.2	0.438 ± 0.010	0.282 ± 0.010	1.05
14.2	0.837 ± 0.018	24.3 ± 0.5	0.339 ± 0.008	0.432 ± 0.004	1.58
21.1	0.532 ± 0.011	14.2 ± 0.3	0.408 ± 0.011	0.339 ± 0.006	0.62
24.0	0.339 ± 0.008	19.4 ± 0.4	0.380 ± 0.008	0.391 ± 0.005	1.93

fluctuation, primarily at 14.2 GeV/c, where the data show much less flattening for larger t, but a (Regge) logarithmic dependence seems a reasonable guess at their variation. Thus, the coupling $\tilde{A}_N$ of a pion to a bound nucleon, despite the complicated dependence on internal deuteron dynamics in (19), seems phenomenologically to have a rather simple and not unfamiliar energy behavior.

V. CONCLUSIONS

It appears that the reaction $\pi^+ d \rightarrow pp$ can be described quite well by the impulse approximation, even though some of the amplitudes may be far from the mass shell. The momentum-transfer dependence of the differential cross section is quite similar to that expected from the deuteron wave function. This result suggests that nuclear effects are quite important in this reaction; even at high energies, the deuteron should not be treated as an elementary particle.

A multiple-scattering series has been derived for processes involving creation or annihilation, using the Lippmann-Schwinger equation, which is applicable to $\pi^+ d \rightarrow pp$ even though the deuteron is not a fixed composite system. Evaluating this series via the linearized propagator technique with diffractively parametrized amplitudes, we find that the theory gives good four-parameter fits to the data. It is hoped that this method will be applicable to other processes as well, e.g., $\bar{p}d \rightarrow \pi^+\pi^- n$.

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