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Azimuthal Correlations and the Nature of the Pomeranchuk Singularity in Models Without Short-Range Order*

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The existence of long-range azimuthal correlations in inclusive data is known to imply that the Pomeranchuk singularity is not a simple pole. A simple independent-emission model, which shares many properties of multiperipheral models except that the produced particles are not ordered along a multiperipheral chain, possesses these long-range correlations. Therefore the leading J-plane singularity in this model is seen to be a cut.

I. INTRODUCTION

The use of s-channel unitarity constraints in high-energy physics takes two distinct and complementary forms. One can either discuss simple models for production processes and investigate the nature of the Regge singularities they generate via unitarity in elastic or other amplitudes, or one can construct a model for the high-energy behavior of an amplitude and use unitarity to constrain production models. In this paper we demonstrate a simple example of the use of unitarity in the first way. From the application by Freedman, Jones, Low, and Young[1] of Mueller's generalized optical theorem,[2] we know that azimuthal correlations in 2-body inclusives can give information about the nature of the leading Regge singularities. We therefore calculate such correlations in a simple independent-emission model (IEM).[3,4] We see that the correlations are of long range and conclude that, in such models, the leading or Pomeranchuk-Regge singularity in the Mueller diagram is invariably a cut. This contrasts with the situation in multiperipheral models (MPM) where, because of a property called short-range order,[5] all correlation lengths are finite, and the leading Regge singularity generated via s-channel unitarity in this manner can be a pole.

In Sec. II we reproduce, for convenience, the arguments leading to the extraction of information about the type of Regge singularity from the azimuthal correlation. In Sec. III we introduce the independent-emission model and in Sec. IV we calculate its predictions for the 2-particle azimuthal distributions. In Sec. V we discuss models with short-range order and briefly review the experimental evidence on 2-particle correlations. In Sec. VI we draw some conclusions about the usefulness of deciding, through models, the nature of the physical Pomeranchuk singularity.

II. THE EXPANSION OF THE FORWARD 8-POINT FUNCTION

The fact that the behavior of inclusive two-particle distributions can give important information about the nature of Regge singularities was first realized by Freedman, Jones, Low, and Young[1] (FJLY). The starting point in their analysis is Mueller's[2] connection between the inclusive spectrum for $ab \to c_1 c_2 X$ and a discontinuity in the (missing mass)^2 of the forward 8-point function

$$\rho_2^{ab\to c_1c_2X}(s;\vec{q}_1,\vec{q}_2)=\frac{\sigma_{\text{tot}}^{-1}\,d\sigma}{(d^3q_1/\omega_1)(d^3q_2/\omega_2)}$$
$$=(\sigma_{\text{tot}}\mathfrak{M}^2)^{-1}\text{disc}_{M^2}$$
$$\times\left[A_{ab\bar{c}_1\bar{c}_2\to ab\bar{c}_1\bar{c}_2}(s;\vec{q}_1\vec{q}_2;\text{all }\Delta^2=0)\right]. \tag{2.1}$$

The discontinuity and the physical variables are shown in Fig. 1. Taking the asymptotic behavior of the discontinuity, FJLY focus on the limit shown in Fig. 2 corresponding to a situation where one of the produced particles is in each fragmentation region. In this limit

$$t_1=(p_a-q_1)^2, \tag{2.2}$$
$$t_2=(p_b-q_2)^2 \tag{2.3}$$

are fixed. The subenergies related to the c.m. Feynman variables $x_1=2q_{1L}/s^{1/2}$, $x_2=2q_{2L}/s^{1/2}$, viz.,

$$s_1/\mathfrak{M}^2=(1-x_1)^{-1}, \tag{2.4}$$
$$s_2/\mathfrak{M}^2=(1+x_2)^{-1}, \tag{2.5}$$
$$s/\mathfrak{M}^2=(1-x_1)^{-1}(1+x_2)^{-1}, \tag{2.6}$$

are fixed and $\mathfrak{M}^2$ is large. The sixth independent variable is chosen to be ϕ, the azimuthal angle between the transverse components of $\vec{q}_1$ and $\vec{q}_2$ in any Lorentz frame collinear with the beam.

The 8-point function illustrated in Fig. 2 is seen to have zero total-momentum transfer between the two sides of the diagram. Lorentz invariance then implies the scattering amplitude, and presumably the relevant $\mathfrak{M}^2$ discontinuity of the scattering amplitude, has an O(3, 1) expansion analogous to the expansion of an equal-mass forward 2-2 scattering amplitude,[6]

$$A_{ab\bar{c}_1\bar{c}_2\to ab\bar{c}_1\bar{c}_2}=\delta_{\lambda\lambda'}\sum_{M,r,r'}\int d\sigma(M^2-\sigma^2)\times A_{ab\bar{c}_1\bar{c}_2}(M,\sigma,r,r')\times e^{i\lambda\phi}d^{M\sigma}_{r,\lambda,r'}(\eta),\quad(2.7)$$

where $d^{M\sigma}_{r\lambda r'}$ is an SO(3, 1) rotation matrix and $\cosh\eta=(\mathfrak{M}^2-t_1-t_2)/2(t_1t_2)^{1/2}$. At zero total-momentum transfer, the leading contribution of $d^{M\sigma}_{r,\lambda,r'}(\eta)$ for large $\mathfrak{M}^2$ is to helicity flip $\lambda=\pm M$,

$$d^{M\sigma}_{r\lambda r'}(\eta)\sim(\mathfrak{M}^2)^{(\sigma-1-|M-\lambda|)}\,.\quad(2.8)$$

The contribution of a Lorentz pole with σ, M to the 8-point amplitude at large $\mathfrak{M}^2$ is then

$$A_{ab\bar{c}_1\bar{c}_2\to ab\bar{c}_1\bar{c}_2}\sim(\mathfrak{M}^2)^{\sigma-1}F_M(t_1,t_2,s_1/\mathfrak{M}^2,s_2/\mathfrak{M}^2)\cos(M\phi)\,.\quad(2.9)$$

A Lorentz pole with Toller quantum number $M\neq 0$ cannot correspond to a simple Regge pole since it contains a mixture of parities. A simple Regge pole with intercept $\alpha(0)$ does not lead to ϕ dependence of the same order, but down by an extra power of $1/\mathfrak{M}^2$:

$$A_{\text{Regge pole}}\sim(\mathfrak{M}^2)^{\alpha(0)}F_0(t_1,t_2,s_1/\mathfrak{M}^2,s_2/\mathfrak{M}^2)\times\left[1+\frac{\cos\phi}{\mathfrak{M}^2}g_1(t_1,t_2,s_1/\mathfrak{M}^2,s_2/\mathfrak{M}^2)+\cdots\right].\quad(2.10)$$

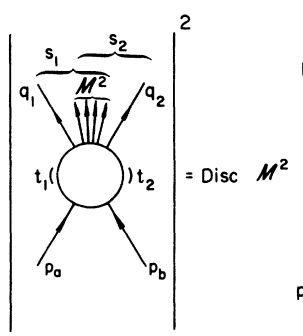

FIG. 1. Mueller's generalized optical theorem relates the two-particle inclusive cross section to a discontinuity of an 8-point function. The variables $t_1=(p_a-q_1)^2$, $t_2=(p_b-q_2)^2$, $\mathfrak{M}^2=(p_a+p_b-q_1-q_2)^2$, $s_1=(p_a+p_b-q_2)^2$, and $s_2=(p_a+p_b-q_1)^2$ are illustrated.

The contribution of a cut in the σ plane would be more complicated, typically involving $\ln\mathfrak{M}^2$ factors, if the discontinuity across the cut can be expanded in a power series. Note that, if cuts are present, the $\mathfrak{M}^2$ discontinuity can have different numbers of logarithmic factors than the amplitude itself, as in the simple example

$$\underset{\mathfrak{M}^2>0}{\text{disc}}\,[\mathfrak{M}^2\ln(\mathfrak{M}^2)\ln(-\mathfrak{M}^2)]=2\pi\mathfrak{M}^2\ln(\mathfrak{M}^2)\,.\quad(2.11)$$

The resolution of logarithmic factors involves asymptotic behavior in the entire $\mathfrak{M}^2$ plane, so we must study crossing constraints. See, for example, a discussion of sum rules in $\mathfrak{M}^2$.[7] The point is, although the presence of logarithmic factors in the two-particle inclusive distribution can signal the presence of a branch point in the σ plane, we cannot immediately conclude anything about the nature of the discontinuity of the branch point in the amplitude.

In using (2.1) and (2.9) to discuss the leading, Pomeranchuk, singularity, we see that the dominant contribution must have Toller quantum number $M=0$ in order that the two-particle distribution be non-negative for all values of ϕ. In the case of a cut, the $M=0$ component can be considered the factorizable (in the sense of Lorentz variables) component. If the Pomeranchuk singularity is a simple pole with intercept greater than the intercept of any cuts, the azimuthal correlations will vanish as a power of $\mathfrak{M}^2$. With some assumptions about the smoothness of the continuation into the central region, this can be translated into rapidities

$$\begin{aligned}F_1(x_1,x_2,t_1,t_2)(\mathfrak{M}^2)^{a_c(0)-a_P(0)}\cos\phi&\sim F_1(x_1,x_2,t_1,t_2)\times\left(\frac{s_{12}(1-x_1)(1+x_2)}{-x_1x_2}\right)^{a_c(0)-a_P(0)}\cos\phi\\&\sim F'(y_1,y_2,\kappa_1,\kappa_2)(\kappa_1\kappa_2)^{1/2}\times\exp\{|y_1-y_2|[a_c(0)-a_P(0)]\}\cos\phi\,,\end{aligned}\quad(2.12)$$

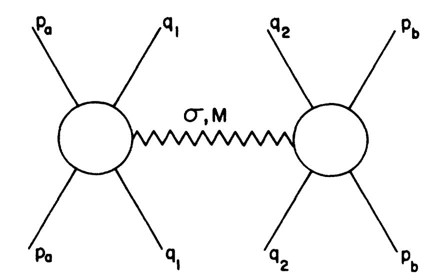

FIG. 2. Exchange of a Lorentz pole with σ, M in the 8-point function.

where $\kappa_i^2 = \mu_i^2 + q_T^2$. In terms of the Feynman gas-liquid analogy, this corresponds to the statement that the azimuthal correlations have a finite correlation length

$$\Lambda = [a_P(0) - a_c(0)]^{-1}. \tag{2.13}$$

If the Pomeranchuk singularity is a cut or a cut coincident with a pole,[8] the azimuthal correlation length will, in general, be infinite. The azimuthal dependence in (2.1) will typically vanish only as an inverse power of $\ln(\mathfrak{M}^2)$.

III. THE INDEPENDENT-EMISSION MODEL AND THE BEHAVIOR OF THE TOTAL CROSS SECTION

In this section we introduce the independent-emission model (IEM)[3,4] and review a method of estimating the phase-space integrals[9] which occur in the model. To illustrate the method, we calculate the total cross section in a specific example of an IEM. We will formulate the model in terms of a process which we will call $pp \to pp + n\pi$ although, for simplicity, we ignore all considerations of spin and internal quantum numbers. The modulus squared of the amplitude for this process is written in the form

$$|A_n|^2 = F(p_1)F(p_2)\prod_{j=1}^{n} f(q_j), \tag{3.1}$$

where the specification of the functions $F(p)$ and $f(q)$ defines the model. These functions are usually chosen to guarantee that the protons behave as "leading particles" and that all particles have limited transverse momenta.[10] Most of what we are going to do is independent of the exact form of these functions, but for illustrative purposes, it is convenient to have a specific form such as that discussed in Ref. 3:

$$F(p) = G e^{2s^{-1/2}\lambda\cdot p} e^{-R^2 p_T^2}, \tag{3.2}$$

$$f(q) = g e^{-R^2 q_T^2}. \tag{3.3}$$

Here λ is a four-vector, which in the c.m. frame has only a time component. From the n-particle rate

$$\Omega_n(P) = \int \left[\prod_{i=1}^{2} \frac{d^3p_i}{2E_i} F(p_i)\right]\left[\prod_{j=1}^{n} \frac{d^3q_j}{2\omega_j} f(q_j)\right] \times \delta^4\!\left(P - p_1 - p_2 - \sum_{j=1}^{n} q_j\right), \tag{3.4}$$

we form the total rate

$$Q(z,P) = \sum_{n=0}^{\infty} \frac{z^n}{n!}\Omega_n(P) \tag{3.5}$$

$$= \int \left[\prod_{i=1}^{2} \frac{d^3p_i}{2E_i} F(p_i)\right] Q_\pi(z, P - p_1 - p_2), \tag{3.6}$$

where the parameter z is introduced for convenience. We can estimate the phase-space integrals in (3.5) and (3.6) using the method of Lurçat and Mazur.[9] This involves taking the Laplace transform

$$Q(z,\beta) = \int d^4P\, Q(z,P) e^{-\beta\cdot P} \tag{3.7}$$

$$= [\psi_N(\beta)]^2 \exp[z\phi_\pi(\beta)], \tag{3.8}$$

where

$$\psi_N(\beta) = \int \frac{d^3p}{2E} e^{-\beta\cdot p} F(p), \tag{3.9}$$

$$\phi_\pi(\beta) = \int \frac{d^3q}{2\omega} e^{-\beta\cdot q} f(q). \tag{3.10}$$

In the case where $F(p)$ and $f(p)$ are given by (3.2) and (3.3) we have

$$\phi_\pi(\beta) = \phi_\pi(\beta_L, \beta_T) = \pi g \int d(q_T^2) e^{-r^2 q_T^2} I_0(\beta_T q_T) K_0(\beta_L \kappa_\pi), \tag{3.11}$$

$$\psi_N(\beta) = \psi_N(\zeta, \beta_T) = \pi G \int d(p_T^2) e^{-R^2 p_T^2} I_0(\beta_T, p_T) K_0(\zeta \kappa_N), \tag{3.12}$$

where $\kappa_\pi = (\mu^2 + q_T^2)^{1/2}$ and $\kappa_N = (m^2 + p_T^2)^{1/2}$ are the transverse masses and the $I_0(x)$ and $K_0(x)$ are modified Bessel functions,

$$\beta_L = (\beta_0^2 - \beta_3^2)^{1/2}, \tag{3.13}$$

$$\beta_T = (\beta_1^2 + \beta_2^2)^{1/2}, \tag{3.14}$$

and

$$\zeta = [(\beta_0 - 2\lambda_0/s^{1/2})^2 - (\beta_3 - 2\lambda_0/s^{1/2})^2]^{1/2}. \tag{3.15}$$

In a large number of models of this type, once the proton factor is chosen to ensure leading particles, no extra dependence on the longitudinal momentum of the pions is inserted. In any case we have a different dependence on the transverse and longitudinal momenta and this is reflected in the nonsymmetric treatment of the components of β [(3.13), (3.14), (3.15)]. The total rate (3.6) is then estimated by using the steepest descents estimate of the inverse transform

$$\frac{Q(z,P)}{Q(z,\beta)} e^{-\beta\cdot P} = \int \frac{d^4t}{(2\pi)^4} e^{-iP\cdot t} \frac{Q(z, \beta - it)}{Q(z,\beta)}, \tag{3.16}$$

valid for positive timelike β. Expanding

$$\ln \frac{Q(z, \beta - it)}{Q(z, \beta)} = -iA \cdot t - \tfrac{1}{2} B^{\mu\nu} t_\mu t_\nu + \cdots , \tag{3.17}$$

the condition $-A^\mu = P^\mu$ determines the point of stationary phase of the integrand. In a frame where $P_3 = 0$ and $|\vec{P}_T|$ is small compared to P_0, the conditions on β_L and β_T are

$$P_0 \cong -\frac{\partial}{\partial \beta_L} \ln Q(z, \beta_L, \beta_T) = s^{1/2} , \tag{3.18}$$

$$P_T \cong -\frac{\partial}{\partial \beta_T} \ln Q(z, \beta_L, \beta_T) . \tag{3.19}$$

To a good approximation, $B^{\mu\nu}$ only contains diagonal components

$$B^{00} = \frac{\partial}{\partial \beta_L{}^2} \ln Q(z, \beta_L, \beta_T) \bigg|_{\beta_T = 0} , \tag{3.20}$$

$$B^{LL} = s^{1/2}/\beta_L , \tag{3.21}$$

$$B^{TT} = (1/\beta_T) \frac{\partial}{\partial \beta_T} \ln Q(z, \beta_L, \beta_T) \bigg|_{\beta_T = 0} . \tag{3.22}$$

In this limit, the approximation is straightforward and the steepest descents estimate gives

$$Q(z, P) = \frac{\exp(\beta_L P_0 - \vec{\beta}_T \cdot \vec{P}_T)}{(2\pi)^2 (\det B)^{1/2}} \times Q(z, \beta_L, 0) [1 + \mathfrak{R}(\beta_L, \beta_T)] . \tag{3.23}$$

Using (3.19) and (3.22),

$$Q(z, P) = \frac{\exp(\beta_L P_0)}{(2\pi)^2 (\det B)^{1/2}} \exp(-P_T{}^2 / 2B_\pi) \times Q(z, \beta_L) [1 + \mathfrak{R}(\beta_L, \beta_T)] . \tag{3.24}$$

With a reasonable choice of $F(p)$ and $f(q)$, the error term, $\mathfrak{R}(\beta_L, \beta_T)$, is small when the energy increases. The nature of the error term depends on the exact form of $F(p)$ and $f(q)$, and is discussed in more detail elsewhere.[3,11,12]

Looking at the high-energy behavior of the total cross section, we can use the optical theorem to infer something about the nature of the Pomeranchuk singularity implied by these models. However, we do not know if the exact behavior of the cross section in the high-energy limit is sensitive to the choice of $F(p)$ and $f(q)$. With the choice of (3.2) and (3.3), we observe that the limit $s \to \infty$ corresponds to small β_L, and we have the approximations[3]

$$\phi_\pi(\beta) = (g\pi/R^2)[-\tfrac{1}{2}\gamma - \ln(\beta/2R) + O(\beta^2 \ln\beta) + O(\mu^2 \ln\mu^2)] , \tag{3.25}$$

$$\psi_N(\zeta) = (G\pi/R^2)[-\gamma - \tfrac{1}{2} e^{R^2 m^2} \Gamma(0, R^2 m^2) - \ln(m\zeta/2) + O(\zeta^2 \ln\zeta)] , \tag{3.26}$$

where γ is Euler's constant and $\Gamma(a, x)$ is an incomplete gamma function. Therefore, Eqs. (3.18) and (3.19) have the solutions

$$\beta_L = 2\lambda s^{-1/2} \left[1 + \frac{4R^2}{zg\pi} \frac{1}{\ln s} + O(\ln \ln s / \ln s)^2) \right] , \tag{3.27}$$

$$\beta_T = 0 . \tag{3.28}$$

The total cross section then approaches the high-energy limit

$$\sigma_{\text{tot}} = Q(z, s)/s \sim \text{const} s^{(zg\pi/2R^2 - 2)} (\ln s) [1 + O(\ln \ln s) + \cdots] . \tag{3.29}$$

In this particular version of the IEM, therefore, the leading J-plane singularity is seen to be a cut with a singular discontinuity (or a cut coincident with a dipole). We have not gained any special insight into the nature of the Pomeranchuk in the general class of IEM's, however, because we can now see that this result depends very strongly on the choice of $F(p)$, (3.2) and $f(q)$, (3.3). In particular, we have a factor

$$[\psi_N(\zeta)]^2 \sim (\pi G \ln s / 2R^2)^2 , \tag{3.30}$$

which can be attributed to the growth in the "longitudinal phase space" available to the leading protons. Alternate versions of the IEM approach can easily be constructed where this type of factor is not present.[4,12] It remains an open question, however, whether a model can be constructed which gives a reasonable leading-particle effect, but which does not contain logarithmic corrections to the leading behavior of the total cross section. Instead, we turn to two-particle azimuthal distributions to find whether in these models the Pomeranchuk singularity can be a pole.

IV. TWO-PARTICLE AZIMUTHAL DISTRIBUTIONS IN THE INDEPENDENT-EMISSION MODEL

The idea that a simple phase-space model can give insight into the nature of diffraction was first discussed by Van Hove.[4] Others[13] have also calculated the shape of the diffraction peak generated by 2-2 unitarity when an ansatz for the phase of the n-particle amplitude is made. The calculation which follows is in the same spirit as these earlier works, except that unitarity is used in the form of the generalized optical theorem.[2]

A reliable indication of the nature of the Pomeranchuk singularity implied by an IEM approach appears through the long-range nature of the two-particle azimuthal correlations. For definiteness, we will calculate proton-proton azimuthal distributions in the model specified above, although the basic features of azimuthal distributions will be seen to be quite insensitive to the details of the model.

The proton-proton distribution function in the model is given from (3.6):

$$\rho_2^{pp\to ppx}(s,\vec{p}_1,\vec{p}_2)=\frac{1}{\sigma_{\rm tot}}\,\frac{F(\vec{p}_1)F(\vec{p}_2)}{s}\times Q_\pi(z,P-p_1-p_2)|_{z=1}\,. \quad (4.1)$$

The contributions due to no pions (elastic scattering) and one pion involve δ functions and can be computed exactly:

$$Q_\pi(z,P-p_1-p_2)=\delta^{(4)}(P-p_1-p_2)+zf(P-p_1-p_2)\delta(\mathfrak{M}^2-\mu^2)+O(z^2)\,, \quad (4.2)$$

where $\mathfrak{M}^2=(P-p_1-p_2)^2$. In what follows, we will exponentiate the sum over final particles, keeping in mind the fact that the terms of zeroth and first order in z calculated in any such result should be replaced by the exact forms above. Let $Q=P-p_1-p_2$; we can now follow the method of Lurçat and Mazur[9] described above and calculate

$$Q_\pi(z,\beta)=\int d^4Q\,Q(z,Q)e^{-\beta\cdot Q}=\exp[z\phi_\pi(\beta)]\,, \quad (4.3)$$

where $\phi_\pi(\beta)$ is given by (3.11) and (3.25). The high-energy limit of the energy-momentum equations (3.18) and (3.19) become

$$\beta_L\cong\pi gz/R^2\mathfrak{M}_L\,, \quad (4.4)$$

$$\beta_T\cong\frac{2R^4}{\ln(2R^3\mathfrak{M}_L/z\pi g)-\frac{1}{2}\gamma-\frac{1}{2}}(\vec{p}_1+\vec{p}_2)_T\,, \quad (4.5)$$

where we have the longitudinal missing mass

$$\mathfrak{M}_L=(Q_0{}^2-Q_L{}^2)^{1/2}\,. \quad (4.6)$$

In the limit as $\mathfrak{M}\to\infty$ with transverse momentum small, the longitudinal missing mass is approximately equal to the total missing mass. Using (3.24), we have at high energy

$$Q_\pi(\mathfrak{M},z)\sim\frac{\rm const}{\ln\mathfrak{M}^2}(\mathfrak{M}^2)^{z\pi g/2R^2-1}\exp\left[\frac{2R^4(p_{1T}{}^2+p_{2T}{}^2+2p_{1T}p_{2T}\cos\phi)}{\ln(2R^3\mathfrak{M}/z\pi g)-\frac{1}{2}\gamma-\frac{1}{2}}\right]. \quad (4.7)$$

In the limit of large $\mathfrak{M}$, in which (2.9) is valid, we therefore expand the exponential to give

$$\rho^{pp\to ppX}(s;\vec{p}_1,\vec{p}_2)\sim\frac{\rm const}{\sigma_{\rm tot}}\frac{(\mathfrak{M}^2)^{z\pi g/2R^2-2}}{\ln\mathfrak{M}^2}s^{-1}\exp[2\lambda(E_1+E_2)s^{-1/2}-R^2(P_{T1}{}^2+P_{T2}{}^2)]\times\left[1+\frac{4R^4(p_{1T}{}^2+p_{2T}{}^2+2p_{1T}p_{2T}\cos\phi)}{\ln\mathfrak{M}^2}+\cdots\right]. \quad (4.8)$$

The power $z\pi g/2R^2-2$ is the same which appears in (3.29). So that, independently of the logarithmic factors which appear in $\sigma_{\rm tot}$, we can rely on the analysis of FJLY[1] discussed above to conclude that the component of the Toller quantum number $M=1$ in the exchange is down from the leading $M=0$ term by only an inverse power of $\ln\mathfrak{M}^2$. The existence of $M=1$, from the discussion in Sec. II, signals unambiguously the presence of a Regge cut. We can identify this leading singularity with the Pomeranchuk singularity since its intercept is the same as the intercept of the singularity which appears in the total cross section. From the discussion above, the fact that the azimuthal correlation vanish only as an inverse power of $\ln\mathfrak{M}^2$ is seen to imply that, in this IEM, we necessarily have an infinite correlation length.

From (3.23), we see that the origin of the azimuthal correlations in the IEM is directly related to the conservation of transverse momentum. It is easy to see physically how transverse momentum conservation gives rise to an asymmetry in azimuthal angle. In a specific n-particle final state, we have

$$\sum_{j=1}^{n}\vec{q}_{jT}=0\,. \quad (4.9)$$

If we assume that when we remove one particle,

$$\vec{q}_{iT}=-\sum_{j\neq i}\vec{q}_{jT}\,, \quad (4.10)$$

the remaining particles are randomly distributed. We have the conditional average

$$\langle\vec{q}_{kT}\rangle|_{\vec{q}_{iT}}=-\vec{q}_{iT}/(n-1)\,, \quad (4.11)$$

so that

$$\langle\vec{q}_{iT}\cdot\vec{q}_{kT}\rangle=-\langle q_{iT}\rangle\langle q_{kT}\rangle/(n-1)\,. \quad (4.12)$$

The distributions calculated [Eq. (4.8)] just reflect this fact. The approximation of Gaussian behavior in $(\vec{p}_1+\vec{p}_2)$ in (4.7) is just valid in the same sense in which the central-limit theorem in mathematical statistics is valid.[14]

On the exclusive level, Foster *et al.*[15] have shown that all the integrations except one azimuthal angle in

$$\int\left(\prod_{j=1}^{n} d^2\vec{q}_{jT}\right)G\left(\sum_{j=1}^{n}\vec{q}_{jT}^{\,2}\right)\delta^{(2)}\left(\sum_{i=1}^{n}\vec{q}_{jT}\right) \qquad (4.13)$$

can be done exactly when $G(\vec{q}^{\,2})$ depends only on $\sum_{j=1}^{n} q_{jT}^2$. The result for the integrated normalized distribution is

$$\frac{d\sigma_n}{d\phi}=\frac{1}{\pi}\frac{n(n-2)}{(n-1)^2}\frac{1}{1-[\cos\phi/(n-1)]^2}\times\left(\frac{\cos\phi[\arcsin(\cos\phi/(n-1))-\frac{1}{2}\pi]}{(n-1)\{1-[\cos\phi/(n-1)]\}^{1/2}}+1\right), \qquad (4.14)$$

which can be shown to approach a form for high n:

$$\lim_{n\to\infty}\frac{d\sigma_n}{d\phi}=\frac{1}{\pi}\left[1-\frac{\pi}{2}\frac{\cos\phi}{(n-1)}+O(1/n^2)+\cdots\right]. \qquad (4.15)$$

The integral (4.13) does not include any constraints of energy conservation to normalize as a function of n except implicitly through the function $G(\sum_{j=1}^{n} q_{jT}^2)$. What the analysis in terms of the IEM shows is that the effect of energy conservation when the exclusive cross sections are summed to give the inclusive spectrum is to introduce a $1/\ln\mathfrak{M}^2$ damping on the asymmetry when the transverse momenta are limited. The connection between the inclusive distribution (4.8) and (4.15) can be seen by noting that the average number of additional particles grows with $\ln\mathfrak{M}^2$ in this model.

This was an illustrative calculation. The basic form of the distribution (4.8)

$$\rho(s;\vec{p}_1,\vec{p}_2)\propto\exp\left(\frac{A\cos\phi}{B+C\ln\mathfrak{M}^2}\right) \qquad (4.16)$$

is repeated when we calculate $\pi\pi$ and πp azimuthal distributions.[3]

The presence of long-range correlations does not arise solely because of the assumption that the modulus squared of the amplitude is written in the strictly factorizable form (3.1). The emission of particles in clusters can be studied in the framework of a cluster expansion for $Q(z,\beta)$ (Ref. 16). It can then be shown that the basic form (4.16) is retained in these cluster emission models due to the fact that there is not an ordering of particles in rapidity space. The thing which is required for the asymmetry to be of finite range, so that the leading singularity in the 8-point function can be a pole, is for the transverse momentum to be conserved semilocally in rapidity space. This is just one aspect of the general feature of short-range order which is built into multiperipheral models.[5] We will discuss these models briefly in Sec. V.

V. MODELS WITH SHORT-RANGE ORDER AND EXPERIMENT

A. The Consequence of Short-Range Order

If we define the momentum variables

$$q_i^{\pm}=q_i^0\pm q_i^3, \qquad (5.1)$$

related to the rapidities by

$$y_i=\ln(q_i^+/\kappa)=-\ln(q_i^-/\kappa), \qquad (5.2)$$

Campbell and Chang[17] define a property they call q factorization by the requirement that the *cross section* can be written

$$\sigma(q_1\cdots q_n)\underset{(q_i^+\cdots q_m^+)>(q_{m+1}^+\cdots q_n^+)}{\sim} f_m(q_1\cdots q_m)f_{n-m}(q_{m+1}\cdots q_n)+O(q_j^+/q_i^+), \qquad (5.3)$$

where $i\in(1,m)$ and $j\in(m+1,\ldots,n)$. This property, often called *short-range order*,[18] is shown to imply that all correlation lengths are finite. This property has been identified by some as a crucial element in general multiperipheral models, and has led to various applications within the context of the Feynman gas-liquid analogy.[18] Because of the assumption of short-range order, the limit $a_P(0)\to1$ is very delicate in the framework of MPM. Since short-range order disappears in this limit, MPM calculations involving Pomeranchukons depend sensitively on the nature of secondary trajectories.[19]

A particular consequence of short-range order is that transverse momentum is conserved, not only in bulk, but semilocally in rapidity space. The arguments leading to (4.16) from (4.15) therefore are not valid. For example, Mak and Tan[20] have examined azimuthal correlations in the framework of a general multiperipheral model containing a leading pole and a branch cut with intercept $a_C(0)<a_P(0)$. They claim that the requirement that the correlation be positive at $\cos\phi=-1$, in agreement with semilocal momentum conservation, determines the sign of the discontinuity across the cut in other processes. Note that, in contrast to our claims, these authors state that an IEM does not lead any to azimuthal asymmetry.

B. Comparison with Experiment

Friedman, Risk, and Smith[21] have done a detailed comparison of data on azimuthal correlations in $pp \to pp\pi^+\pi^+\pi^-\pi^-$ at 23 GeV/c with a simple statistical model and a multiperipheral model. While these data are not strictly inclusive, the authors claim the general features are typical of an inclusive distribution. Their statistical model was similar to the one discussed in Sec. III except the proton factors were given by

$$F(p)=e^{g(p_a-p_1)^2}. \tag{5.4}$$

The multiperipheral model chosen was a simple multi-Regge form,

$$|A_n|^2=C\prod_{i=1}^{n-1}\left(\frac{b+s_i}{b}\right)^{2\alpha_i(t_i)}\beta^2(t_i). \tag{5.5}$$

Their results show that the statistical model pretty well fits the experimental distribution over a wide range of rapidity gaps. Both the correlation between neighboring pions and between protons was well approximated by the statistical model, while the multi-Regge model overestimated the correlation between neighboring pions and underestimated the correlation between the protons. Figure 3, taken from their paper,[21] illustrates these findings.

The evidence, of course, is not conclusive in support of the infinite-range azimuthal correlations necessary to conclude that the leading Pomeranchuk singularity is a cut. All the comparison really shows is that the correlation length is much larger than that predicted by the simplest type of multi-Regge model with ρ exchange. These came in only to order $\alpha_\rho - 1$. A more complicated multiperipheral model with a moderately realistic spectrum of output singularities such as that envisioned by Mak and Tan[20] would have a very large correlation length, so that it would really be necessary to do experiments at ultra-high energies in order to distinguish between the two cases.

Stone, Ferbel, Slattery, and Werner[22] have also noted that experimental azimuthal distribution in K^+p and π^+d interactions are well represented by models without ordering assumptions.

Further indirect evidence in support of the absence of strong ordering features in experimental ϕ distributions came from Foster *et al*,[15] who studied the dependence of azimuthal dependence on charged prong number in pp and pd collisions at 28 GeV/c and K^-p collisions at 9 GeV/c. They did not make a direct test of the dependence of the correlations on the rapidity gap, but their results on n dependence can be related to this as discussed in Sec. IV.

The field is certainly open for more thorough analyses of data on azimuthal correlations. They are easy in the sense that the one-particle spectra do not have to be subtracted. A study of azimuthal correlations between leading particles at high energies would be very instructive.

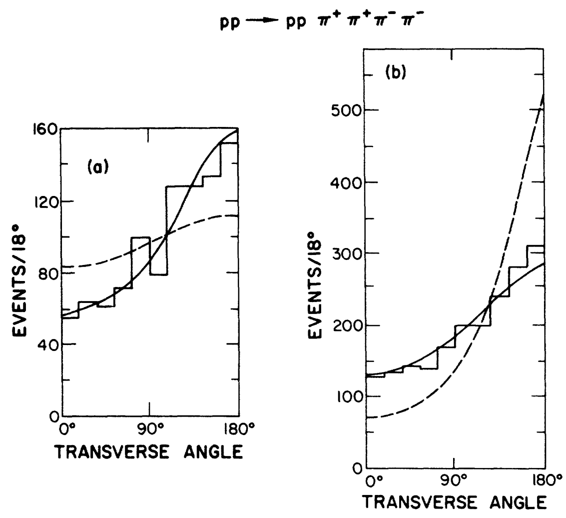

FIG. 3. Figure taken from Ref. 21 where further details can be found. The data are on $pp \to pp\pi^+\pi^+\pi^-\pi^-$ at 23 GeV/c. Figure 3(a) shows pp azimuthal distributions, and Fig. 3(b) shows the azimuthal distribution between protons and pions with the largest c.m. longitudinal momentum. Solid curve is statistical model and dashed curve is multi-Regge model.

C. Diffraction

Another important experimental consideration is the azimuthal dependence of those diffractive fragmentation cross sections which have the same energy dependence, within powers of logarithms, as the total cross section. Berger[23] points out that asymmetry in ϕ is an experimentally established fact at currently measured energies for those 2-3 or 2-4 processes commonly labeled diffractive. Deck-model calculations or simple phase-space arguments suggest these asymmetries persist at high energies. If this is true, the presence of $M=1$ in the leading singularity is again inevitable as a special example of a general statement of the necessity of long-range correlations in the presence of diffractive dissociation.[24]

Specific diffractive models also have long-range azimuthal correlations. In the nova model,[25] this is seen to be a consequence of the assumption that the decay of an excited state is isotropic in its own rest frame. These models also have a leading cut.

VI. CONCLUSIONS

We have seen that in the framework of models without short-range order the presence of long-range azimuthal correlations indicates that the leading Pomeranchuk singularity is a cut rather than a pole. IEM's form a class of very simple models which share many properties with MPM's but which do not have short-range order. These models, therefore, provide a counterexample to s-channel unitarity calculations which produce a leading J-plane pole.

In an IEM, factorization properties of the Pomeranchuk can be valid only in the limit that logarithmic factors are neglected. This may allow the possibility for experimental factorization tests to distinguish between the two cases. On the other hand, there is some experimental evidence[21-23] which supports the idea of long-range azimuthal correlations, but the situation cannot easily be distinguished from the possibility of a large but finite correlation length such as that predicted by an MPM with a leading pole only slightly above a cut.

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