g_2 as given by Eq. (A4) is in the reduced mass μ_j, $j=1,2$. Because the channel-1 ($\Sigma^- p$) and channel-2 ($\Sigma^0 n$) reduced masses are so close, in order to look at the dependence of g_Σ on λ_3 it makes sense to write

$$g_2 = g_1 + \delta g \tag{A8}$$

and consider $|\delta g| \ll |g_1|$. Substitution of Eq. (A8) into the right-hand side of Eq. (A3) and expansion of the result in powers of δg yields

$$g_\Sigma = g_1 + \tfrac{1}{3}\delta g - \frac{2}{9}\frac{\lambda_3}{(1-\lambda_3 g_1)}(\delta g)^2 + O(\delta g^3) + \cdots .$$

Thus terms that depend on λ_3 give at most a second-order change in g_Σ compared to the value it would have if only isospin-$\frac{1}{2}$ parts of the ΣN interaction were used; i.e., if λ_3 were zero. Since, from Eqs. (A1) through (A6), it is only through g_Σ that λ_3 affects the determination of the parameters β_Λ, λ_Λ, λ_Σ, and λ_x, these parameters also are very insensitive to the value of λ_3.

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[2]L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in Quantum Scattering Theory* (Davey, New York, 1965).

[3]Such a mesh size results in numerical values of the three-body cross sections accurate to within 1%.

[4]M. M. Nagels, T. A. Rijken, and J. J. de Swart, University of Nijmegen report (unpublished). The masses of the hyperons used here are just those given in Table III of this reference: 1115.59, 1189.42, 1192.51, and 1197.37 MeV for the Λ, Σ^+, Σ^0, and Σ^- mass, respectively.

[5]See, for example, F. Eisele, H. Filthuth, W. Fohlisch, V. Hepp, and G. Zech, Phys. Lett. 37B, 204 (1971).

[6]The $^3S_1$ $\Sigma^+ p$ phase shift for a repulsive interaction is in very good agreement with the results of Ref. 4 up to 300 MeV/c, so that the choice of momentum at which to fit the cross section does not appear critical. The value of 170 MeV/c is within the range covered by the experiments described in Ref. 5.

[7]See D. Cline, R. Laumann, and J. Mapp, Phys. Rev. Lett. 20, 1452 (1968).

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[9]L. H. Schick, in *Few Particle Problems in the Nuclear Interaction*, edited by Ivo Šlaus, Steven A. Moszkowski, Roy P. Haddock, and W. T. H. Van Oers (North-Holland, Amsterdam, 1972), p. 910.

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Tritium Beta Decay and the Neutrino Mass*

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A method of analysis is proposed which may provide a better limit for the neutrino mass when analyzing Kurie plots of the beta spectrum from tritium beta decay near the end-point energy. By determining the sign of the second derivative of the Kurie plot, it is possible to place an *upper* or *lower* limit to the neutrino mass of about 35 eV. However, the method demands data of sufficient accuracy.

The most favored method to detect the presence of a finite neutrino rest mass ν (we shall not distinguish between ν and $\bar{\nu}$) seems to be to study the behavior of a β spectrum in the vicinity of the end point. In fact, a most recent exhaustive study by Bergkvist[1] on the β spectrum from tritium decay places an upper limit of $\nu = 60$ eV after taking into account various corrections.

Essentially, for an allowed β decay of the bare tritium nucleus, the spectrum shape is given by

$$N(p)dp \propto F(Z,E)p^2(E_0-E)[(E_0-E)^2-\nu^2]^{1/2}dp \tag{1}$$

for an assumed $V-A$ form.[2] E_0 is the end-point energy corresponding to the case $\nu=0$. Usually the Kurie plot is used,

$$\begin{aligned} K(E) &= [N/p^2F]^{1/2} \\ &= \{(E_0-E)[(E_0-E)^2-\nu^2]^{1/2}\}^{1/2}, \end{aligned} \tag{2}$$

and the data are fitted as well as possible for different values of ν near the end point. However, an alternative method suggests itself. The plot deviates from a straight line near the end point if $\nu \neq 0$; in fact

$$C \equiv \frac{1}{K(E)}\frac{d^2K}{dE^2} = -\frac{\nu^2[\nu^2 + 2(E_0 - E)^2}{4K^2(E_0 - E)[(E_0 - E)^2 - \nu^2]^{1/2}} \tag{3}$$

is zero for $\nu = 0$ and negative for $\nu \neq 0$. Thus a determination of the sign of C will tell us whether the mass ν is zero or not. However, this will not give a quantitative limit to ν.

Most experiments are performed with the tritium atom, where atomic effects tend to obscure the simplicity of Eq. (1). Surprisingly, this has its compensations, as we shall show below, as it allows one to give a quantitative upper limit to ν.

For β decay of a free tritium atom, we can build up the composite β spectrum (neglecting recoil and nuclear effects) using the formalism developed for autoionization.[3] In the β decay of the tritium atom in its ground state, the atomic electron can find itself in any of the nS states of the final He^+ ion. The formalism in Ref. 3 gives the β spectrum for this process as (in units of $\hbar = c = m = 1$)

$$N_1(p)dp \propto p^2dp \sum_{n=1}^{\infty} ((W_0 - 1 - W_p - \Delta_n)[(W_0 - 1 - W_p - \Delta_n)^2 - \nu^2]^{1/2} \{|\langle e_n' | e_K\rangle|^2 F(Z', W_p) + |\langle e_p' | e_K\rangle|^2 (Z'\alpha)^3/n^3\pi - |\langle e_n' | e_K\rangle\langle e_K | e_p'\rangle| [F(Z', W_p)(Z_\alpha')^3/n^3\pi]^{1/2}\}), \tag{4}$$

where W_p is the electron energy, and

$$\Delta_n = B_K(\mathrm{He}^+)(1 - 1/n^2), \quad W_0 = W_{\beta\,\max} + 1 + \nu, \quad Z' = 2.$$

The overlap integrals $\langle e_n' | e_K\rangle$ and $\langle e_p' | e_K\rangle$ are evaluated in Ref. 3 for Dirac wave functions. We will use their nonrelativistic forms, however.

To Eq. (4) we must add another contribution, namely, that in which both electrons are ejected into the continuum. This shakeoff spectrum is given by

$$N_2(p)dp \propto p^2dp \int_0^{s_0(p)} \frac{s^2ds}{\pi^2}(W_0 - \Delta_\infty - W_p - W_s)[(W_0 - \Delta_\infty - W_p - W_s)^2 - \nu^2]^{1/2} \times \{|\langle e_p' | e_K\rangle|^2 F(Z', W_s) + |\langle e_s' | e_K\rangle|^2 F(Z', W_p) - |\langle e_s' | e_K\rangle\langle e_K | e_p'\rangle| [F(Z', W_p)F(Z', W_s)]^{1/2}\}, \tag{5}$$

with

$$s_0(p) = [(W_0 - \Delta_\infty - W_p - \nu)^2 - 1]^{1/2}.$$

In the above expressions, $B_K(\mathrm{He}^+)$ is the K-shell binding energy of the He^+ ion (0.054 keV). These expressions neglect final-state interactions between the two electrons. One could crudely allow for this by reducing $B_K(\mathrm{He}^+)$, since the interaction would, to a first approximation, reduce the binding energy. Furthermore, since we are interested in the spectrum near the end point, we expect the distortion due to final-state interaction to be small.

A Kurie plot of the composite spectrum in the vicinity of the end point is shown in Fig. 1, using $W_0 - 2 = 18.61$ keV, $B_K(\mathrm{He}^+) = 0.054$ keV, and $\nu = 0$, 60 eV. It shows generally that for $\nu = 0$ the almost straight portion turns upward to meet the end point at 18.61 keV, while the $\nu = 60$ eV case turns downward to meet the end point at 18.55 keV. If a straight line were extrapolated from the almost straight portion, it would intercept the energy axis at about 12 eV below the end point for the $\nu = 0$ case. In other words, if $\nu = 0$, the linear extrapolation causes a systematic error of 12 eV, as was first pointed out by Scott.[4]

Let us consider fitting a polynomial to the theoretical curves of the form

$$P_n(E) = \sum_{i=0}^{n} C_i^{(n)} E^i \tag{6}$$

for the energy range 17.6–18.5 keV. In Fig. 2, we depict the coefficients $C_2^{(2)}$, $C_2^{(3)}$, $C_3^{(3)}$ versus the neutrino mass. The interesting feature that emerges is that the coefficients C's change sign for a particular value of ν. In fact, for a quadratic fit, $C_2^{(2)}$ is positive for $\nu < 35$ eV and negative for $\nu > 35$ eV. The cubic fit shows a similar behavior, the coefficients $C_2^{(3)}, C_3^{(3)}$ crossing at the point $\nu = 34$ eV. The curves labeled 1, 2, and 3 correspond to a choice of $B_K(\mathrm{He}^+)$ of 0.054 keV, while those labeled 4 and 5 correspond to 0.025 and 0.100 keV, respectively. Furthermore, a shift of 100 eV in the value of W_0 produced less than 1 eV shift in the crossover points. Curve 4 represents the crude way of taking final-state interaction between the electrons into account, and it shifts the crossover point to

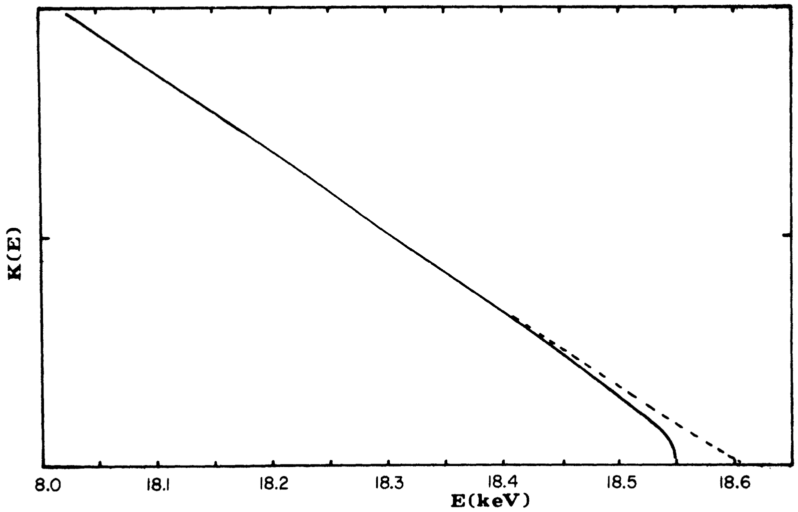

FIG. 1. Kurie plot of composite β spectrum in the vicinity of the end point. --- $\nu=0$; — $\nu=60$ eV. Ordinate in arbitrary units.

a lower value (~23 eV). Curve 5 indicates that the crossover point shifts to larger values (~58 eV) if we assume an unrealistic value for the binding energy. For reasonable parameters, one would expect the crossover point to be 35 eV or lower.

Thus if any quadratic energy fit to the experimental data produces a positive sign for $C_2^{(2)}$, it would automatically place an upper limit of 35 eV on the neutrino mass. Conversely, it would also place a lower limit on the neutrino mass if $C_2^{(2)}$ turned out to be negative. Similar results are obtained for the cubic and higher-order fits, where the uncertainty in the crossover point for these higher-order fits is about 1 eV.

Although the method seems very attractive, one has to approach it with caution. Even assuming the above model to be free from defects, the experimental situation is not clear-cut. In the results of Langer and Moffat,[5] it is found that the β spectrum is distorted due to surface effects. This distortion creeps towards the end point, depending on the experimental setup.

Our choice of the energy range is not arbitrary. It corresponds to the data range of Bergkvist.[1] Furthermore, it avoids the region near the end point where the statistics are poor and hopefully also avoids the region where spectrum distortion due to extraneous effects is large. It appears that even in the case of Bergkvist's data, the curvature may be attributed to extraneous effects.[1] In fact, Bergkvist linearizes the Kurie plot by applying a function

$$\Delta = c(E_0 - E)^2$$

to his data over the energy range 17.9–18.5 keV (470Ω–480Ω in his notation), which produces a value

$$c = -8\%(\text{keV})^{-2}.$$

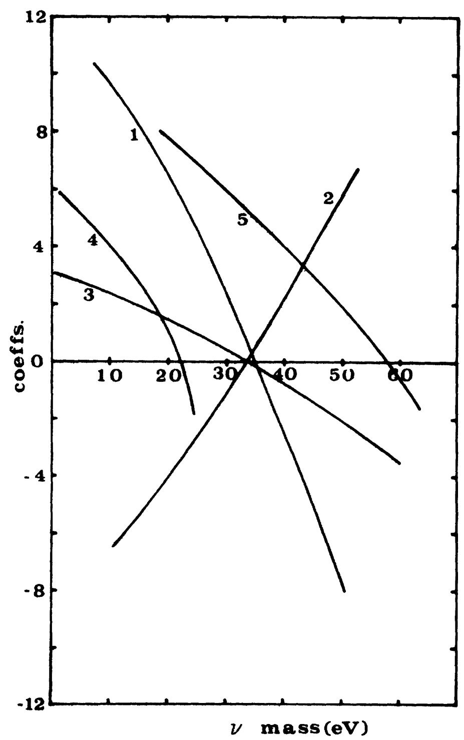

FIG. 2. Polynomial coefficients versus neutrino mass. 1: $C_2^{(2)}$, $B_K(\text{He}^+)=0.054$ keV; 2: $C_2^{(3)}$, $B_K(\text{He}^+)=0.054$ keV; 3: $C_3^{(3)}$, $B_K(\text{He}^+)=0.054$ keV; 4: $C_2^{(2)}$, $B_K(\text{He}^+)=0.025$ keV; 5: $C_2^{(2)}$, $B_K(\text{He}^+)=0.100$ keV. Ordinate in arbitrary units.

He finds that this correction for the energy range $475\Omega-480\Omega$, which he then uses for the neutrino-mass hypothesis with Eq. (1), is very minor. If one could take the sign of his c with confidence and if it can be shown that the distortion is not all due to extraneous effects, then one could put an *upper limit* of 35 eV on the neutrino mass. However, one must be very cautious, as surface effects which distort the β spectrum in this region are not known.

To take full advantage of the method, a way of minimizing distortion effects on the β spectrum by using different experimental arrangements will have to be found. The chief advantage of the method proposed seems to be that one can use data away from the end point, where statistics are better.

Of course, a traditional fit of W_0, ν, $B_K(\mathrm{He}^+)$ as parameters which may be varied to fit the data directly as well as possible is not ruled out. This method will supply, in addition to the neutrino mass, the end-point energy.

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Multiperipheral Model of Meson and Baryon Multiplicities*

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We construct a simple coupled-channel multiperipheral model of mesons and baryons in order to calculate their respective multiplicities at high energy. Requiring total cross sections to have their usual Regge behavior constrains the baryon multiplicity to be small, with an upper bound not far from the experimental value. More precise results are difficult to obtain in this context. Arguments are given that this result will be valid in more realistic models.

I. INTRODUCTION

One striking feature of the CERN Intersecting Storage Rings data is the small magnitude and steep rise with energy of the multiplicity of baryons with respect to mesons (see Fig. 1 and Refs. 1 and 2). The increase with energy can be understood intuitively in terms of the large ratio of masses: The multiplicity is expected to become appreciable only when the energy is high enough to produce many particles, and much more energy is needed to create baryons – especially via $B\bar{B}$ pairs – than mesons. In the framework of multiperipheral models, one can be more specific and say that the limiting $\ln s$ behavior of the average multiplicity will occur only when $\ln s$ is large enough for many partial cross sections to contribute, and at a given energy more of these are accessible to lower-mass particles. In this paper we will be more concerned with the small limiting[3] ratio, which may also be anticipated in this context for two reasons. First, it is a characteristic of multiperipheral models that the average rapidity interval between clusters of produced particles increases with the mass of the cluster. In the ABFST (Amati-Bertocchi-Fubini-Stanghellini-Tonin) model,[4] for example, it is found[5,6] that the average interval, w, in $\ln s$ between links in the multiperipheral chain is roughly given by

$$\cosh w \approx 1 + s_0/2\,|t_{\max}|\,, \qquad (1)$$

where $(s_0)^{1/2}$ is the mass emitted at each link and $|t_{\max}|$ is the effective upper limit of momentum transfer. Since baryons are likely to be emitted from more massive clusters than mesons, these clusters will be more widely spaced in rapidity, and thus have a smaller average multiplicity. By cluster, we mean, for example, the $\Delta(1236)$ and