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Low-Energy $\Sigma^- d$ Scattering with Explicit and Implicit Λ Channel*

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A Faddeev formalism and two-body S-wave central separable potentials are used to calculate $\Sigma^- d$ quartet elastic and reaction cross sections for $\Sigma^- d$ lab momentum in the range 30–150 MeV/c. A three-channel ($\Sigma^- p$, $\Sigma^0 n$, Λn) hyperon-nucleon potential is used, $\Sigma^- d$ cross sections being calculated with both an explicit and an implicit treatment of the Λ channel. The implicit Λ channel approximation is found to be excellent when the $\Sigma^- n$ interaction is repulsive, but breaks down in part for an attractive $\Sigma^- n$ interaction.

I. INTRODUCTION

In an earlier work[1] low-energy quartet $\Sigma^- d$ cross sections were calculated in a crude model that incorporated the assumption that the Λ channel could be treated implicitly. In this paper the validity of the implicit Λ-channel approximation (ILCA) for this model is demonstrated.

The basic model used here is the same as that of Ref. 1, but more recent experimental data and theoretical analysis of the hyperon-nucleon interaction are used here. The original motivation for Ref. 1 was an interest in the sensitivity of low-energy $\Sigma^- d$ scattering to the existence of a spin-1 Λn resonance just below the $\Sigma^0 n$ threshold. To make the most of this sensitivity, only quartet $\Sigma^- d$ cross sections were calculated. Here too only spin-$\frac{3}{2}$ $\Sigma^- d$ interactions are considered so that only spin-triplet two-body interactions are needed. The two-body interactions are all taken to be $^3S_1$ central potentials. The single-channel np and $\Sigma^- n$ interactions and each matrix element of the three-channel ($\Sigma^- p$, $\Sigma^0 n$, Λn) interaction are represented by an S-wave nonlocal separable (NLS) potential. Coulomb forces are neglected. $\Sigma^- d$ elastic and reaction cross sections are calculated from the elastic scattering amplitude for incident Σ^- lab momentum $p_- = 30, 60, 90, 120, 150$ MeV/c (the threshold for deuteron breakup being $p_- = 93.4$ MeV/c) using a Faddeev[2] type of formalism. Most of these cross sections are calculated both with an explicit Λ channel and with the ILCA.

If the Λ channel is treated correctly (i.e., explicitly) then terms must be included in the $\Sigma^- d$ elastic amplitude for which, after the Λ is produced via $(\Sigma^- p)n \rightarrow (\Lambda n)n$ or $(\Sigma^- p)n \rightarrow (\Sigma^0 n)n \rightarrow (\Lambda n)n$, the Λ scatters off of the other neutron and/or converts back and forth to either Σ (which themselves may scatter off the nucleons) before finally converting back to a Σ^-. Because the Λ produced in $\Sigma N \rightarrow \Lambda n$ is quite energetic (i.e., $M_\Sigma - M_\Lambda \approx 80$ MeV) the Λ should leave the interaction volume too quickly for such Λn interactions to play much of a role. In the ILCA all such terms involving Λ scatterings and conversions that result in a linking of the two nucleons are dropped from consideration. Thus the only role played by the Λn channel in the $(\Sigma^- p, \Sigma^0 n, \Lambda n)$ interaction is as an absorptive energy-dependent contribution to the $\Sigma^- p \rightarrow \Sigma^- p$, $\Sigma^- p \rightarrow \Sigma^0 n$, and $\Sigma^0 n \rightarrow \Sigma^0 n$ amplitudes. In the determination of the potential parameters of the $(\Sigma^- p, \Sigma^0 n, \Lambda n)$ interaction from experimental results a full 3-channel formalism is used, but in employing this interaction in the $\Sigma^- d$ problem all *explicit* reference to the Λ channel is dropped and only the two-channel $(\Sigma^- p, \Sigma^0 n)$ interaction appears. For quartet $\Sigma^- d$ scattering described here the number of coupled integral equations to be solved is one less than in the exact treatment. For doublet $\Sigma^- d$ scattering, in which both spin-zero and spin-one S-wave two-body interactions are needed, if both Λn channels could be treated with the ILCA, there would be a saving of two coupled integral equations; i.e., there would be a reduction in the number of equations from 12 to 10. Based on a need for at least 24 points on a mesh side per integral equation for an adequate representation of the functions involved,[3] in the doublet scattering case with only S-wave interactions this reduction already means a saving of more than 25 000 words of computer storage. The validity of the ILCA from this very practical point of view is therefore certainly of great interest.

In Sec. II the details of the NLS two-body potentials are given and the determination of their parameters is described. Section III contains the results obtained for the $\Sigma^- d$ cross sections. These indicate that when the $\Sigma^- n$ interaction is repulsive, the ILCA is very good indeed, but when the $\Sigma^- n$ interaction is attractive, the ILCA breaks down in calculations of the reaction cross section at very low values of the incident Σ^- momentum. The dependence of the size of the $\Sigma^- d$ cross sections on the Λn resonance parameters and the sign of the $\Sigma^- n$ interaction is also covered.

II. TWO-BODY POTENTIALS

The np potential was taken to have the same NLS form as that of Ref. 1. In a relative momentum-space representation, the matrix element of the np potential-energy operation is

$$\langle \vec{k}' | V_{np} | \vec{k} \rangle = \lambda_N v_N(k') v_N(k) \quad , \tag{1}$$

where

$$v_N(k) = 1/(k^2 + \beta_N^2) \ . \tag{2}$$

From the deuteron binding energy $\epsilon = 2.226$ MeV, the neutron and proton masses $M_n = 939.5527$ MeV and $M_p = 938.259$ MeV, and the triplet scattering length $a_N = 5.39$ F, it follows that

$$\lambda_N = -84.3957 \times (2\pi/10)^3 \mathrm{F}^{-2}, \tag{3}$$

$$1/\beta_N = 0.698\,342 \ \mathrm{F} \ . \tag{4}$$

For the $\Sigma^- n$ potential the same basic form as that of the np potential was used:

$$\langle \vec{k}' | V_{-n} | \vec{k} \rangle = \lambda_3 \, v_-(k') v_-(k) \ , \tag{5}$$

$$v_-(k) = 1/(k^2 + \beta_-^2) \ . \tag{6}$$

This potential was assumed to be a pure isospin-$\frac{3}{2}$ interaction, the same as the nuclear part of the $\Sigma^+ p$ interaction. The parameters λ_3 and β_- were determined by fitting the $^3S_1$ $\Sigma^+ p$ nuclear scattering length a_+ and effective range r_{0+} values obtained by Nagels *et al.*[4]; namely $a_+ = 0.63$ F and $r_{0+} = -0.76$ F. The results were

$$\lambda_3 = 62.7456 \times (2\pi/10)^3 \ \mathrm{F}^{-2} \ , \tag{7}$$

$$\beta_- = 1.52627 \ \mathrm{F}^{-1} \ . \tag{8}$$

Since the sign of the low-energy $\Sigma^+ p$ amplitude has not yet been definitely determined directly from experiment,[5] a second value for λ_3 was determined. Using the values of λ_3 and β_- given in Eqs. (7) and (8), the $^3S_1$ $\Sigma^+ p$ cross section was calculated for a lab momentum of 170 MeV/c. The parameter β_- was kept fixed and a new $\lambda_3 < 0$ was determined by the condition that it yield this same cross section.[6] The result was

$$\lambda_3 = -24.6995 \times (2\pi/10)^3 \ \mathrm{F}^{-2} \ , \tag{9}$$

which with the value of β_- from Eq. (8) yields

$$a_+ = -0.75 \ \mathrm{F}, \qquad r_{0+} = 4.25 \ \mathrm{F} \ . \tag{10}$$

As shown in Table I, four different $\Sigma^- p$ potentials were constructed for both the repulsive and attractive $\Sigma^+ p$ interactions.

The 3-channel $\Sigma^- p$ NLS potential was given a simple form by the assumptions that the $\Sigma^- p$, $\Sigma^0 n$, and $\Sigma^+ p$ shapes and range parameters were the same and that the strengths of the potentials involved were just those that would occur if the Σ and N mass multiplets were each degenerate. In a relative momentum-space representation, the matrix elements of this potential could then be written as

$$\langle \vec{k}_i' | V_{ij} | \vec{k}_j \rangle = \lambda_{ij} \ v_i(k_i') v_j(k_j), \qquad i, j = 1, 2, 3. \tag{11}$$

With channel 1 the $\Sigma^- p$ channel, channel 2 the $\Sigma^0 n$ channel, and channel 3 the Λn channel,

$$\beta_1 = \beta_2 = \beta_- \equiv \beta_\Sigma \ , \tag{12}$$

$$\lambda_{11} = \tfrac{1}{3}(2\lambda_\Sigma + \lambda_3), \qquad \lambda_{21} = \tfrac{1}{3}\sqrt{2}(\lambda_3 - \lambda_\Sigma) \tag{13}$$

$$\lambda_{22} = \tfrac{1}{3}(\lambda_\Sigma + 2\lambda_3) \ , \qquad \lambda_{31} = (\tfrac{2}{3})^{1/2}\lambda_x, \tag{14}$$

$$\lambda_{32} = -(\tfrac{1}{3})^{1/2}\lambda_x, \qquad \lambda_{33} = \lambda_\Lambda . \tag{15}$$

TABLE I. *YN* input parameters and the resulting $\Sigma^- p$ scattering lengths. For all cases the additional inputs $a_\Lambda = -1.02$ F and $r_{0\Lambda} = 2.55$ F were used.

Potential No.	a_+ (F)	r_{0+} (F)	E_0 (MeV)	Γ (MeV)	A_- (F)
1	0.63	−0.76	70.92	10.0	$2.0437 - i0.7355$
2	0.63	−0.76	70.92	5.0	$2.1769 - i0.6087$
3	0.63	−0.76	73.92	10.0	$2.0128 - i1.0601$
4	0.63	−0.76	73.92	5.0	$2.3299 - i0.9436$
5	−0.75	4.25	70.92	10.0	$1.6541 - i1.3735$
6	−0.75	4.25	70.92	5.0	$1.8621 - i1.2528$
7	−0.75	4.25	73.92	10.0	$1.4943 - i1.7736$
8	−0.75	4.25	73.92	5.0	$1.9496 - i1.7405$

Here λ_Σ, λ_x, and λ_Λ are the $T=\frac{1}{2}$ $\Sigma N \leftrightarrow \Sigma N$, $\Sigma N \leftrightarrow \Lambda N$, and $\Lambda N \leftrightarrow \Lambda N$ strength parameters, respectively. The sign of λ_x is of course arbitrary.

For this $\Sigma^- p$ potential there are 6 potential parameters, λ_Σ, λ_x, λ_Λ, λ_3, β_Λ, and β_Σ, of which two, λ_3 and β_Σ, have already been determined. The other four parameters were fixed by the requirements that this potential yield the ${}^3S_1$ Λn scattering length a_Λ and effective range $r_{0\Lambda}$ given in Ref. 4, namely $a_\Lambda = -1.02$ F and $r_{0\Lambda} = 2.55$ F, and that a Λn resonance exist at a c.m. energy E_0 with width Γ, the values of E_0 and Γ being just those given in Table I. The existence, or nonexistence, of such a resonance again has not been completely determined by experiment, but, if it does exist, the values of E_0 (lying 3 and 6 MeV below the $\Sigma^0 n$ threshold, respectively) and Γ used here are certainly reasonable.[7]

With the use of the expressions given in the Appendix, the 8 sets of input parameters given in Table I were found to yield the corresponding 8 sets of potential parameters given in Table II as well as the corresponding values for the ${}^3S_1$ $\Sigma^- p$ scattering length A_- shown in the last column of Table I. It should be noted that for corresponding cases (i.e., potentials 1 and 5, 2 and 6, 3 and 7, 4 and 8) the values obtained in Table II for β_Λ, λ_Λ, λ_x, and λ_Σ are almost completely independent of λ_3. As shown in the Appendix, this is a result of the almost degenerate $\Sigma^- p$ and $\Sigma^0 n$ thresholds. For these corresponding cases, however, Eqs. (13) through (15) indicate that the $\Sigma^- p \leftrightarrow \Sigma^- p$, $\Sigma^- p \leftrightarrow \Sigma^0 n$, and $\Sigma^0 n \leftrightarrow \Sigma^0 n$ strengths do change quite radically as λ_3 is changed from its repulsive to its attractive value. In particular Table I shows that A_- becomes much more absorptive.

III. $\Sigma^- d$ CALCULATIONS

As in Ref. 1 the application of the Faddeev formalism paralleled that used by Hetherington and Schick[8] in their treatment of $K^- d$ elastic scattering. Here, however, the Λ channel was treated both implicitly and explicitly. In the former case four coupled one-dimensional integral equations were obtained and in the latter five such equations were obtained. These equations were solved numerically on the University of Wyoming SDS $\Sigma 7$ using a mesh size of $24N \times 24N$ points, where N was the number of coupled equations. The $\Sigma^- d$ elastic cross section σ_{el} was calculated directly from the elastic scattering amplitude $f_{-d}(\theta)$. The total cross section was obtained from $f_{-d}(\theta)$ using the optical theorem. The reaction cross section σ_{re} was then obtained by subtraction.

The $\Sigma^- d$ cross sections were calculated at incident Σ^- lab momenta p_- of 30, 60, 90, 120, and 150 MeV/c for which the relative $\Sigma^- d$ momenta are 18.31, 36.62, 54.93, 73.24, and 91.55 MeV/c, respectively. The corresponding energies available in the $\Sigma^- d$ and $\Sigma^0 nn$ systems are -2.00 and 1.56 MeV, -1.30 and 2.26 MeV, -1.62 and 3.40 MeV, 1.44 and 5.01 MeV, and 3.51 and 7.08 MeV, respectively. The higher two p_- values lie above the threshold for deuteron breakup, whereas for the lower three values of p_-, σ_{re} includes only the processes $\Sigma^- d \to \Sigma^0 nn$ and $\Sigma^- d \to \Lambda nn$.

Table III contains the results obtained for σ_{el} and σ_{re} with the use of the YN potentials 1–4, while Table IV contains these same results for potentials 5–8. In both Tables III and IV, the column labeled im show the results using the implicit Λ channel approximation whereas those columns headed ex show the results obtained with an explicit Λ channel. In the latter case, because of the longer computer time needed to complete each calculation, cross sections were calculated only for $p_- = 30$, 90, and 150 MeV/c.

From Table III it is immediately obvious that the implicit Λ channel approximation is extremely good when the $\Sigma^- n$ potential is repulsive. In all cases σ_{re} calculated with this approximation lies within 1% of the result obtained when the Λ channel is treated explicitly. Similarly, the results obtained for σ_{el} using this approximation are also very close to the exact results, the error being less

TABLE II. YN potential parameters for the potentials of Table I. For all potentials $\beta_\Sigma = 1.526\,27$ F^{-1}.

Potential No.	β_Λ (F^{-1})	$\lambda_3/(\frac{1}{5}\pi)^3$ (F^{-2})	$-\lambda_\Lambda/(\frac{1}{5}\pi)^3$ (F^{-2})	$-\lambda_\Sigma/(\frac{1}{5}\pi)^3$ (F^{-2})	$-\lambda_x/(\frac{1}{5}\pi)^3$ (F^{-2})
1	1.708 63	62.7456	29.1527	112.3833	49.0518
2	1.832 65	62.7456	48.9613	113.6106	36.8598
3	1.685 58	62.7456	24.6015	101.0973	54.5268
4	1.821 35	62.7456	46.2080	102.2253	41.2338
5	1.706 94	−24.6995	28.8918	112.1803	49.2363
6	1.831 80	−24.6995	48.7998	113.4076	37.0151
7	1.680 42	−24.6995	23.8418	100.7234	55.0116
8	1.818 77	−24.6995	45.7255	101.8327	41.6619

TABLE III. $\Sigma^- d$ elastic and reaction cross sections using implicit (im) and explicit (ex) Λ-channel calculations for potentials 1–4.

p_- (MeV/c)	Potential 1 im	Potential 1 ex	Potential 2 im	Potential 2 ex	Potential 3 im	Potential 3 ex	Potential 4 im	Potential 4 ex
				σ_{el} (mb)				
30	901	923	1116	1127	738	817	1200	1221
60	661		819		542		781	
90	486	489	598	605	383	399	524	534
120	361		438		278		364	
150	269	275	321	324	205	212	258	263
				σ_{re} (mb)				
30	1640	1630	1553	1541	2035	2048	2273	2266
60	777		759		900		995	
90	523	520	520	518	583	582	642	640
120	394		396		429		468	
150	314	311	318	317	334	332	361	360

than 5% in all cases. From Table IV, when the $\Sigma^- n$ potential is attractive, although the ILCA is very good for $p_- \geq 90$ MeV/c – errors in the cross sections $\lesssim 10\%$ being typical – for σ_{re} at $p_- = 30$ MeV/c the errors range up to 40%. Furthermore, it is now σ_{el} for which the discrepancies are generally smaller rather than σ_{re} as was the case for the results shown in Table III. An argument can be made that an attractive $\Sigma^- n$ potential pulls the neutron closer to the proton while a repulsive $\Sigma^- n$ potential breaks up the bound np system. Thus, when $\Sigma^- p \rightarrow \Lambda n$ occurs, a rescattering of the Λ by the original neutron is more likely in the former case than it is in the latter. As it is just these Λn rescatterings which are neglected in the implicit Λ-channel approximation, it makes sense that this approximation works better for a repulsive $\Sigma^- n$ potential (Table III results) than it does with an attractive $\Sigma^- n$ potential (Table IV results). However, why this approximation is so very good for the cases shown in Table III and why it tends to break down for some particular calculations of σ_{re} shown in Table IV still remain to be explained.

To check that the validity of the implicit Λ-channel approximation for a repulsive $\Sigma^- n$ potential was not confined to cases for which a Λn resonance was present close to but below the $\Sigma^0 n$ threshold, σ_{re} and σ_{el} were calculated for 3 further cases. In all three of these, a_Λ, $r_{0\Lambda}$, a_+, and r_{0+} were kept the same as the value used to determine potentials 1–4. In the first, however, the additional param-

TABLE IV. $\Sigma^- d$ elastic and reaction cross sections using implicit (im) and explicit (ex) Λ-channel calculations for potentials 5–8.

p_- (MeV/c)	Potential 5 im	Potential 5 ex	Potential 6 im	Potential 6 ex	Potential 7 im	Potential 7 ex	Potential 8 im	Potential 8 ex
				σ_{el} (mb)				
30	986	974	1061	1043	1112	1131	1299	1276
60	810		897		860		1035	
90	638	659	719	723	645	699	786	803
120	481		547		469		572	
150	354	374	404	411	336	373	407	425
				σ_{re} (mb)				
30	681	493	416	334	1045	723	685	523
60	412		317		549		451	
90	334	291	293	273	403	339	379	344
120	290		273		327		327	
150	257	242	254	246	276	254	284	273

eters' fit were the real and imaginary parts of the $\Sigma^- p$ scattering length A_- at the value $A_- = (1.25 - i\,1.00)$ F. With these input parameters the real part of the Λn phase shift passes through $\frac{1}{2}\pi$ between the $\Sigma^0 n$ and $\Sigma^- p$ thresholds. Similarly, the values $A_- = (0.50 - i\,1.00)$ F and $A_- = (-1.00 - i\,1.00)$ F were also used, both of these yielding potentials with nonresonant Λn amplitudes. For all three of these further cases, the values obtained for σ_{el} and σ_{re} with Σ^- incident lab momentum in the range $30 \leq p_- \leq 150$ MeV/c using the ILCA were in excellent agreement with the values obtained treating the Λ channel correctly. The percentage errors in these $\Sigma^- d$ cross sections were never greater than 7% for $p_- = 30$ MeV/c, while for $p_- = 90$ or 150 MeV/c the errors varied from 4% down to less than 1%.

To a large degree the sensitivity of the $\Sigma^- d$ cross sections calculated here to the resonance parameters is greater than, but parallels, the sensitivity obtained in Ref. 1, where the method of determining the potential parameters was somewhat different. If the resonance width is fixed and its position is varied from 3 MeV to 6 MeV below the $\Sigma^0 n$ threshold, variations of up to 30% in σ_{el} and up to 50% in σ_{re} are obtained, with most of the larger variations occurring at the lowest values of p_-. It is clear from Tables III and IV that because of the interplay between the effects of changing E_0 and the effects of changing Γ, if a Λn resonance near the $\Sigma^0 n$ threshold exists, low-energy $\Sigma^- d$ experimental cross sections (even the $S = \frac{3}{2}$ cross sections calculated here) could not be used to determine both of these parameters. Even determining one of the resonance parameters given the other might be very difficult when the effects of doublet $\Sigma^- d$ scattering are included in the theoretical calculations.

On the other hand, as pointed out by Schick,[9] the $\Sigma^- d$ reaction cross sections are very sensitive in a systematic way to the sign of the $\Sigma^- n$ interaction; the lower the energy the greater the difference in σ_{re} obtained from Tables III and IV at the same values of E_0, Γ, and p_-. Calculations of σ_{re} performed with $\Sigma^- p$ potentials adjusted to give the same values of A_-, rather than the same values of E_0 and Γ, confirm that this sensitivity is not due to the difference in the values of A_- of potentials 1 and 5, 2 and 6, 3 and 7, or 4 and 8. It might be possible to use low-energy $\Sigma^- d$ reaction cross sections, rather than Coulomb interference in low-energy $\Sigma^+ p$ angular distributions, to determine the sign of the low-energy isospin-$\frac{3}{2}$ ΣN potential. Whether the $\Sigma^- d$ reaction cross section remains sensitive to the sign of the spin-1 isospin-$\frac{3}{2}$ ΣN potential when $\Sigma^- d$ doublet scattering is included in the theoretical calculations is presently under investigation.

APPENDIX

The calcuation of the Λn phase shift δ_Λ proceeds in the manner of the analogous calculation in Ref. 1. The results presented below are given for the sake of completeness.

As in Ref. 1, the 3-channel hyperon-nucleon NLS potential discussed in Sec. II leads to

$$k_3 \cot \delta_\Lambda = -\frac{2\pi(-g_3^{\,p} + 1/\gamma_3)}{\mu_3 v_3^{\,2}(k_3)}\,, \tag{A1}$$

where

$$\gamma_3 = \lambda_\Lambda + \lambda_\chi^{\,2} g_\Sigma/(1-\lambda_\Sigma g_\Sigma)\,, \tag{A2}$$

μ_j is the reduced mass in channel j, and k_3 is the relative momentum in channel 3; i.e., the Λn channel. Here, however, because the isospin-$\frac{3}{2}$ contribution to the ΣN interaction is included,

$$g_\Sigma = \frac{\frac{1}{3}(2g_1 + g_2) - \lambda_3 g_1 g_2}{1 - \frac{1}{3}\lambda_3(g_1 + 2g_2)}\,, \tag{A3}$$

where

$$g_j = -\frac{\mu_j}{\pi^2}\int_0^\infty \frac{q^2 v_j^{\,2}(q)\,dq}{q^2 - k_j^{\,2} - i\eta}\,, \quad \eta \to 0^+ \tag{A4}$$

and $g_j^{\,p}$ is the principal part of g_j. In the limit as $\lambda_3 \to 0$, Eq. (A3) reduces to the result obtained in Ref. 1.

From Eq. (A1) the condition for the existence of a Λn resonance at a Λn-channel energy $E_3 = E_0$ is

$$1 - g_3^{\,p}\gamma_3 = 0\,, \tag{A5}$$

where the left-hand side is evaluated at $E_3 = k_3^{\,2}/2\mu_3 = E_0$. The width Γ of such a resonance is given by[1]

$$\Gamma = -(k_0/\mu_3)\Big/\frac{d}{dk_3^{\,2}}(k_3 \cot\delta_\Lambda)\bigg|_{k_3 = k_0}\,, \tag{A6}$$

i.e., k_0 is the value of k_3 at the resonance.

In the limit $k_3 \to 0$, Eq. (A1) becomes identical with

$$k_3 \cot\delta_\Lambda = -1/a_\Lambda + \tfrac{1}{2} r_{0\Lambda} k_3^{\,2}\,. \tag{A7}$$

The equations in this appendix together with the equations and input parameters given in Sec. II are sufficient to determine the hyperon-nucleon potential parameters.

Now consider Eq. (A3) for the case used in the text; namely, the only difference between g_1 and

g_2 as given by Eq. (A4) is in the reduced mass μ_j, $j=1,2$. Because the channel-1 ($\Sigma^- p$) and channel-2 ($\Sigma^0 n$) reduced masses are so close, in order to look at the dependence of g_Σ on λ_3 it makes sense to write

$$g_2 = g_1 + \delta g \tag{A8}$$

and consider $|\delta g| \ll |g_1|$. Substitution of Eq. (A8) into the right-hand side of Eq. (A3) and expansion of the result in powers of δg yields

$$g_\Sigma = g_1 + \tfrac{1}{3}\delta g - \frac{2}{9}\frac{\lambda_3}{(1-\lambda_3 g_1)}(\delta g)^2 + O(\delta g^3) + \cdots .$$

Thus terms that depend on λ_3 give at most a second-order change in g_Σ compared to the value it would have if only isospin-$\frac{1}{2}$ parts of the ΣN interaction were used; i.e., if λ_3 were zero. Since, from Eqs. (A1) through (A6), it is only through g_Σ that λ_3 affects the determination of the parameters β_Λ, λ_Λ, λ_Σ, and λ_x, these parameters also are very insensitive to the value of λ_3.

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[1]L. H. Schick and P. S. Damle, Phys. Rev. D 5, 2773 (1972).

[2]L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in Quantum Scattering Theory* (Davey, New York, 1965).

[3]Such a mesh size results in numerical values of the three-body cross sections accurate to within 1%.

[4]M. M. Nagels, T. A. Rijken, and J. J. de Swart, University of Nijmegen report (unpublished). The masses of the hyperons used here are just those given in Table III of this reference: 1115.59, 1189.42, 1192.51, and 1197.37 MeV for the Λ, Σ^+, Σ^0, and Σ^- mass, respectively.

[5]See, for example, F. Eisele, H. Filthuth, W. Fohlisch, V. Hepp, and G. Zech, Phys. Lett. 37B, 204 (1971).

[6]The $^3S_1$ $\Sigma^+ p$ phase shift for a repulsive interaction is in very good agreement with the results of Ref. 4 up to 300 MeV/c, so that the choice of momentum at which to fit the cross section does not appear critical. The value of 170 MeV/c is within the range covered by the experiments described in Ref. 5.

[7]See D. Cline, R. Laumann, and J. Mapp, Phys. Rev. Lett. 20, 1452 (1968).

[8]J. H. Hetherington and L. H. Schick, Phys. Rev. 137, B935 (1965).

[9]L. H. Schick, in *Few Particle Problems in the Nuclear Interaction*, edited by Ivo Šlaus, Steven A. Moszkowski, Roy P. Haddock, and W. T. H. Van Oers (North-Holland, Amsterdam, 1972), p. 910.

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Tritium Beta Decay and the Neutrino Mass*

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A method of analysis is proposed which may provide a better limit for the neutrino mass when analyzing Kurie plots of the beta spectrum from tritium beta decay near the end-point energy. By determining the sign of the second derivative of the Kurie plot, it is possible to place an *upper* or *lower* limit to the neutrino mass of about 35 eV. However, the method demands data of sufficient accuracy.

The most favored method to detect the presence of a finite neutrino rest mass ν (we shall not distinguish between ν and $\bar{\nu}$) seems to be to study the behavior of a β spectrum in the vicinity of the end point. In fact, a most recent exhaustive study by Bergkvist[1] on the β spectrum from tritium decay places an upper limit of $\nu = 60$ eV after taking into account various corrections.

Essentially, for an allowed β decay of the bare tritium nucleus, the spectrum shape is given by

$$N(p)dp \propto F(Z,E)p^2(E_0-E)[(E_0-E)^2-\nu^2]^{1/2}dp \tag{1}$$

for an assumed $V-A$ form.[2] E_0 is the end-point energy corresponding to the case $\nu=0$. Usually the Kurie plot is used,

$$\begin{aligned} K(E) &= [N/p^2F]^{1/2} \\ &= \{(E_0-E)[(E_0-E)^2-\nu^2]^{1/2}\}^{1/2}, \end{aligned} \tag{2}$$