Structure Effects in the Decays $K_{L,S} \to \gamma$+Dalitz Pair*

L. M. Sehgal
III. Physikalisches Institut, Technische Hochschule, Aachen, Germany
(Received 8 September 1972)

We investigate the form factors associated with the vertices $K_{1,2} \to \gamma\gamma$ when one of the photons is virtual and their influence on the rate and spectrum of the Dalitz pair reactions $K_{1,2} \to \gamma\, l^+ l^-$. We find that while the form factor in $K_2 \to \gamma\gamma$ produces a small uniform enhancement, that in $K_1 \to \gamma\gamma$ suppresses the relative probability of lepton pairs of large invariant mass ($\sqrt{t}$), an effect that should be most noticeable in the decay $K_1 \to \gamma\mu^+\mu^-$ for muon pairs of $t > 4$(pion mass)^2.

I. INTRODUCTION

The decay $K_L \to \gamma\gamma$ is known to occur at a rate of $(0.95 \pm 0.08) \times 10^4\ \text{sec}^{-1}$,[1] while the decay $K_S \to \gamma\gamma$ currently being searched for[2] is expected on conventional models to occur with a somewhat higher rate.[3] These rates are by no means low, which encourages the hope that the associated Dalitz processes $K_{L,S} \to \gamma l^+ l^-$ ($l = e, \mu$) will eventually come under measurement. These Dalitz processes have a special significance, as they act as probes of the structure that underlies the $K_{L,S} \to \gamma\gamma$ vertices – a structure about which our notions may well be inaccurate. The object of this paper is to predict, on the basis of generally accepted models, the structure effects that are to be expected in the decays $K_{L,S} \to \gamma l^+ l^-$.

Besides the Dalitz mechanism, an amplitude for $K_{L,S} \to \gamma l^+ l^-$ can, in principle, arise as the result of bremsstrahlung from the reactions $K_{L,S} \to l^+ l^-$. In the conventional theory, however, the latter processes are allowed only in order Ge^4 or G^2, in which case the bremsstrahlung effect should be negligible. Experimental limits on $K_L \to l^+ l^-$ confirm the smallness of this amplitude,[4] and while the present limits on $K_S \to l^+ l^-$ are not very stringent, we shall assume this amplitude to be likewise small.

Our calculation will be based on the assumption of CP invariance, and we shall identify the states K_S and K_L with the CP eigenstates K_1 and K_2, respectively. Some comments on the possible consequences of CP violation will be made in Sec. VI.

II. DECAYS $K_{1,2} \to \gamma l^+ l^-$ IN THE ABSENCE OF FORM FACTORS

A calculation of the decay $K_{1,2} \to \gamma l^+ l^-$ involves a knowledge of the $K_{1,2} \to \gamma\gamma$ vertex when one of the photons is off the mass shell. The dependence of this vertex on t, the invariant (mass)^2 of the virtual photon, can be expressed by means of a form factor $F_{1,2}(t)$. In the limit in which $F_{1,2}(t)$ is constant, the Dalitz process can be completely expressed in terms of the two-photon decay. The differential decay rate with respect to t [which is the invariant (mass)^2 of the lepton pair] is given by[5]

$$\frac{d\Gamma(K_{1,2} \to \gamma l^+ l^-)/dt}{\Gamma(K_{1,2} \to \gamma\gamma)} = \frac{2\alpha}{\pi}\left(1 - \frac{t}{M_K^2}\right)^3 \left(1 + \frac{2m_l^2}{t}\right)\left(1 - \frac{4m_l^2}{t}\right)^{1/2} \frac{1}{t}, \tag{1}$$

where M_K (m_l) is the K-meson (lepton) mass, and $\alpha = e^2/4\pi \approx \frac{1}{137}$. The branching ratio is

$$R_{1,2}(l) \equiv \frac{\Gamma(K_{1,2} \to \gamma l^+ l^-)}{\Gamma(K_{1,2} \to \gamma\gamma)} = \frac{2\alpha}{3\pi}\left[(1 - \tfrac{9}{8}a^2 + \tfrac{1}{8}a^3)\ln\left(\frac{1 + (1-a)^{1/2}}{1 - (1-a)^{1/2}}\right) + (1-a)^{1/2}(-\tfrac{7}{2} + \tfrac{13}{4}a + \tfrac{1}{4}a^2)\right], \tag{2}$$

where $a = 4m_l^2/M_K^2$. Numerically, $R_{1,2}(\mu) = 4.1 \times 10^{-4}$, and $R_{1,2}(e) = 1.6 \times 10^{-2}$. We shall refer to Eqs. (1) and (2) as the canonical predictions for the decays $K_{1,2} \to \gamma l^+ l^-$.

Deviations from the results (1) and (2) will occur if the form factors $F_{1,2}(t)$ have a significant variation over the allowed range of t, $4m_l^2 < t < M_K^2$. We proceed to discuss these form factors and their effects in the next two sections.

III. STRUCTURE EFFECTS IN $K_1 \to \gamma l^+ l^-$

The invariant amplitude of $K_1 \to \gamma\gamma$ has the general form

$$\mathcal{A}(K_1 \to \gamma\gamma) = F_1(0)(\epsilon \cdot \epsilon' k \cdot k' - \epsilon \cdot k' \epsilon' \cdot k), \tag{3}$$

where (ϵ, k) and (ϵ', k') are the (polarization, momentum) of the two photons. In terms of $F_1(0)$, the decay rate of $K_1 \to \gamma\gamma$ is

$$\Gamma(K_1 \to \gamma\gamma) = M_K{}^3 |F_1(0)|^3/(64\pi). \tag{4}$$

If the photon (ϵ', k') converts internally into a lepton pair (Fig. 1), the amplitude for the Dalitz process is

$$\mathcal{A}(K_1 \to \gamma l^+ l^-) = e\,\frac{F_1(t)}{t}\,\epsilon_\mu[\,\bar{u}(p_-)\gamma_\nu v(p_+)](g_{\mu\nu} k \cdot k' - k'_\mu k_\nu), \tag{5}$$

where p_+ (p_-) is the momentum of l^+ (l^-), $k' = p_+ + p_-$, and $t = k'^2 = (p_+ + p_-)^2$.

To calculate $F_1(t)$, we shall assume that the transition $K_1 \to \gamma\gamma$ proceeds via a $\pi^+\pi^-$ intermediate state, and that we may take a point coupling for the $K_1 \to \pi^+\pi^-$ vertex, and minimal electromagnetic coupling for the pions. An interesting feature of this model is that $F_1(t)$ is complex, since the two-pion intermediate state gives an absorptive part to the $K_1 \to \gamma\gamma$ amplitude.

For the two-photon process, we have[6]

$$\begin{aligned} \mathrm{Im}F_1(0) &= \frac{ge^2}{2\pi}\,\frac{\mu^2}{s^2}\,\ln\frac{1+(1-4\mu^2/s)^{1/2}}{1-(1-4\mu^2/s)^{1/2}}\,, \\ \mathrm{Re}F_1(0) &= \frac{1}{\pi}\,\frac{ge^2}{2\pi}\,\frac{1}{s}\left[\frac{2\mu^2}{s}\left(\tfrac{1}{2}\pi^2 - \tfrac{1}{2}\ln^2\frac{1+(1-4\mu^2/s)^{1/2}}{1-(1-4\mu^2/s)^{1/2}}\right) - 1\right], \end{aligned} \tag{6}$$

where μ is the pion mass, s is the square of the K-meson mass, and g is the $K_1 \to \pi^+\pi^-$ coupling constant, defined by

$$\Gamma(K_1 \to \pi^+\pi^-) = \frac{|g|^2}{16\pi M_K}\left(1 - \frac{4\mu^2}{M_K{}^2}\right)^{1/2}. \tag{7}$$

The decay rate of $K_1 \to \gamma\gamma$ corresponding to (6) is 2.2×10^4 sec^{-1}, which is about twice the decay rate of $K_2 \to \gamma\gamma$.

To determine $F_1(t)$ for nonzero t, we must obtain the real and imaginary parts of the $K_1 \to \gamma\gamma$ vertex when one photon is virtual. We do this using the techniques of dispersion theory.[7] The imaginary part is obtained by setting the internal pions on the mass shell. This is illustrated in Fig. 2. Notice that there are two possible ways of "cutting" the internal lines. The cut 1 gives an absorptive part for all values of t, while the cut 2 gives a nonvanishing absorptive part only for $t > 4\mu^2$. Accordingly the imaginary part of $F_1(t)$ has the general form

$$\mathrm{Im}F_1(s,t) = [A(s,t) - A'(s,t)\theta(t - 4\mu^2)]\,\theta(s - 4\mu^2), \tag{8}$$

where we have inserted an explicit dependence on $s = M_K{}^2$, so as to be able to treat it as a dispersion variable $[F_1(s,t) \equiv F_1(t)]$. The term $A(s,t)$ is calculable in terms of the amplitudes for $K_1 \to \pi^+\pi^-$ and $\pi^+\pi^- \to \gamma\gamma$, while the term $A'(s,t)$ is determined in terms of $K_1 \to \pi^+\pi^-\gamma$ and $\pi^+\pi^- \to \gamma$. Explicitly, we find

$$\begin{aligned} A(s,t) &= \frac{ge^2}{2\pi}\,\frac{1}{(s-t)^2}\left(\mu^2 \ln\frac{1+\beta}{1-\beta} - \tfrac{1}{2}\beta t\right), \\ A'(s,t) &= \frac{ge^2}{2\pi}\,\frac{1}{(s-t)^2}\left(\mu^2 \ln\frac{1+\beta'}{1-\beta'} - \tfrac{1}{2}\beta' t\right), \end{aligned} \tag{9}$$

where

$$\beta = [1 - (4\mu^2/s)]^{1/2}, \qquad \beta' = |1 - (4\mu^2/t)|^{1/2}. \tag{10}$$

Note that while A and A' are singular at $t = s$ (this is the configuration in which the real photon is soft) this "infrared" singularity disappears from $\mathrm{Im}F_1(s,t)$, as it should.

We now obtain the real part $\mathrm{Re}F_1(s,t)$ by writing an unsubtracted dispersion relation in the variable s, holding t fixed:

$$\mathrm{Re}F_1(s=M_K{}^2,t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{ds'}{s'-M_K{}^2}\,\mathrm{Im}F_1(s',t)\,. \tag{11}$$

The integration can be carried out explicitly, and the result is

$$\mathrm{Re}F_1(t)=\frac{1}{\pi}\,\frac{ge^2}{2\pi}\begin{cases}\mu^2 I_1(t)-\frac{1}{2}tI_2(t), & \text{for } t<4\mu^2\,,\\ \mu^2 J_1(t)-\frac{1}{2}tJ_2(t), & \text{for } t>4\mu^2\,,\end{cases} \tag{12}$$

where

$$\begin{aligned}
I_1&=-\frac{2}{(s-t)}\,\frac{1}{\beta' t}\tan^{-1}\frac{1}{\beta'}+\frac{1}{(s-t)^2}\left[\left(\tfrac{1}{2}\pi^2-\tfrac{1}{2}\ln^2\frac{1+\beta}{1-\beta}\right)-2\left(\tan^{-1}\frac{1}{\beta}\right)^2\right],\\
I_2&=\frac{1}{(s-t)}\left(\frac{1}{t}-\frac{4\mu^2}{t^2}\,\frac{1}{\beta'}\tan^{-1}\frac{1}{\beta'}\right)+\frac{1}{(s-t)^2}\left(\beta\ln\frac{1-\beta}{1+\beta}+2\beta'\tan^{-1}\frac{1}{\beta'}\right),\\
J_1&=\frac{1}{(s-t)}\left(\frac{1}{t\beta'}\ln\frac{1+\beta'}{1-\beta'}-\frac{1}{t\beta'^2}\ln\frac{1+\beta'}{1-\beta'}-\frac{1}{t\beta'}\ln\frac{t-4\mu^2}{s-4\mu^2}\right)\\
&\quad+\frac{1}{(s-t)^2}\left(\tfrac{1}{2}\ln^2\frac{1+\beta'}{1-\beta'}-\tfrac{1}{2}\ln^2\frac{1+\beta}{1-\beta}-\ln\frac{1+\beta'}{1-\beta'}\ln\frac{t-4\mu^2}{s-4\mu^2}\right),\\
J_2&=\frac{1}{(s-t)}\left(\frac{1}{t}+\frac{2\mu^2}{t^2\beta'}\ln\frac{1+\beta'}{1-\beta'}-\frac{2\mu^2}{t^2}\,\frac{1}{\beta'}\ln\frac{t-4\mu^2}{s-4\mu^2}-\frac{1}{t\beta'}\right)\\
&\quad+\frac{1}{(s-t)^2}\left(\beta\ln\frac{1-\beta}{1+\beta}-\beta'\ln\frac{1-\beta'}{1+\beta'}-\beta'\ln\frac{t-4\mu^2}{s-4\mu^2}\right).
\end{aligned} \tag{13}$$

The formidable-looking expression for $\mathrm{Re}F_1(t)$, together with the function $\mathrm{Im}F_1(t)$, are plotted in Fig. 3. Observe that while $\mathrm{Im}F_1(t)$ falls monotonically with t, $-\mathrm{Re}F_1(t)$ increases up to $t=4\mu^2$, and falls rapidly thereafter, plunging to zero at $t=M_K{}^2$. Fig. 4 shows a plot of the structure function $|F_1(t)/F_1(0)|$; the important feature is that this function is close to unity for $t<4\mu^2$, but decreases significantly for larger t. The effect on the dilepton spectrum is shown in Fig. 5. Note the considerable suppression of lepton pairs with $t>4\mu^2$. The branching ratios $R_1(\mu)$ and $R_2(e)$ are shown in Table I.

IV. STRUCTURE EFFECTS IN $K_2\to\gamma l^+l^-$

The invariant amplitude for $K_2\to\gamma\gamma$ has the form

$$\mathfrak{a}(K_2\to\gamma\gamma)=F_2(0)\epsilon_{\mu\nu\rho\sigma}\epsilon_\mu\epsilon'_\nu k_\rho k'_\sigma\,, \tag{14}$$

where, as before, (ϵ,k), (ϵ',k') label the two photons. For the Dalitz process in which the

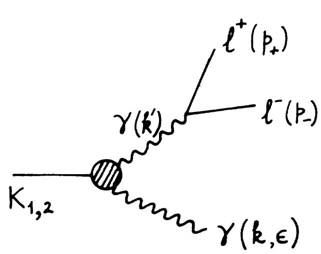

FIG. 1. Kinematic notation for the decays $K_{1,2}\to\gamma l^+l^-$.

photon (ϵ',k') is internally converted, the amplitude is

$$\mathfrak{a}(K_2\to\gamma l^+l^-)=\frac{eF_2(t)}{t}\,\epsilon_\mu(\bar{u}(p_-)\gamma_\nu v(p_+))\epsilon_{\mu\nu\rho\sigma}k_\rho(p_++p_-)_\sigma\,. \tag{15}$$

It is commonly believed[8] that the most important intermediate states in the decay $K_2\to\gamma\gamma$ are the π^0 and η^0 mesons. Accepting that, the structure of the form factor $F_2(t)$ will be determined by the variation of the $\pi^0\to\gamma\gamma$ and $\eta^0\to\gamma\gamma$ vertices when one photon is virtual. (Of course, the π^0 and η^0 are also off the mass shell, having the (mass)^2 $M_K{}^2$, but the dependence on the meson mass cancels when one computes the ratios $[d\Gamma(K_2\to\gamma l^+l^-)/dt]/\Gamma(K_2\to\gamma\gamma)$ or $\Gamma(K_2\to\gamma l^+l^-)/\Gamma(K_2\to\gamma\gamma)$.) An analysis of the reactions $\pi^0\to\gamma\gamma$ and $\eta^0\to\gamma\gamma$ by Pratap and Smith[9] suggests that the likely structure of these amplitudes is a nucleon loop. The form factor resulting from this model is

$$F_2(t)=F_2(0)\,\frac{1-\left(\sin^{-1}\dfrac{\sqrt{t}}{2M}\right)^2\Big/\left(\sin^{-1}\dfrac{M_K}{2M}\right)^2}{1-t/M_K{}^2}\,, \tag{16}$$

where M is the nucleon mass. In the linear approximation, this is

$$F_2(t)\simeq F_2(0)\left(1+\frac{1}{12}\,\frac{t}{M^2}\right). \tag{17}$$

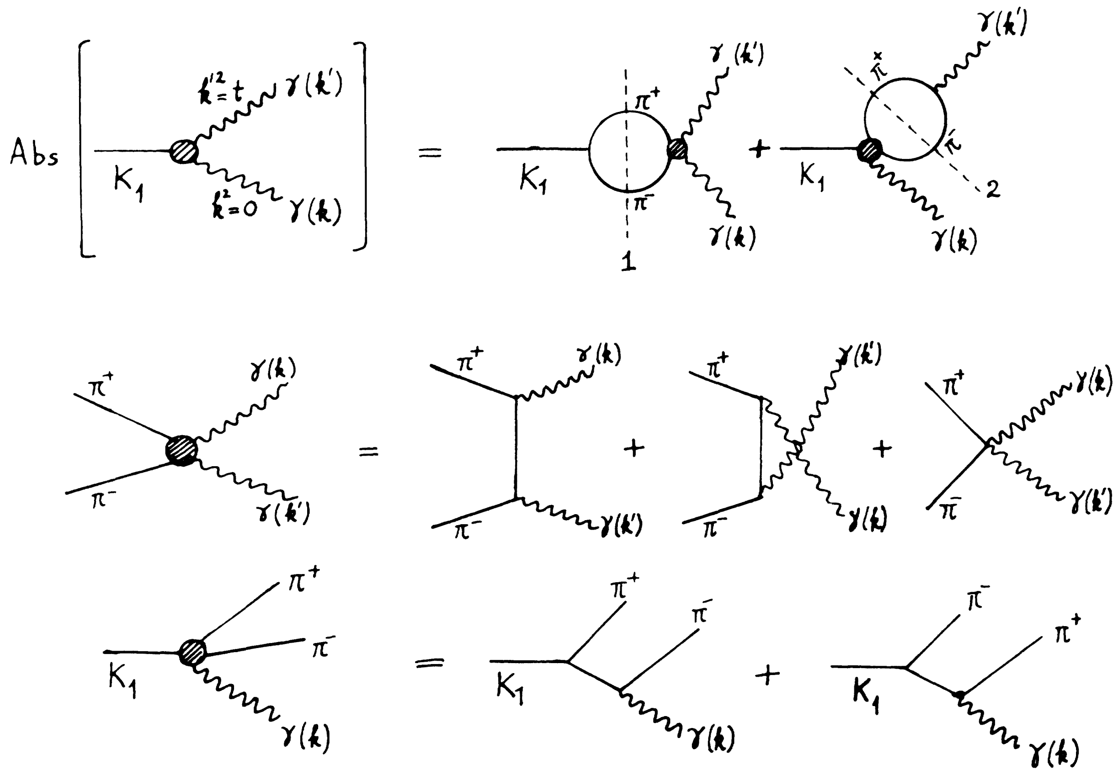

FIG. 2. Model for the absorptive part of the $K_1 \to \gamma\gamma$ vertex with one photon virtual.

Thus the form factor in $K_2 \to \gamma l^+ l^-$ is expected to be a slowly *increasing* function of t. This should be contrasted with the behavior of $F_1(t)$ shown in Fig. 4. The effects of the function $F_2(t)$ on the spectrum of $K_2 \to \gamma l^+ l^-$ and on the ratios $R_2(l)$ are quite negligible.

As an alternative model, we may assume the dominance of $F_2(t)$ by a vector meson of mass M_V (of the order of the ρ mass). In this case,

$$F_2(t) = \left(1 - \frac{t}{M_V{}^2}\right)^{-1}, \tag{18}$$

which is also an increasing function of t. The effect on the spectrum of $K_2 \to \gamma \mu^+ \mu^-$ is shown in Fig. 6 and the ratios $R_2(l)$ in Table I. Note that the Dalitz reaction is uniformly enhanced, in contrast to the structure effect in $K_1 \to \gamma l^+ l^-$.

V. DECAY OF A K_1-K_2 SUPERPOSITION

Since, in practice, a pure K_1 beam is not obtainable, we must consider what would be observed if the state decaying into $\gamma l^+ l^-$ is a superposition of K_1 and K_2. Let us suppose that the decaying state is $|\psi\rangle = (|K_2\rangle + \rho|K_1\rangle)(1 + |\rho|^2)^{-1/2}$. If CP invariance is maintained, there can be no interference between the K_1 and K_2 components as far as the lepton pair distribution is concerned. Accordingly,

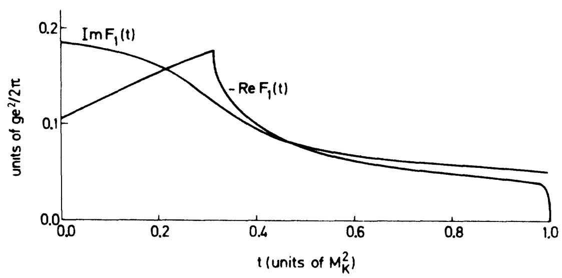

FIG. 3. Real and imaginary parts of $F_1(t)$.

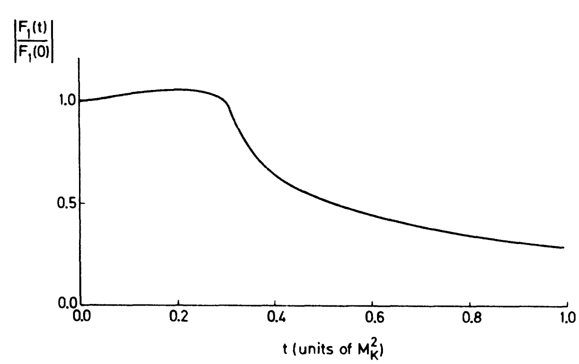

FIG. 4. A plot of the form factor $|F_1(t)/F_1(0)|$.

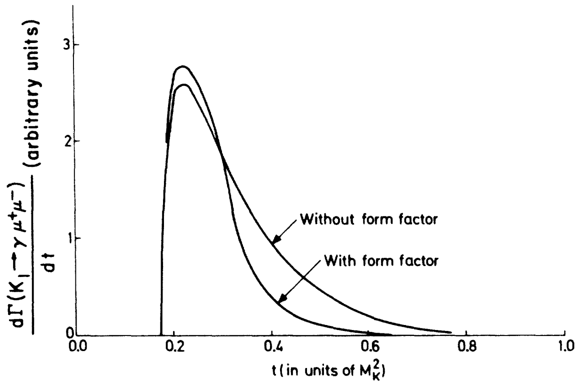

FIG. 5. Lepton-pair spectrum in $K_1 \to \gamma l^+ l^-$.

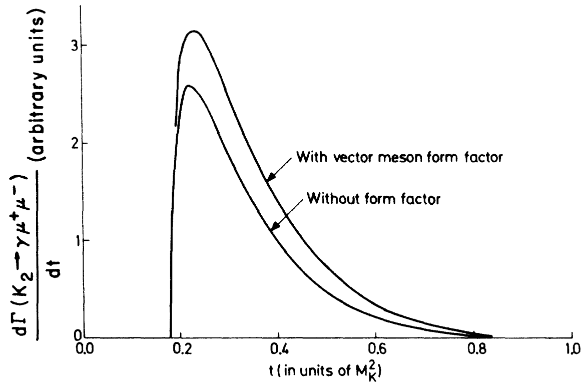

FIG. 6. Lepton-pair spectrum in $K_2 \to \gamma l^+ l^-$.

we should have the spectrum

$$\frac{d\Gamma(\psi)}{dt} = \left(\frac{d\Gamma(K_2)}{dt} + |\rho|^2 \frac{d\Gamma(K_1)}{dt}\right)(1+|\rho|^2)^{-1}. \quad (19)$$

The branching ratio will be

$$R_\rho(l) = \frac{\Gamma(\psi\to\gamma l^+ l^-)}{\Gamma(\psi\to\gamma\gamma)} = \frac{\Gamma(K_2\to\gamma l^+ l^-) + |\rho|^2\Gamma(K_1\to\gamma l^+ l^-)}{\Gamma(K_2\to 2\gamma) + |\rho|^2\Gamma(K_1\to\gamma\gamma)}. \quad (20)$$

An interesting case is $\rho = \pm 1$ (i.e., a pure K^0 or $\bar{K}^0$ state). The branching ratios for this situation are given in Table I.

Note that if structure effects were absent, the ratio R_ρ would be independent of $|\rho|$, the admixture of K_1 present in the decaying state. Thus a variation of R_ρ with ρ would be clear evidence that the structure effects are playing a role in the Dalitz processes.

VI. EFFECTS OF *CP* VIOLATION

Our predictions for the decays $K_{1,2}\to\gamma l^+ l^-$ will be affected if there occurs an appreciable violation of CP invariance in these amplitudes. We will restrict ourselves to the following general remarks.

(i) If CP violation in the decays $K_{1,2}\to\gamma l^+ l^-$ originates only in a violation in the decays $K_{1,2}\to\gamma\gamma$, and if all form factor effects are ignored, the spectrum and branching ratio will be exactly those given by Eqs. (1) and (2), for an arbitrary mixture of K_1 and K_2 and for an arbitrary degree of violation.[10] To the extent that the form factors associated with the CP-violating amplitudes in $K_{1,2}\to\gamma\gamma$ are constant, the general effect will be a *dilution* of the structure effects predicted above; in any event, no major deviation from the canonical predictions can occur as a result of this type of CP violation.

(ii) It is possible to construct models of CP violation which give rise to a CP-violating amplitude for the decays $K_{1,2}\to\gamma l^+ l^-$ which is unrelated to the decays $K_{1,2}\to\gamma\gamma$, the latter being CP-in-

TABLE I. Branching ratios of the Dalitz reactions relative to the two-photon decays. The symbols (I) and (II) refer to the predictions of the nucleon-loop model and vector-meson model, respectively.

Decaying state	$R(\mu)$ Without form factor	$R(\mu)$ With form factor	$R(e)$ Without form factor	$R(e)$ With form factor
K_1	2.3×10^{-4}, $t<4\mu^2$ 1.8×10^{-4}, $t>4\mu^2$ 4.1×10^{-4}, all t	2.5×10^{-4}, $t<4\mu^2$ 0.8×10^{-4}, $t>4\mu^2$ 3.3×10^{-4}, all t	1.59×10^{-2}	1.59×10^{-2}
K_2	4.1×10^{-4}	4.15×10^{-4} (I) 5.6×10^{-4} (II)	1.59×10^{-2}	1.59×10^{-2}(I) 1.65×10^{-2}(II)
$K^0,\bar{K}^0$	4.1×10^{-4}	3.56×10^{-4} (I) 4.0×10^{-4} (II)	1.59×10^{-2}	1.59×10^{-2} (I) 1.62×10^{-2} (II)

variant. A model in which such an amplitude arises for the decay $K_2 \to \gamma l^+ l^-$ has been proposed by Wolfenstein[11] in connection with attempts to resolve the $K_L \to \mu^+ \mu^-$ puzzle, and its implications have been studied by Singh.[12] The general effect of such a violation (ignoring effects of order ϵ) will be to *enhance* the decay rates of $K_{L,S} \to \gamma l^+ l^-$ over those found by us, and, possibly, to distort the dilepton spectrum. Especially for the decays $K_{L,S} \to \gamma e^+ e^-$, where the structure effects in the Dalitz amplitude are quite small, an appreciable enhancement over the canonical predictions could be interpreted as evidence for a CP-violating contribution of this type.

VII. COMMENTS

The essential conclusion of our analysis is that while the form-factor effects in $K_2 \to \gamma l^+ l^-$ produce a minor enhancement of the Dalitz reaction, those in $K_1 \to \gamma l^+ l^-$ tend to suppress the rate of this process, particularly for lepton pairs of large t. The different behavior of the form factor in the two cases can be roughly understood as follows. In the case of the vertex $K_2 \to \gamma\gamma$, with one photon off the mass shell, the intermediate state appearing in the virtual-photon channel has a (mass)^2 that is *large* compared with $M_K{}^2$, the maximum value allowed for t. In the case $K_1 \to \gamma\gamma$ on the other hand, the state appearing in the virtual-γ channel is a 2π state whose (mass)^2 is *small* compared to $M_K{}^2$. Thus the two form factors tend to have slopes of opposite sign.

It is possible that the models used by us are too simple. Particularly for $K_1 \to \gamma l^+ l^-$, some modification is to be expected because of the strong interaction between the pions in the intermediate state, and because of the pion charge form factor. One index of the importance of strong-interaction effects will be the absolute decay rate of $K_1 \to \gamma\gamma$ for which the model makes a prediction of 2.2×10^4 sec^{-1}. For the form factor in $K_2 \to \gamma\gamma$, also, one can think of a more complicated model, but it is difficult to imagine a structure that would change the slowly varying character of $F_2(t)$. Finally, we must allow for the possibility that our ideas about the $K^0 \to \gamma\gamma$ vertices are incomplete, and that, for instance, there might exist a direct coupling of K^0, $\bar{K}^0$ to a photon pair. A measurement of the Dalitz reactions would throw interesting light on these questions.

---

*Work supported by the German Bundesministerium für Bildung und Wissenschaft.

[1]Particle Data Group, Phys. Letters 39B, 1 (1972).

[2]M. Banner *et al.*, Phys. Rev. Letters 29, 237 (1972); private communications from M. Holder and E. Radermacher.

[3]B. R. Martin, E. de Rafael, and J. Smith, Phys. Rev. D 2, 179 (1970); Y. Kohara, Progr. Theoret. Phys. (Kyoto) 48, 261 (1972) and University of Tokyo Report No. UT-172, 1972 (unpublished).

[4]A. R. Clark *et al.*, Phys. Rev. Letters 26, 1667 (1971).

[5]R. H. Dalitz, Proc. Phys. Soc. (London) A64, 667 (1951); N. M. Kroll and W. Wada, Phys. Rev. 98, 1355 (1955); T. Miyazaki, Lett. Nuovo Cimento 5, 125 (1972); S. Okubo and M. Baĉe, Nucl. Phys. B40, 541 (1972).

[6]Martin *et al.*, Ref. 3; K. Nishijima, *Fields and Particles* (Benjamin, New York, 1969), pp. 351–359.

[7]K. Nishijima, Ref. 6, Chap. 8.

[8]M. Moshe and P. Singer, Phys. Rev. Letters 27, 1685 (1971); R. F. Sarraga and H. J. Munczek, Phys. Rev. D 4, 2884 (1971).

[9]M. Pratap and J. Smith, Phys. Rev. D 5, 2020 (1972).

[10]T. Miyazaki, Ref. 5.

[11]L. Wolfenstein, *Particles and Fields –1971*, Proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Melissinos and P. F. Slattery (American Institute of Physics, New York, 1971).

[12]S. K. Singh, Phys. Rev. D 6, 2646 (1972).