$$R_s = m + mp[1+\tfrac{1}{4}(x_s{}^2-1)]^{1/2}\ . \tag{10}$$

Closed timelike curves will occur where the norm of ∂_ϕ changes sign. Using the components of Eq. (1), the norm is given by

$$g_{\phi\phi} = m^2(1-y^2)(4AB)^{-1}[16q^2C^2(1-y^2) - p^2(x^2-1)B^2]\ .$$

There is one root in the equatorial plane, $y=0$, lying in the ergosphere. In the region where the norm became positive, the circles $t=$const, R $=$const, and $\Theta=\frac{1}{2}\pi$ are closed timelike lines.

Finally, we note that the T-S solution is of Petrov type I by the theorem of Lind,[10] which states that any nonradiative asymptotically flat vacuum solution (with singularities bounded away from infinity) is either Kerr-Newman or type I.

ACKNOWLEDGMENTS

The author is grateful to Dr. Yavuz Nutku for many stimulating discussions, and Dr. David Robinson for valuable suggestions which improved the final manuscript.

[1]A. Tomimatsu and H. Sato, Phys. Rev. Letters 29, 1344 (1972).
[2]J. Winicour, A. Janis, and E. T. Newman, Phys. Rev. 176, 1507 (1968).
[3]J. Bardeen and R. Wagoner, Astrophys. J. 167, 359 (1971).
[4]R. Penrose, Proc. Roy. Soc. (London) A284, 159 (1965).
[5]R. Penrose, in *Battelle Rencontres, 1967,* edited by C. M. DeWitt and J. A. Wheeler (Benjamin, New York, 1968).
[6]L. Tamburino and J. Winicour, Phys. Rev. Letters 15, 601 (1965).
[7]L. Tamburino and J. Winicour, Phys. Rev. 150, 1039 (1966).
[8]A. Komar, Phys. Rev. 127, 1411 (1962).
[9]An event horizon is a nonsingular surface defined as the past boundary of future null infinity. It is the boundary of the region from which particles or photons can escape to future infinity. See Ref. 5 for further discussion.
[10]R. Lind, report, 1972 (unpublished).

PHYSICAL REVIEW D VOLUME 7, NUMBER 10 15 MAY 1973

Comment on Weak and Electromagnetic Mass Differences*

T. C. Yang
Department of Physics, University of California, La Jolla, California 92037

(Received 29 November 1972)

We discuss weak and electromagnetic mass differences of pions in the framework of (I) current algebra and (II) a renormalizable Lagrangian model.

I. CURRENT-ALGEBRA FRAMEWORK

If one hypothesizes that weak interactions are mediated by massive intermediate vector bosons, the electromagnetic and weak mass differences can be written as

$$\Delta m_\pi{}^2 = H_A + H_Z + H_W\,, \tag{1}$$

where

$$H_A = e^2\,\frac{2E_\pi}{4\pi}\int d^4k\,\frac{g^{\mu\nu}}{k^2-i\epsilon}\left[\int d^4x\,e^{ikx}\langle\pi^+|T^*(j_\mu^{\rm em}(x)j_\nu^{\rm em}(0))|\pi^+\rangle - (\pi^+\to\pi^0)\right], \tag{2}$$

$$H_W = g_W{}^2\,\frac{2E_\pi}{4\pi}\int d^4k\,\frac{(g^{\mu\nu}-k^\mu k^\nu/m_W{}^2)}{k^2-m_W{}^2-i\epsilon}\left[\int d^4x\,e^{ikx}\langle\pi^+|T^*(j_\mu^+(x)j_\nu^-(0)+j_\mu^-(x)j_\nu^+(0))|\pi^+\rangle - (\pi^+\to\pi^0)\right], \tag{3}$$

$$H_Z = g_Z{}^2\,\frac{2E_\pi}{4\pi}\int d^4k\,\frac{(g^{\mu\nu}-k^\mu k^\nu/m_Z{}^2)}{k^2-m_Z{}^2-i\epsilon}\left[\int d^4x\,e^{ikx}\langle\pi^+|T^*(j_\mu^Z(x)j_\nu^Z(0))|\pi^+\rangle - (\pi^+\to\pi^0)\right], \tag{4}$$

where the following three massive vector bosons are considered[1]: charged vector bosons W of mass m_W

coupled to the charged hadronic current $j_\mu^+ = (V-A)_\mu^+$ with coupling constant g_W, and a neutral vector boson Z of mass m_Z coupled to the neutral hadronic current j^Z (defined below) with coupling constant g_Z. The electromagnetic current is related to the hadronic current via $j^{\rm em} = V^3 + (1/\sqrt{3})V^8$. By assuming the above form, we can discuss the divergence problem if certain properties of the hadronic two-point functions are known.

Electromagnetic mass differences have been widely discussed in the literature.[2,3] The on-mass-shell amplitude has been found to be divergent in model calculations.[3] One hopes that the weak mass-difference divergence will cancel the electromagnetic mass-difference divergence. In forms (1)–(4), one assumes equal bare masses for the pion multiplet.

The hadronic symmetry is usefully discussed in terms of SU(2)⊗SU(2) symmetry-breaking parameters; thus we assume that it makes sense to talk about first-order and higher-order corrections in the symmetry-breaking parameter ϵ $[O(\epsilon) = O(m_\pi^2)]$.

(a) First, in the zeroth-order approximation of SU(2)⊗SU(2) symmetry breaking, we can treat $\partial_\mu j_\mu^\pm = 0$, $\partial_\mu j_\mu^3 = 0$, and $m_\pi^2 = 0$. Assuming also $\partial_\mu j_\mu^{0,8} = 0$, we have the logarithmically divergent part[4]:

$$\begin{aligned}(\Delta m^2)_{\rm div} = \frac{2E_\pi}{4\pi}\,{\rm Re}\int\frac{d^4k}{k^2-i\epsilon}\int d^4x\, e^{ikx} g^{\mu\nu}[\, & e^2\langle\pi^+|T^*(j_\mu^{\rm em}(x)j_\nu^{\rm em}(0))|\pi^+\rangle \\ & + g_W^2\langle\pi^+|T^*(j_\mu^+(x)j_\nu^-(0)+j_\mu^-(x)j_\nu^+(0))|\pi^+\rangle \\ & + g_Z^2\langle\pi^+|T^*(j_\mu^Z(x)j_\nu^Z(0))|\pi^+\rangle - (\pi^+\to\pi^0)] + O(m_\pi^2)\,. \end{aligned} \tag{5}$$

In this approximation, one can justifiably use the soft-pion method.[2] What is calculated is usually referred to as an off-shell amplitude. Since all the currents are conserved, one can replace the covariant T^* product by the T product if one consistently neglects Schwinger terms. One has[2]

$$(\Delta m^2)_{\rm div} \underset{\text{soft pion}}{=} \frac{2E_\pi}{4\pi}\,{\rm Re}\int\frac{d^4k}{k^2-i\epsilon}\,\frac{2}{f_\pi^2}\,g^{\mu\nu}[e^2+g_Z^2(a^2-b^2)][\Delta_{\mu\nu}^A(k)-\Delta_{\mu\nu}^V(k)] + O(m_\pi^2)\,, \tag{6}$$

where

$$\Delta_{\mu\nu}^j(k) = i\int d^4x\, e^{ikx}\langle 0|T(j_\mu^+(x)j_\nu^-(0))|0\rangle\,,$$

and

$$j^Z = aV^3 + bA^3 + \text{hypercharge current } (j^8) + \text{singlet current } (j^0)\,.$$

If an unsubtracted dispersion relation is assumed for the two-point spectral functions, and use is made of the first and second sum rules of Weinberg, it is found[2] that the electromagnetic mass difference is finite by itself. It is evident from (6) that the same conclusion can be drawn for the weak contributions.[5] If the second Weinberg sum rule is not assumed, then one can see that the requirement of no divergence in the mass difference reads

$$e^2 = -g_Z^2(a^2-b^2)\,. \tag{7}$$

This relation is interesting, for it is satisfied in Weinberg's unified model of weak and electromagnetic interactions[6] as summarized in the following semileptonic Lagrangian:

$$\begin{aligned}\mathcal{L}' = & \frac{g}{2\sqrt{2}}(W_\mu^- j_\mu^+ + W_\mu^+ j_\mu^-) \\ & + \tfrac{1}{2}(g^2+g'^2)^{1/2} Z_\mu j_\mu^Z - eA_\mu V_\mu^{\rm em}\,,\end{aligned} \tag{8}$$

where

$$\begin{aligned} j^Z &= (V-A)^3 - \frac{2g'^2}{g^2+g'^2}V^{\rm em} \\ &= \frac{g^2-g'^2}{g^2+g'^2}V^3 - A^3 - \mathcal{G}\,\frac{2g'^2}{g^2+g'^2}\,,\end{aligned}$$

and where $e = gg'(g^2+g'^2)^{-1/2}$, and

$$V_\mu^{\rm em} = i\bar N\gamma_\mu(t^3+\tfrac{1}{2})N\,,$$

$$V_\mu^3 = i\bar N t^3\gamma_\mu N\,,\quad A_\mu^3 = -i\bar N t^3\gamma_5 N\,,$$

$$\mathcal{G}_\mu \equiv \tfrac{1}{2}i\bar N\gamma_\mu N\,.$$

We find that $g_Z = \frac{1}{2}(g^2+g'^2)^{1/2}$, $a = (g^2-g'^2)/(g^2+g'^2)$, and $b=-1$; thus (7) is satisfied. (For the Lagrangian model of the pion, see Sec. II.)

(b) Next, we include first-order terms in ϵ (thus on the mass shell). We work with the following two assumptions (this section has nothing to do with the soft pion; it can equally deal with the proton-neutron mass difference):

(1) The σ term is an isoscalar up to second order of ϵ.

(2) The light-cone region (or deep-inelastic region of current-pion scattering) is relevant for the divergence in the mass difference. In this region, we can use the bilocal algebra of Fritzsch and Gell-Mann.[7]

By assumption (1), one can see that quadratically divergent contribution to the mass difference vanishes: Using a Ward identity,[8] $k^\mu k^\nu$ terms [in (3) and (4)] can be reduced to the sum of the time-ordered product of two divergences of currents, which is of second order in ϵ, plus the σ term, which cancels between π^+ and π^0 states, and plus an equal-time commutator term, $k^\nu\langle\pi|\epsilon_{abc}V_\nu^c|\pi\rangle = k\cdot pF(0)$, which is linearly divergent.

Next, consider the logarithmically divergent part from $g^{\mu\nu}$ terms. Since currents are not conserved in this order approximation, one can write five invariant amplitudes for the two-point function

$$\int d^4x\, e^{ikx}\langle p|T^*(j_\mu^a(x)j_\nu^b(0))|p\rangle = Ap_\mu p_\nu + B(p_\mu k_\nu + p_\nu k_\mu) + Ck_\mu k_\nu + Dg_{\mu\nu} + E\epsilon_{\mu\nu\sigma\rho}k^\sigma p^\rho\,, \tag{9}$$

where j is a vector or axial-vector current. Using the Ward identity

$$k^\nu\int d^4x\, e^{ikx}\langle p|T(j_\mu^a(x)j_\nu^b(0))|p\rangle = i\int d^4x\, e^{ikx}\langle p|T(j_\mu^a(x)\partial_\nu j_\nu^b(0))|p\rangle + i\int d^4x\, e^{ikx}\langle p|[j_0^b(0), j_\mu^a(x)]\delta(x_0)|p\rangle\,,$$

we obtain

$$\begin{aligned} &A\nu + Bk^2 = Q + F\,,\\ &B\nu + Ck^2 + D = L\,, \end{aligned} \tag{10}$$

where $\nu = k\cdot p$ and Q, L, F are invariant amplitudes defined by

$$i\int d^4x\, e^{ikx}\langle p|T^*(j_\mu^a(x)\partial_\nu j_\nu^b(0))|p\rangle = Qp_\mu + Lk_\mu \tag{11}$$

and

$$\begin{aligned} i\int d^4x\, e^{ikx}\langle p|[j_0^b(0), j_\mu^a(x)]\delta(x_0)|p\rangle &= \epsilon_{abc}\langle p|j_\mu^c(0)|p\rangle + \text{S.T.}\\ &\equiv F(0)p_\mu + \text{S.T.}\,, \end{aligned}$$

where S.T. stands for Schwinger terms. Using (10), Eq. (9) can be written

$$\begin{aligned} \int d^4x\, e^{ikx}\langle p|T^*(j_\mu^a(x)j_\nu^b(0))|p\rangle &= \left(p - \frac{k\cdot p}{k^2}k\right)_\mu\left(p - \frac{k\cdot p}{k^2}k\right)_\nu A(\nu, k^2) + \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)D(\nu, k^2)\\ &\quad + \left(\frac{p_\mu k_\nu + p_\nu k_\mu}{k^2} - \frac{k\cdot p}{k^4}k_\mu k_\nu\right)(Q+F) + \frac{k_\mu k_\nu}{k^2}L + E\epsilon_{\mu\nu\sigma\rho}k^\sigma p^\rho, \end{aligned}$$

and thus

$$g^{\mu\nu}\int d^4x\, e^{ikx}\langle p|T^*(j_\mu^a(x)j_\nu^b(0))|p\rangle = A\left(p^2 - \frac{(k\cdot p)^2}{k^2}\right) + 3D + F\frac{k\cdot p}{k^2} + \left(L + Q\frac{k\cdot p}{k^2}\right). \tag{12}$$

The third term on the right-hand side of (12) is linearly divergent and presumably one does not have to worry about it. We can see that the last term cancels between π^+ and π^0 states, and thus does not contribute to the mass difference. To see this, we multiply (11) by k_μ; the left-hand side by the Ward identity reduces to the σ term plus $O(\epsilon^2)$; on the right-hand side we find that $Q(k\cdot p/k^2) + L \sim O(\epsilon^2)$ for the difference between π^+ and π^0 states by assumption (1). Thus one has

$$\begin{aligned} g^{\mu\nu}\int d^4x\, e^{ikx}[\langle\pi^+(p)|T^*(j_\mu^a(x)j_\nu^b(0))|\pi^+(p)\rangle &- (\pi^+\to\pi^0)]\\ &= \left(p^2 - \frac{(k\cdot p)^2}{k^2}\right)T_2^{ab}(k^2,\nu) + 3T_1^{ab}(k^2,\nu) + O(\epsilon^2). \end{aligned}$$

T_2 and T_1 are the differences of A and D between π^+ and π^0 states, thus both satisfy unsubtracted dispersion relations. Consider the contribution of T_2 to the mass difference[9]

$$\begin{aligned} T_2(k^2,\nu) &= \frac{1}{2\pi}\int_0^\infty d\nu'^2\,\frac{W_2(k^2,\nu')}{\nu'^2 - \nu^2}\\ &= \frac{2\omega^2}{\pi k^2}\int_0^1 d\omega'\,\frac{F_2(\omega',k^2)}{\omega'^2 - \omega^2}\,, \end{aligned}$$

where $\omega = -k^2/2\nu \equiv -k^2/2kp$ and $F_2(\omega, k^2) \equiv \nu W_2(k^2,\nu)$; thus

$$\begin{aligned} [\delta m_2]_{\text{div}}^{ab} &\sim m_\pi^2\int_0^\infty\frac{dk_E^2}{k_E^2}\int_0^{k_E}dk_4(k_E^2 - k_4^2)^{3/2}\\ &\qquad\times T_2^{ab}(-k_E^2, im_\pi k_4)\\ &\sim m_\pi^2\int_0^\infty\frac{dk_E^2}{k_E^2}\int_0^\infty d\omega\, F_2^{ab}(\omega, -k_E^2)\\ &\qquad\times I\left(\frac{k_E^2}{4m_\pi^2\omega^2}\right), \end{aligned} \tag{13}$$

where

$$I(x) \equiv x \int_0^1 dz \frac{(1-z)^{3/2}}{x+z^2} \xrightarrow[x\to\infty]{} \text{const}.$$

Now, by assumption (2), we know[7] that F_2 scales in the Bjorken limit:

$$F_2^{ab}(\omega, -k_E^2) \xrightarrow[\substack{k_E^2\to\infty \\ \omega \text{ fixed}}]{} F_2^{ab}(\omega)$$

and

$$F_2^{ab}(\omega) = \tfrac{1}{2}\omega\{if_{abc}S^c_{\pi^+}(\omega) + d_{abc}\mathcal{A}^c_{\pi^+}(\omega)\} - (\pi^+ \to \pi^0), \tag{14}$$

where S and $\mathcal{A}$ are Fourier transforms defined in Ref. 10. From (1), (13), and (14), the logarithmic divergent part becomes[11]

$$[\delta m_2]_{\text{div}} \sim m_\pi^2 \int_0^\infty \frac{dk_E^2}{k_E^2} \int_0^\infty \omega\, d\omega\, \tfrac{2}{3}[e^2 + g_Z^2(ac+bd)] \times (\mathcal{A}^3_{\pi^+} - \mathcal{A}^3_{\pi^0}) + O(\epsilon^2), \tag{15}$$

where a, b, c, d are the constants in the neutral current

$$j^Z = aV^3 + bA^3 + \tfrac{1}{3}(cV^8 + dA^8) + \text{singlet current}.$$

A similar calculation can be done for $(\delta m_1)_{\text{div}}$. Note that $F_2(\omega) = 2\omega F_1(\omega)$ by bilocal algebra. It turns out that δm_1 does not contribute to the logarithmic divergence. Thus, from (15), there will be no logarithmic divergence if

$$e^2 = -g_Z^2(ac+bd). \tag{16}$$

We should note that (16) cannot be checked in Weinberg's semileptonic Lagrangian (8), because (8) is based on the group $SU(2)_L \otimes U(1)$, and hence one does not know the SU(3) classification of the current $\mathcal{J}_\mu$, which can be an arbitrary mixture of V^0 and V^8. If one makes the SU(3) assumption $j^{\text{em}} = V^3 + (1/\sqrt{3})V^8$ and writes

$$j^Z = \frac{g^2 - g'^2}{g^2 + g'^2} V^3 - A^3 - \frac{1}{\sqrt{3}} \frac{2g'^2}{g^2+g'^2} V^8,$$

then

$$e^2 + g_Z^2(ac+bd) = \frac{1}{2\cos^2\theta_W} e^2,$$

where e, g_Z, a, and b were given before, and $c = -2g'^2/(g^2+g'^2)$, $\tan\theta_W = g'/g$. One finds that the total mass difference is still logarithmically divergent, differing in magnitude by a factor $(2\cos^2\theta_W)^{-1}$ from the electromagnetic mass-difference divergence. Such a conclusion[12] is actually a result of the classification of the current $\mathcal{J}_\mu$.

In summary, we have discussed the convergence property of the lowest-order contribution in the weak and electromagnetic couplings to the mass difference. For the hadronic two-point function, we assume that perturbation series in the $SU(2) \otimes SU(2)$ symmetry-breaking parameter ϵ make sense, at least for practical purposes. Two completely independent methods are applied: The light-cone analysis agrees with the soft-pion result[13] that the off-mass-shell electromagnetic mass difference is divergence-free, but the on-mass-shell mass difference is found finite only if (16) is true in the first order of ϵ approximation. Equation (13) is the basis for assuming that the light cone is relevant in the divergence problem of mass differences. This may not be true. Within the framework of soft-pion technique alone, one also notes that a semileptonic Lagrangian like (8) ensures a finite mass difference even if the second Weinberg sum rule is invalid.

II. RENORMALIZABLE LAGRANGIAN MODEL

A Lagrangian model for the pion with spontaneous symmetry breakdown can be constructed: We use the $SU(2)_L \otimes U(1)$ group of Weinberg.[1] After spontaneous symmetry breakdown, the neutral vector bosons are

$$Z_\mu = (g^2 + g'^2)^{-1/2}(gA^3_\mu + g'B_\mu),$$
$$A_\mu = (g^2 + g'^2)^{-1/2}(-g'A^3_\mu + gB_\mu).$$

We assign[14] $\vec{\psi} = \vec{\Sigma} + \vec{\pi}$ to an SU(2) triplet with $Y_L = (T_3 - Q)_L = 0$. $(\Sigma - \pi)^+$, $(\Sigma-\pi)^0$, and $(\Sigma-\pi)^-$ are then three singlets, with $Y_R = -Q_R$ taken to be -1, 0, and $+1$, respectively ($T_{3R} = 0$). The Lagrangian for Σ and π is

$$-\tfrac{1}{2}|\partial_\mu(\vec{\Sigma}+\vec{\pi}) - ig\vec{A}\times(\vec{\Sigma}+\vec{\pi})|^2 - \tfrac{1}{2}|\partial_\mu(\Sigma^+ - \pi^+) - ig'B_\mu(\Sigma^+ - \pi^+)|^2 - \tfrac{1}{2}|\partial_\mu(\Sigma^0 - \pi^0)|^2 - \tfrac{1}{2}|\partial_\mu(\Sigma^- - \pi^-) + ig'B_\mu(\Sigma^- - \pi^-)|^2. \tag{17}$$

The interaction for the neutral vector boson is then

$$\tfrac{1}{2}gA_\mu^3[(\Sigma+\pi)^-\partial_\mu(\Sigma+\pi)^+-(\Sigma+\pi)^+\partial_\mu(\Sigma+\pi)^-]+\tfrac{1}{2}g'B_\mu[(\Sigma-\pi)^-\partial_\mu(\Sigma-\pi)^++(\Sigma-\pi)^+\partial_\mu(\Sigma-\pi)^-]$$
$$=\tfrac{1}{2}(g^2+g'^2)^{1/2}Z_\mu\left(\frac{g^2-g'^2}{g^2+g'^2}J_\mu^V+J_\mu^A\right)-eA_\mu J_\mu^{\rm em}, \tag{18}$$

where

$$J_\mu^V=J_\mu^{\rm em}$$
$$\equiv(\pi^-\partial_\mu\pi^+-\pi^+\partial_\mu\pi^-)+(\pi\to\Sigma),$$
$$J_\mu^A=(\Sigma^-\partial_\mu\pi^+-\Sigma^+\partial_\mu\pi^-)+(\Sigma\rightleftarrows\pi).$$

The semileptonic Lagrangian agrees with (8), but in addition there are seagull terms. By the representation content of the pion, we have equal bare masses for pions, since the scalar meson ϕ of Weinberg does not couple to pions. From the Feynman diagrams and seagull terms, we find that quadratic divergence is not present in the lowest-order g^2, g'^2, gg' calculation; this observation was made in a more general context by Weinberg.[15] Noted for the mechanism of spontaneous symmetry breaking, the above Lagrangian is believed to be renormalizable in all orders of g and g'. However, in such models the hadrons interact in a pointlike fashion. How to incorporate the strong interaction and still be able to renormalize is a difficult problem. The difficulty can be seen in (18), where in order to have a two-body pion semileptonic decay Σ must develop a vacuum expectation value, raising doubts about renormalizability.

One customarily divides the Hamiltonian into four parts:

$$\mathcal{H}=\mathcal{H}_{\rm free}+\mathcal{H}_{\rm st}^{\rm Int}(\epsilon)+\mathcal{H}_w^{\rm Int}(g,g')+\mathcal{H}_{\rm em}^{\rm Int}(g,g'),$$

where $\mathcal{H}_{\rm st}^{\rm Int}$ breaks $SU(2)\otimes SU(2)$ symmetry [e.g., by the $(\frac{1}{2},\frac{1}{2})$ representation], and $\mathcal{H}_w^{\rm Int}, \mathcal{H}_{\rm em}^{\rm Int}$ also break $SU(2)\otimes SU(2)$ symmetry but probably in a different fashion [for example, $\mathcal{H}_w^{\rm Int}, \mathcal{H}_{\rm em}^{\rm Int}$ of (8) are only $SU(2)_L$-invariant, but not $SU(2)_R$-invariant in the usual sense]. Thus perturbation series in ϵ and in g, g' are completely independent series, although both contribute to symmetry breaking. Taking this point of view, the (renormalizable) Lagrangian model provides a perturbation series in g, g', but since the strong interaction is absent, it can be classified as zeroth-order in ϵ, for example $\partial_\mu j_\mu=O(g,g')$. On the other hand, in the current-algebra formalism, one makes some assumption on the nature of weak and electromagnetic interactions. The defect is that the theory is not guaranteed to be renormalizable in orders of g, g', unless relations between coupling constants and masses are imposed. For each order in g, g' to be finite, one will expect the corresponding constraints. The total number of constraints is hopefully finite in view of the Lagrangian model. Thus in form (1) we are calculating only the lowest-order contributions in e^2, $g_W{}^2$, and $g_Z{}^2$. The advantage is that perturbation series in ϵ can be studied in principle. We learned from the light-cone analysis, for example, that the strong interaction imposes in first order of ϵ calculation a relation (16) which does not seem to be required in the zeroth order of ϵ calculation.

In the (renormalizable) Lagrangian model (where $\mathcal{H}_{\rm st}^{\rm Int}=0$) one talks about symmetry group of $\mathcal{H}_w^{\rm Int}, \mathcal{H}_{\rm em}^{\rm Int}$ [for example, $SU(2)_L\otimes U(1)$], while in current-algebra formalism one talks about symmetry of the strong interaction. Can one put those two pictures together? A workable way is to start with the renormalizable Lagrangian, for example (17), where we know the mass difference is finite[16] in all orders of g, g' and to zeroth order in ϵ. Next, we introduce the strong interaction by replacing the pointlike interaction by currents which manifest the strong interaction in form factors. The third step is to ask, in each order of g, g' and ϵ, what the condition is for it to be renormalizable. This program is not guaranteed to succeed, but we learned from the above calculations that if we start with a renormalizable Lagrangian where (16) is generated by way of spontaneous symmetry breaking, then we find that in first-order ϵ and g^2, g'^2, gg' the mass difference is finite if assumptions (1) and (2) are assumed for the strong-interaction part. Unfortunately (16) cannot be checked in Weinberg's model, since the SU(3) property of the current $\mathcal{J}_\mu$ is not known, as we discussed before. It will be interesting to learn whether (16) is satisfied in a more complete (renormalizable) model. The fact that (7) is satisfied in Weinberg's semileptonic Lagrangian (8) is very encouraging.

It is also possible that a renormalizable theory for strong, weak, and electromagnetic interactions should come purely from spontaneous symmetry breaking. One presumably writes down a Lagrangian including strong, weak, and electromagnetic interactions, and then spontaneously breaks the given symmetry. In order for this theory to be renormalizable, all symmetry breaking must be completely due to a spontaneous symmetry-breaking mechanism because any added term will destroy the renormalizability. It might happen then that the perturbation series in g, g' and ϵ are not two independent series, but one series. In that case, the above discussion may not be very useful. In this kind of theory, in order to incorporate strong interaction, the smallest group to start with will be $SU(2)_L \otimes SU(2)_R \otimes U(1) \otimes U(1)$. After symmetry breaking, the $SU(2)_L \otimes U(1)$ group will presumably be broken as, for example, prescribed by Weinberg's model. What about $SU(2)_R \otimes U(1)$? What is the particle representation? These questions will have to be answered.

After the paper was completed, we received a preprint by Dicus and Mathur[17] who have made the same observation on the soft-pion mass difference.

ACKNOWLEDGMENTS

I would like to thank Professor N. Kroll for his interest and for discussions. I thank Dr. S. Y. Lee for many discussions of mass differences in the Weinberg model. I would also like to thank Dr. J. Rawls for reading the manuscript.

---

*Work supported in part by the U. S. Atomic Energy Commission.

[1]We consider three vector bosons as suggested by Weinberg's model [S. Weinberg, Phys. Rev. Letters 19, 1264 (1967)]. More general cases can also be discussed.

[2]T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters 18, 759 (1967).

[3]J. D. Bjorken, Phys. Rev. 148, 1467 (1966); G. C. Wick and B. Zumino, Phys. Letters 25B, 479 (1967); J. Schwinger, Phys. Rev. Letters 19, 1154 (1967); M. B. Halpern and G. Segrè, *ibid.* 19, 611 (1967); I. S. Gerstein, B. W. Lee, H. T. Nieh, and H. J. Schnitzer, *ibid.* 19, 1064 (1967). For more complete references, see A. Zee, Phys. Reports 3C, 127 (1972).

[4]$k_\mu k_\nu$ terms in the mass difference reduce by the Ward identity to

$$k^\mu \langle p|[j_0^b(0), j_\mu^a(x)]\,\delta(x_0)|p\rangle = F(0)k\cdot p + \text{S.T.},$$

which is linearly divergent. Customarily one does not worry about linearly divergent terms for $\int (d^4k/k^4)k\cdot p = 0$.

[5]J. Rawls, Nucl. Phys. B10, 323 (1969).

[6]S. Weinberg, Phys. Rev. D 5, 1412 (1972).

[7]R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970); R. Jackiw and H. Schnitzer, *ibid.* 5, 2008 (1972); H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone,* 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2.

[8]We assume that the T^* product can be replaced by the T product if one consistently forgets about Schwinger terms, as is often done in the literature.

[9]Jackiw, Van Royen, and West, Ref. 7; see Appendix O of the review article by Zee, Ref. 3. Note that T_1 for the difference of π^+ and π^0 states is assumed unsubtracted, which is different from the unsubtracted T_L assumed by Jackiw, Van Royen, and West. The former assumption has the consequence that $G_L(\omega) \equiv G_L^{\pi^+}(\omega) - G_L^{\pi^0}(\omega)$ does not contribute to the mass difference, where $G_L^\pi(\omega)$ is defined by

$$W_L^\pi \sim -\frac{1}{2\omega}\left[F_L^\pi(\omega) + \frac{G_L^\pi(\omega)}{k^2} + \cdots\right].$$

Two alternatives regarding $G_L^\pi(\omega)$ have been considered in the literature: $G_L^\pi(\omega) = 0$, $G_L^\pi(\omega) = 2\omega F_2^\pi(\omega)$. See Jackiw, Van Royen, and West, Ref. 7; H. Pagels, Phys. Rev. D 3 (1971); 4, 1932(E) (1971); T. D. Lee, in *Proceedings of the Amsterdam International Conference on Elementary Particles, 1971,* edited by A. G. Tenner and M. Veltman (North-Holland, Amsterdam, 1972). Either relation in the framework of Jackiw, Van Royen, and West will produce the same conclusion (16). More discussions on $G_L(\omega)$ are given by K. Morita [Lett. Nuovo Cimento 4, 385 (1972)] and R. Jackiw and H. Schnitzer [this issue, Phys. Rev. D 7, 3116 (1973)].

[10]
$$\mathcal{A}_\pi^c(\omega) = \int e^{i\omega(p\cdot z)}\,\mathcal{A}_\pi^c(p\cdot z)\,d(p\cdot z),$$

$$S_\pi^c(\omega) = \int e^{i\omega(p\cdot z)}\,S_\pi^c(p\cdot z)\,d(p\cdot z),$$

where $\mathcal{A}_\pi^c(p\cdot z)$ and $S_\pi^c(p\cdot z)$ are defined by

$$\langle\pi(p)|\mathcal{F}_\rho^c(x,y) - \mathcal{F}_\rho^c(y,x)|\pi(p)\rangle = 2\,\mathcal{A}_\pi^c(p\cdot z)p_\rho + \text{trace terms},$$

$$\langle\pi(p)|\mathcal{F}_\rho^c(x,y) + \mathcal{F}_\rho^c(y,x)|\pi(p)\rangle = 2\,S_\pi^c(p\cdot z)p_\rho + \text{trace terms},$$

where $\mathcal{F}$ is the bilocal current and $z = x - y$.

[11]Note that here we use π^+ and π^0 to denote particle states. The result we obtain is general for any isospin multiplet. For example, one would like to talk about the proton-neutron mass difference, where the $\Delta I = 1$ part is more relevant. For actual pions, one knows that $\Delta I = 1$ part does not contribute (i.e., $\mathcal{A}_\pi^3 \equiv 0$); thus the pion mass difference is divergence-free in this model.

[12]Fayyazuddin and Riazuddin, Phys. Rev. D (to be published).

[13]That is, if T_2 does not contain a $1/m_\pi^2$ factor.

[14]Let ψ, Ψ be triplet representations of $SU(2)_L$ and $SU(2)_R$, respectively; then Σ and π can be assigned $\vec{\Sigma} = \vec{\psi} + \vec{\Psi}$, $\vec{\pi} = \vec{\psi} - \vec{\Psi}$. Note that $\Psi = \Sigma - \pi$ transforms under $SU(2)_L$ as a singlet.

[15]S. Weinberg, Phys. Rev. Letters 29, 388 (1972).

[16]Since there is no relation among the coupling constants and masses of the scalar and vector bosons, cancellation of divergences must occur among contributions from vector bosons alone if the model is renormalizable.

[17]D. A. Dicus and V. S. Mathur, Phys. Rev. D 7, 525 (1973).