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Upper Bounds on the Values of Masses in Unified Gauge Theories*

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Upper bounds are found for the masses of most of the new, unobserved particles in unified gauge theories by requiring that the partial-wave amplitudes satisfy unitarity bounds.

It is well known that in the usual description of weak interactions by an unrenormalizable theory, there exists a unitarity cutoff, Λ_u; otherwise, at least some partial-wave amplitudes would grow indefinitely with energy, contradicting unitarity. The numerical value of the cutoff Λ_u depends on the process one is considering and the particular theory being used. For the intermediate-vector-boson (IVB) theory (with $W^{\pm}$ vector bosons), the process $\nu+\bar{\nu}\to W^+ + W^-$ in lowest-order perturbation theory leads to $\Lambda_u{}^2 = 24\pi/G$; in general, Λ_u is typically of the order of a few thousand GeV. It has been argued by Gell-Mann *et al.*[1] that there must exist another, much lower cutoff in the theory (the effective cutoff, $\Lambda_{\rm eff}$) so that the violations of strong selection rules, weak selection rules, and universality arising from higher-order weak effects stay small. For, if $\Lambda_{\rm eff}$ does not exist, the higher-order weak interactions would produce a correction of order $G\Lambda_u{}^2/16\pi^2 \sim 3/2\pi$. In fact estimates based on the K_L-K_S mass difference, and the upper bound on $K\to\mu\bar{\mu}$ decay do indicate the existence of a substantially lower cutoff than Λ_u.[2]

In this note, we seek an interpretation of these cutoffs in the renormalizable, unified gauge theories of weak and electromagnetic interactions.[3] The smaller bound $\Lambda_{\rm eff}$, which makes perturbation theory a valid procedure, is clearly the one of interest, but we do not know of any method, short of explicitly calculating higher-order terms for many processes, to determine it. We can, however, find unitarity bounds by calculating in lowest order. Later we shall show how the effective bounds can be estimated from the unitarity bounds. Weinberg[4] has shown, by considering explicit examples, how the large energy dependence of the amplitudes cancels between graphs in the $SU_L(2)\times Y_L$ gauge theory, so that there is no need for a unitarity cutoff in the energy. The amplitude then goes as GM^2 where G is the Fermi coupling constant and M is some effective mass in the process. Thus, even though the amplitude is finite, it may violate unitarity unless there are constraints on the masses of the particles. In particular, the form GM^2 suggests an upper bound on the mass M.[5] In the gauge theories, therefore, these bounds on the masses are the analog of the unitarity cutoff.

New, unobserved, particles are required in the construction of unified gauge theories. In some of these theories, theoretical and experimental constraints already exist on the masses of some of these particles; for example, in Weinberg's theory[6] the W and Z bosons possess theoretical lower bounds $m_W > 37.3$ GeV, and $m_Z > 74.6$ GeV, whereas the electron-neutrino scattering data require that m_W be larger than 65 GeV.[7] Thus it is also of interest to check if these and similar constraints in other gauge theories are compatible with the constraints arising from unitarity and especially from the considerations that perturbation theory makes sense.

We now turn our attention to calculating the unitarity bounds, and consider the three popular theories of Weinberg,[6] Lee, Prentki, and Zumino[8] (LPZ), and Georgi and Glashow.[9] We treat only two-particle-to-two-particle reactions and deal only with the *leptonic part* of the theory. The only

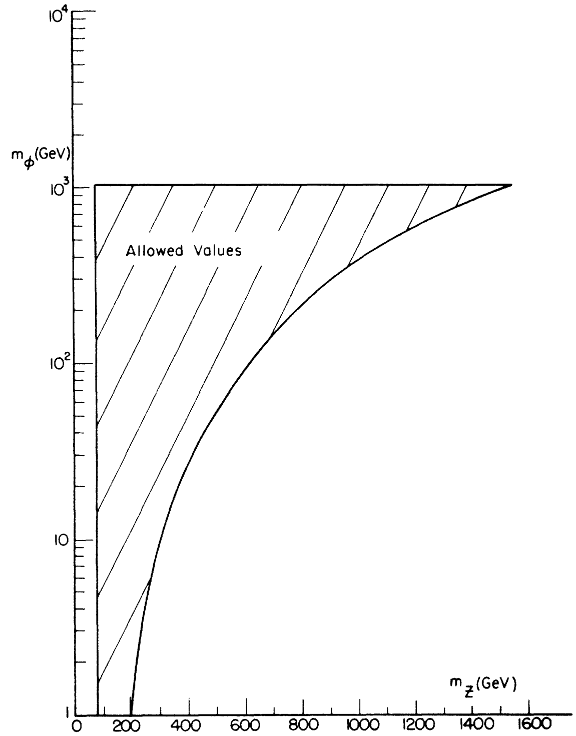

FIG. 1. The allowed values for the masses of m_ϕ and m_Z in Weinberg's theory. The bounds come from the process $ZZ \to ZZ$, with all Z longitudinally polarized.

concession we make to hadrons is to allow two scalars with nonzero vacuum expectation values in the LPZ theory. Lacking a clear-cut criterion for deciding which reaction would yield the best bound, we consider several different reactions in each theory (but clearly not all possible reactions), and, where warranted, consider different helicity amplitudes of a given process. We assume, as usual, that perturbation theory makes sense and impose the unitarity requirement, so that the partial-wave amplitude, in the lowest order of perturbation, is bounded by unity. Note that the lowest-order partial-wave amplitude for some special processes diverges like $\ln(E/M)$.[10] This is presumably not inconsistent with renormalizability, and indeed one encounters such a behavior in quantum electrodynamics. However, for sufficiently large E, it would invalidate the perturbation expansion. We do not make use of the particular processes which lead to such a behavior. The unitarity bounds on masses are then obtained by maximizing the partial-wave amplitude with respect to energy. In many reactions, the amplitudes contain two or more unknown masses. In those cases where two unknown masses appear, the unitarity bound leads to a two-dimensional region of allowed values of masses (see, for example, Fig. 1) from which absolute bounds may be read off.

In the Weinberg theory,[6] we consider the processes $\nu\bar{\nu} \to WW$, $ZZ \to ZZ$, $\nu_e\bar{\nu}_e \to \nu_\mu\bar{\nu}_\mu$, $WZ \to WZ$, and $e^+e^- \to \mu^+\mu^-$. The best bounds come from the process $ZZ \to ZZ$ with all the Z longitudinal. We find

$$m_Z < 1550 \text{ GeV},$$

$$m_\phi < 1020 \text{ GeV}.$$

m_W is also bounded by 1550 GeV since, for mass values this large, it is approximately equal to m_Z. For some values of m_Z there is also a lower bound on m_ϕ; the allowed values of m_Z and m_ϕ are shown in Fig. 1.

The fact that some of the processes, which depend on more than one unknown mass (like, for example, $ZZ \to ZZ$) give absolute bounds is interesting. It could have given, instead of Fig. 1, a curve which ran from small m_ϕ, small m_Z, to large m_ϕ, large m_Z, without ever closing. The process $ZZ \to ZZ$ with some of the helicities transverse does give such a curve.

The process $WZ \to WZ$ is interesting for quite another reason besides providing bounds on the masses. The relevant diagrams are W exchange in the s and t channels, ϕ exchange in the t channel, and a contact term. For all helicities zero the W-exchange terms and the contact term go as E^4 for large E. When these three diagrams are added together, the result goes as E^2 and it is only when the ϕ term is included that the total amplitude does not violate unitarity. This shows, in a simple way, that the ϕ plays an essential role in making the theory finite.

In the LPZ[8] theory the reaction $\nu\bar{\nu} \to WW$ establishes a bound on the mass of the heavy lepton l. The process $e^+e^- \to \mu^+\mu^-$ bounds the mass of the Z, and $ZZ \to ZZ$ gives a bound on m_W:

$$m_l < 1475 \text{ GeV},$$

$$m_W < 1100 \text{ GeV},$$

$$m_Z < 2640 \text{ GeV}.$$

We did not consider any processes that would put bounds on any of the physical scalars in this theory although the ratio of m_{χ^0} to m_Z is related to m_W as shown in Fig. 2. χ^0 is the neutral member of the physical triplet.

In the Georgi-Glashow theory[9] we consider the processes $e^+e^- \to \mu^+\mu^-$, $\nu\bar{\nu} \to WW$, and $l^0\bar{l}^0 \to l^0\bar{l}^0$, where l^0 is the neutral heavy lepton. The neutrino-W process limits the mass of the charged heavy lepton

$$m_{l^+} < 1475 \text{ GeV}.$$

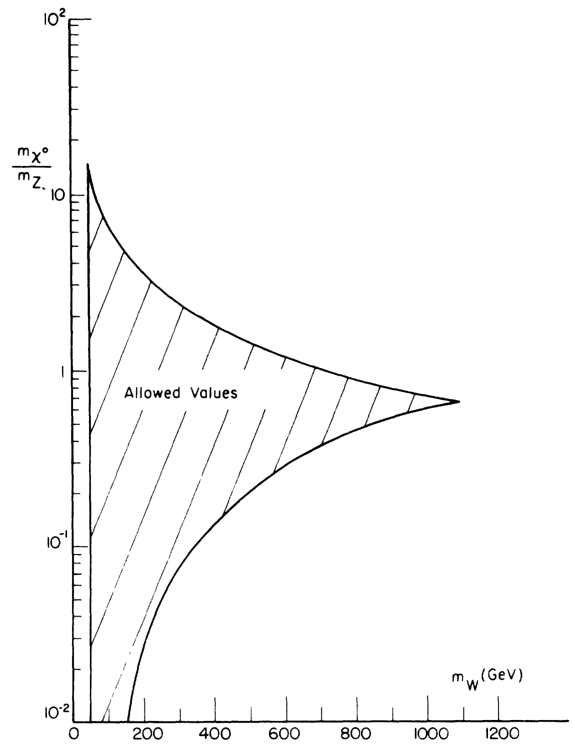

FIG. 2. The allowed values of the W-meson mass and the ratio of the mass of the scalar χ^0 to the mass of the Z in the LPZ theory. The bounds come from the process $ZZ \rightarrow ZZ$, with all Z longitudinal.

The neutral-lepton process gives

$$m_{l^0} < 2500 \left(\frac{m_0}{m_W}\right) \text{GeV},$$

where $m_0 = 53$ GeV is the maximum value of m_W. These bounds hold for both electron-type heavy leptons (usually denoted X^+, X^0) and muon-type (Y^+, Y^0). In the reactions considered here we do not find a bound on the mass of the physical scalar.

By considering a limited number of processes, we have found upper bounds on the masses of all the particles in the three theories, except for some of the scalars. There is, however, no reason to expect that any particle might escape the unitarity bound altogether; presumably, one has simply to consider more reaction processes. It is clear, however, that the bounds are typically around 1500 GeV, and, as discussed before, replace the bound on the energy in the old IVB theory. These bounds are rather generous and not very useful by themselves, from an experimental point of view. However, the fact that bounds of any value exist, especially for the scalars, is interesting, since it shows that these particles cannot be effectively eliminated from the theory by taking their mass to be arbitrarily large.

More interesting perhaps is the impact of these bounds on the validity of the perturbation approximation. It is clear from our comments on the old IVB theory that when the masses of the particles attain their upper bound in the gauge theories, the higher-order weak interactions would, in general, produce a correction of order unity. Thus for precisely the same reasons which led Gell-Mann *et al.*[1] to propose the effective cutoff in the IVB theory, the masses of the various particles in the gauge theories should actually never exceed a fraction of their unitarity bounds. As a rough estimate, if one requires that the correction to the lowest-order perturbation result should be no more than 1%, the effective upper bound on the masses should be an order of magnitude smaller than the unitarity bound,[11] or typically around 150 GeV.[12] Bounds of this order of magnitude may play an important role in the phenomenological search for effects typical to gauge theories.

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[2]B. L. Ioffe and E. P. Shabalin, Yad. Fiz. 6, 828 (1967) [Sov. J. Nucl. Phys. 6, 603 (1968)]; R. N. Mohapatra, J. S. Rao, and R. E. Marshak, Phys. Rev. Letters 20, 1081 (1968).

[3]For a review of all aspects of these theories see B. W. Lee, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972 (unpublished). Also see B. Zumino, in *Cargèse Summer Institute Lectures, 1972*, edited by M. Lévy and J. Sucher (Gordon and Breach, New York, 1972).

[4]S. Weinberg, Phys. Rev. Letters 27, 1688 (1971).

[5]If the process involves photons then there will be terms which go as e^2 as well as terms which go as GM^2. Such terms must, of course, be taken into account, but they do not prevent us from finding bounds on M.

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[7]H. H. Chan and B. W. Lee, Phys. Rev. D 5, 1874 (1972).

[8]B. W. Lee, Phys. Rev. D 6, 1188 (1972); J. Prentki and B. Zumino, Nucl. Phys. B47, 99 (1972).

[9]H. Georgi and S. L. Glashow, Phys. Rev. Letters 28, 1494 (1972).

[10]Examples of processes whose partial-wave amplitudes

have $\ln E$ behavior are $\nu+\nu \rightarrow \nu+\nu$ and, if all particles have zero helicity, $WZ \rightarrow WZ$.

[11]This statement may not apply to the masses of the scalars since they never seem to appear in the coupling constants.

[12]Processes like $K \rightarrow \mu\bar{\mu}$ probably require even further suppression; the effective cutoff estimated in Refs. 1 and 2 was around 5 GeV. In the quark model, for any process involving hadrons, this may come about if the matrix element depends on the quark mass difference.

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Asymptotic Operators for Coulomb Scattering*

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The connection between Zwanziger's prescription for asymptotic fields for charged particles and the Dollard-Kulish-Faddeev formalism is pointed out.

The work of Chung[1] and Kulish and Faddeev[2] has stimulated renewed interest in the problem of infrared and Coulomb phase divergences in quantum electrodynamics.[3,4,5] In particular, Zwanziger[3] has proposed a weak asymptotic limit for a charged field which yields a perturbative Feynman calculus that is free of Coulomb divergences order by order. The purpose of this paper is to point out that Zwanziger's definition of the asymptotic field for the charged particles is precisely the one that would be obtained when adapting the Dollard[6]-Kulish-Faddeev[2] approach to the formal definition of asymptotic field as

$$\psi_{\text{in out}}(\vec{q}) = \Omega_{\mp}\psi(\vec{q})\Omega_{\mp}^{-1},$$

where $\Omega_{\pm}$ are the Møller wave operators.[7] We outline here the procedure for the case of nonrelativistic particles interacting through Coulomb forces. For that system the interaction Hamiltonian is given by

$$H_I(t) = \tfrac{1}{2}e^2\int d^3p\int d^3q\, \frac{d^3k}{(2\pi)^3}\, \frac{1}{\vec{k}^2} \times \exp[i(E(\vec{p}+\vec{k})+E(\vec{q}-\vec{k})-E(\vec{p})-E(\vec{q}))t] \times \psi^*(\vec{p}+\vec{k})\psi^*(\vec{q}-\vec{k})\psi(\vec{q})\psi(\vec{p}), \quad (1)$$

where $E(\vec{p}) = \vec{p}^2/2m$. In the limit $t \rightarrow \pm\infty$, there is a contribution of order $1/t$ to $H_I(t)$ coming from the $\vec{k}\rightarrow 0$ range of $\vec{k}$ values for all values of $\vec{p}$ and $\vec{q}$. This contribution defines the asymptotic Hamiltonian

$$H_I^{\text{as}}(t) = \tfrac{1}{2}e^2\int d^3p\int d^3q\, K\left(|\vec{p}-\vec{q}|\,\frac{t}{m}\right) \times \psi^*(\vec{p})\psi^*(\vec{q})\psi(\vec{q})\psi(\vec{p}), \quad (2)$$

with[8]

$$K\left(|\vec{p}-\vec{q}|\,\frac{t}{m}\right) = \int \frac{d^3k}{(2\pi)^3\vec{k}^2}\exp\left[i\vec{k}\cdot(\vec{p}-\vec{q})\frac{t}{m}\right] \quad (3a)$$

$$= \frac{1}{4\pi}\,\frac{m}{|\vec{p}-\vec{q}|\,|t|}. \quad (3b)$$

The Møller matrix as defined by Dollard[6] is

$$\Omega_{\pm}^{C} = \lim_{t\rightarrow\pm\infty} e^{iHt}U_C^{\text{as}}(t,t_0), \quad (4)$$

where $U_C^{\text{as}}(t,t_0)$ is the propagator for the asymptotic motion

$$U_C^{\text{as}}(t,t_0) = e^{-iH_0t}\exp\left[-i\int_{t_0}^{t}H_I^{\text{as}}(t')dt'\right]. \quad (5)$$

It can be shown – at least formally – that $\Omega_{\pm}^{C}$ are isometric operators which satisfy the intertwining relations

$$e^{iHt}\Omega_{\pm}^{C} = \Omega_{\pm}^{C}e^{-iH_0t}\exp\left[-i\int_{t_0}^{t}H_I^{\text{as}}(t')dt'\right], \quad (6)$$

where H is the full Hamiltonian, $H = H_0 + H_I$.

The definition given in Ref. 7 for the in-field is formally equivalent to defining this asymptotic operator as the solution of the equation

$$i\hbar d_t\psi_{\text{in}}(\vec{q},t) = [H_{0\text{in}} + H_{I\text{in}}^{\text{as}}(t), \psi_{\text{in}}(\vec{q},t)] = E(\vec{q})\psi_{\text{in}}(\vec{q},t) + \frac{e^2}{4\pi}\,\frac{m}{|t|}\int\frac{d^3p}{|\vec{p}-\vec{q}|}\psi_{\text{in}}^*(\vec{p},t)\psi_{\text{in}}(\vec{p},t)\psi_{\text{in}}(\vec{q},t). \quad (7)$$

This equation is, of course, valid only asymptotically for $t \rightarrow -\infty$. The time independence of $\psi_{\text{in}}^*(\vec{p},t)\times\psi_{\text{in}}(\vec{p}\,t)$ – which is readily deduced from the defining equation (7) – allows one to write the solution of (7) as