High-Energy Theorems and Structure of SU(3)⊗SU(3)-Symmetry Breaking

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Consequences of Weinberg's postulative assumption requiring that rapidly growing pole terms which contribute to the forward-scattering amplitude must cancel among themselves and not with the continuum in order to preserve the Regge behavior are examined within the framework of dispersion relations. Twice-subtracted dispersion relations with soft-meson theorems as known subtraction constants are used to determine the transformation properties of the SU(3)⊗SU(3)-symmetry-breaking term in the mass spectrum of hadrons.

I. INTRODUCTION

Over the last few years a large amount of interest has been devoted to the algebraic structure derived from the sum of tree-graph contributions for the forward meson-hadron scattering generated by any chiral-invariant Lagrangians. In particular, Weinberg[1] obtained algebraic relations involving axial-vector-coupling matrices and masses of hadrons. In the case of pion-hadron processes, these relations take on the following elegant form:

$$[X^\alpha, X^\beta] = i\epsilon^{\alpha\beta\gamma} I^\gamma, \tag{1.1}$$

$$[X^\alpha, [m^2, X^\beta]] = \tfrac{1}{3}\delta^{\alpha\beta}[X^\gamma, [m^2, X^\gamma]], \tag{1.2}$$

where $\alpha, \beta = 1, 2, 3$ are pion isovector indices. The meaning of the various symbols in these two equations is the following: I^α is the isospin-generator matrix of the isospin group SU(2) and m^2 is the diagonal mass-squared matrix. X^α denotes the matrix in the space of internal quantum numbers of hadrons (like the spin, isospin, parity, etc.) while its dependence on external quantum numbers such as the helicity λ and momentum p has been already singled out. The matrix element $(X^\alpha)_{ba}$ is related to the invariant Feynman amplitude $M^\alpha_{ba}(p', q, p)$ describing any collinear (helicity-conserving) transition process $a(p,\lambda) \to \pi^\alpha(q) + b(p', \lambda')$ of massless pions by

$$M^\alpha_{ba}(p', q, p) = 2if_\pi^{-1}(m_a{}^2 - m_b{}^2)(X^\alpha)_{ba}\delta_{\lambda\lambda'}, \tag{1.3}$$

where $a(p,\lambda)$ and $b(p',\lambda')$ denote hadrons with the masses m_a and m_b, momenta p and p', and helicities λ and λ', respectively. f_π is the pion decay amplitude, approximately equal to 95 MeV.

The algebraic relation (1.1) completed with the commutators involving the isospin generator matrices I^α forms the Lie algebra of the SU(2)⊗SU(2) group. This implies that the hadron states which Weinberg included in his tree graphs must for each helicity be assigned to an irreducible or reducible representation of the chiral SU(2)⊗SU(2) group. The second algebraic structure (1.2) then specifies that the squared-mass matrix m^2 behaves as the sum of a chiral scalar and a component of a chiral four-vector with respect to commutation relations with I^α and X^α. One easily sees that the method demonstrated by Weinberg provides a simple and attractive scheme for calculating decay rates and mass spectra of hadrons and that is why this program became such a popular starting point for several works.[2]

The essential assumptions leading to the derivation of the algebraic strucutres (1.1) and (1.2) are:

(i) Even if individual tree graphs do have a high-energy behavior worse than that expected for the actual scattering amplitude, their sum grows no faster at high energy than the asymptotic behavior prescribed by Regge-pole theory. This means that rapidly growing pole terms contributed by tree graphs must cancel among themselves and not with the continuum in order to preserve the proper Regge behavior at high energy.

(ii) There should be no exotic states. This implies that intercepts of the Regge trajectories having exotic quantum numbers must be less than zero.

The truthfulness of the last assumption may be checked directly with existing experimental data. However, the first assumption, which is a most crucial one, must be taken as a postulate. It may be justified *a posteriori* by reasonable results following from it. So far these two assumptions have been systematically applied only to tree-graph contributions for the forward scattering amplitude generated by the chiral-invariant Lagrangians. Their implications on the continuum part of the scattering amplitude were always left out of consideration.

The present work has the purpose to show that Weinberg's algebraic structures (1.1) and (1.2) can be, in fact, rederived merely by assuming the hypothesis of partial conservation of axial-vector current (PCAC).

In order to carry out this purpose, use is made of current-algebra hypotheses and dispersion relations for the forward-scattering amplitude. We apply Weinberg's assumptions for twice-subtracted dispersion relations, where soft-meson theorems play the role of known subtraction constants.

Using the Lehmann-Symanzik-Zimmermann (LSZ) reduction technique and current-algebra hypotheses we present an expression for the scattering amplitude of massless mesons with hadrons in Sec. II. After the kinematical preliminaries of Sec. III and a brief exposition in Sec. IV in which we derive twice-subtracted dispersion relations for the forward scattering amplitude, we present in Sec. V the details of the applications of Weinberg's assumptions which yield the algebra of the axial-vector coupling matrices X^α and m^2. Section VI is devoted to a discussion of how the SU(3) $\otimes$SU(3) symmetry is broken.

II. CURRENT-ALGEBRA HYPOTHESES AND THE SCATTERING AMPLITUDE

Consider a completely general meson-hadron transition process,

$$\pi^\alpha(q)+a(p)\rightarrow\pi^\beta(q')+b(p'), \tag{2.1}$$

where $a(p)$ and $b(p')$ denote hadrons with the respective four-momenta p and p', $\pi^\alpha(q)$ and $\pi^\beta(q')$ are mesons of momenta q and q', respectively, and α and β are meson SU(3) indices running over 1, 2, 3, ..., 8. This process is described by the Feynman invariant amplitude $M_{ba}^{\beta\alpha}(p',q',p,q)$ defined by

$$^{(\text{out})}\langle\beta,q',b,p'\,|\,\alpha,q,a,p\rangle^{(\text{in})}=1+i(2\pi)^4\delta(p+q-p'-q')M_{ba}^{\beta\alpha}(p',q',p,q)\,. \tag{2.2}$$

We shall calculate $M_{ba}^{\beta\alpha}(p',q',p,q)$ by adhering throughout to the approximation of neglecting meson masses, so that

$$q^2=q'^2=0\,. \tag{2.3}$$

Our basic dynamical assumption is the PCAC hypothesis, i.e.,

$$\partial^\mu A_\mu^\alpha(x)=f_\alpha m_\alpha{}^2\varphi^\alpha(x), \tag{2.4}$$

where $A^\alpha(x)$ is the phenomenological axial-vector current, $\varphi^\alpha(x)$ is the interpolating meson field, m_α is the appropriate meson mass, and f_α denotes the decay amplitude of the α meson. In our normalization of (2.4)

$$f_\pi=0.69m_\pi \tag{2.5}$$

and

$$f_K=0.87m_\pi\,, \tag{2.6}$$

which are the values corresponding to a Cabibbo angle with $\tan\theta_c=0.21$. The covariant normalization of states,

$$\langle a,p\,|\,b,p'\rangle=(2\pi)^3 2p_0\delta_{ab}\delta^3(\vec{p}-\vec{p}\,'), \tag{2.7}$$

is used for both baryons and mesons.

Our first step is in deriving the expression for the scattering amplitude by employing the LSZ reduction formula along with (2.3) and (2.4). This yields the S matrix in the form

$$\langle\beta,q',b,p'\,|\,S-1\,|\,\alpha,q,a,p\rangle=-(f_\beta f_\alpha)^{-1}\int d^4x\,e^{iq'x}\int d^4y\,e^{-iqy}\theta(x_0-y_0)\left\langle b,p'\left|\left[\frac{\partial}{\partial x_\mu}A_\mu^\beta(x),\frac{\partial}{\partial y_\nu}A_\nu^\alpha(y)\right]\right|a,p\right\rangle\,. \tag{2.8}$$

Next we express the retarded commutator entering (2.8) by the well-known identity

$$\begin{aligned}\theta(x_0-y_0)\left[\frac{\partial}{\partial x_\mu}A_\mu^\beta(x),\frac{\partial}{\partial y_\nu}A_\nu^\alpha(y)\right]\equiv{}&\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}\{\theta(x_0-y_0)[A_\mu^\beta(x),A_\nu^\alpha(y)]\}\\&+\frac{1}{2}\frac{\partial}{\partial x_\mu}\{\delta(x_0-y_0)[A_\mu^\beta(x),A_0^\alpha(y)]\}-\frac{1}{2}\frac{\partial}{\partial y_\nu}\{\delta(x_0-y_0)[A_0^\beta(x),A_\nu^\alpha(y)]\}\\&-\frac{1}{2}\delta(x_0-y_0)\left\{\left[A_0^\beta(x),\frac{\partial}{\partial y_\nu}A_\nu^\alpha(y)\right]+\left[A_0^\alpha(y),\frac{\partial}{\partial x_\mu}A_\mu^\beta(x)\right]\right\}.\end{aligned} \tag{2.9}$$

We employ in (2.9) the assumptions of Gell-Mann[3] about the equal-time commutators between axial-vector currents and axial-vector divergences, namely,

$$\delta(x_0-y_0)[A_\mu^\beta(x),A_0^\alpha(y)]=if^{\beta\alpha\gamma}V_\mu^\gamma(x)\delta^4(x-y), \tag{2.10}$$

$$\delta(x_0-y_0)\left[A_0^\beta(x),\frac{\partial}{\partial y_\nu}A_\nu^\alpha(y)\right]=-i\Sigma^{\beta\alpha}(y)\delta^4(x-y)\,, \tag{2.11}$$

where $f^{\beta\alpha\gamma}$ are the SU(3) structure constants, $V_\mu^\gamma(x)$ is a conserved phenomenological vector current, and $\Sigma^{\beta\alpha}(x)$ is a symmetric SU(3) tensor. Since one does not want to admit operators of isospin or hypercharge 2 into the commutator (2.11), $\Sigma^{\beta\alpha}(x)$ must not contain any part transforming like the $\underline{27}$ representation of the SU(3) group. Making the substitutions (2.10) and (2.11) in (2.9) and then (2.9) in (2.$\overline{8}$) one gets the invariant Feynman amplitude in the form

$$M_{ba}^{\beta\alpha}(p',q',p,q)=i(f_\beta f_\alpha)^{-1}\{i\langle b,p'|\Sigma^{\beta\alpha}(0)|a,p\rangle+\tfrac{1}{2}(q^\mu+q'^\mu)f^{\beta\alpha\gamma}\langle b,p'|V_\mu^\gamma(0)|a,p\rangle$$
$$+q^\nu q'^\mu\int d^4z\,e^{-iqz}\theta(-z_0)\langle b,p'|[A_\mu^\beta(0),A_\nu^\alpha(z)]|a,p\rangle\} \tag{2.12}$$

by integrating the divergences by parts and using the translation invariance of matrix elements. Introducing a complete set of states $|n,p_n\rangle$ between $A_\mu^\beta(0)$ and $A_\nu^\alpha(z)$, integrating over z, and making judiciously covariant groups of noncovariant terms, Eq. (2.12) becomes

$$M_{ba}^{\beta\alpha}(p',q',p,q)=(f_\beta f_\alpha)^{-1}\Bigg\{-\langle b,p'|\Sigma^{\beta\alpha}(0)|a,p\rangle+\tfrac{1}{2}i(q^\mu+q'^\mu)f^{\beta\alpha\gamma}\langle b,p'|V_\mu^\gamma(0)|a,p\rangle$$
$$-q^\nu q'^\mu\sum_n\left[\frac{\langle b,p'|A_\mu^\beta(0)|n,p_n\rangle\langle n,p_n|A_\nu^\alpha(0)|a,p\rangle}{p_n^2-s}\right.$$
$$\left.+\frac{\langle b,p'|A_\nu^\alpha(0)|n,p_n\rangle\langle n,p_n|A_\mu^\beta(0)|a,p\rangle}{p_n^2-u}\right]\Bigg\}\,, \tag{2.13}$$

where $s=(p+q)^2$ and $u=(p'-q)^2$ are Mandelstam variables. A general sum over n represents the summation over pole terms and integration over the continuum. The last relation will be applied for the forward scattering of massless mesons by hadrons in two intersecting storage rings.

III. TWO INTERSECTING BEAMS: KINEMATICS AND PRELIMINARIES

Consider the forward-scattering process of the type (2.1) in two intersecting storage rings. By "forward" is meant only that

$$t=(q-q')^2$$
$$=0\,. \tag{3.1}$$

In such a process all momenta p, q, p', and q' are collinear and can be defined as in Ref. 1,

$$q^\mu=\omega n^\mu\,,\quad q'^\mu=\omega' n^\mu\,, \tag{3.2a}$$

$$\vec{p}=-\vec{n}|\vec{p}|\,,\quad \vec{p}'=-\vec{n}|\vec{p}'|\,, \tag{3.2b}$$

$$p_0=(\vec{p}^{\,2}+m_a^{\,2})^{1/2}\,,\quad p_0'=(\vec{p}'^{2}+m_b^{\,2})^{1/2}\,, \tag{3.2c}$$

where

$$n_0=|\vec{n}|=1\,. \tag{3.3}$$

The conservation of total four-momentum yields

$$n^\mu p_\mu=p_0+|\vec{p}|$$
$$=n^\mu p'_\mu$$
$$=p'_0+|\vec{p}'|$$
$$\equiv E\,, \tag{3.4}$$

$$\omega'=\omega+(m_a^{\,2}-m_b^{\,2})/2E\,, \tag{3.5}$$

and the angular momentum conservation implies that hadron helicities λ_a and λ_b are the same. The invariant Mandelstam variables s and u become

$$s=m_a^{\,2}+2E\omega$$
$$=m_b^{\,2}+2E\omega'\,, \tag{3.6a}$$

$$u=m_a^{\,2}-2E\omega'$$
$$=m_b^{\,2}-2E\omega\,. \tag{3.6b}$$

We will study the properties of the forward-scattering amplitude $M_{ba}^{\beta\alpha}$ deduced from (2.13) as a function of the initial meson energy ω, with the quantity E defined by (3.4) held fixed. Note that for the forward scattering, where there is no spin flip of hadrons, the term containing the matrix element of the vector current $V_\mu^\gamma(0)$ from (2.13) is given by

$$(q'^\mu+q^\mu)\langle b,p'|V_\mu^\gamma(0)|a,p\rangle=2E(\omega'+\omega)(F^\gamma)_{ba}\,, \tag{3.7}$$

where F^γ is the SU(3) generator matrix. Next we introduce the following abbreviations for matrix elements of the operators $A_\mu^\alpha(0)$ and $\Sigma^{\beta\alpha}(0)$ taken between *single*-particle states at $t=0$,

$$n^\mu\langle b, p'|A_\mu^\alpha(0)|a, p\rangle = 2iE(X^\alpha)_{ba}, \tag{3.8}$$

$$\langle b, p'|\Sigma^{\beta\alpha}(0)|a, p\rangle = (\Sigma^{\beta\alpha})_{ba}, \tag{3.9}$$

where X^α and $\Sigma^{\beta\alpha}$ are independent on E or ω. Separating the pole terms and continuum contributions from (2.13) we get

$$M_{ba}^{\beta\alpha}(\omega) = (f_\beta f_\alpha)^{-1}\Big\{ if^{\beta\alpha\gamma}(F^\gamma)_{ba}E(\omega+\omega') - (\Sigma^{\beta\alpha})_{ba} + 4E^2\omega\omega'\sum_n\Big[\frac{(X^\beta)_{bn}(X^\alpha)_{na}}{m_n^2-m_a^2-2E\omega} + \frac{(X^\alpha)_{bn}(X^\beta)_{na}}{m_n^2-m_b^2+2E\omega}\Big]$$
$$- \omega\omega'\int\Big[\frac{D_{ba}^{\beta\alpha}(v)}{v-m_a^2-2E\omega} + \frac{D_{ba}^{\alpha\beta}(v)}{v-m_b^2+2E\omega}\Big]d\mu(v)\Big\}, \tag{3.10}$$

where

$$D_{ba}^{\beta\alpha}(v) = n^\mu n^\nu\langle b, p'|A_\mu^\beta(0)|n, p_n\rangle\langle n, p_n|A_\nu^\alpha(0)|a, p\rangle,$$
$$v = p_n^2, \tag{3.11}$$

and $\mu(v)$ denotes an integration measure over multiparticle states. The dependence of $M_{ba}^{\beta\alpha}(\omega)$ on the fixed quantities E and λ will be suppressed throughout. It is obvious from (3.10) that the forward-scattering amplitude $M_{ba}^{\beta\alpha}(\omega)$ is indeed an analytic function of ω and one is allowed to write down a dispersion relation for it. It obeys the crossing symmetry between s and u channels,

$$M_{ba}^{\beta\alpha}(\omega) = M_{ba}^{\alpha\beta}(-\omega'). \tag{3.12}$$

The structure of (3.10) reminds a dispersion relation with two subtractions which fix the values of $M_{ba}^{\beta\alpha}(\omega)$ at $\omega=0$ and $\omega'=0$. Relations determining these two values are referred to as soft-meson theorems or low-energy theorems. These theorems will be used as the known subtraction constants for writing down the dispersion relation for the forward-scattering amplitude. One can ask if only two subtractions are sufficient for writing down a fully meaningful dispersion relation in question. The answer is, of course, affirmative due to the Froissart bound[4] and the work of Eden[5] who has shown from axiomatic field theory that a scattering amplitude for fixed momentum transfer t grows no faster than $s^{2-\epsilon}$, where ϵ is positive.

Instead of writing down the twice-subtracted dispersion relation for $M_{ba}^{\beta\alpha}(\omega)$ it is more convenient to divide M into antisymmetric and symmetric parts in the meson indices α and β

$$M_{ba}^{\beta\alpha(-)}(\omega) = E^{-1}(\omega+\omega')^{-1}[M_{ba}^{\beta\alpha}(\omega) - M_{ba}^{\alpha\beta}(\omega)], \tag{3.13}$$

$$M_{ba}^{\beta\alpha(+)}(\omega) = \tfrac{1}{2}[M_{ba}^{\beta\alpha}(\omega) + M_{ba}^{\alpha\beta}(\omega)], \tag{3.14}$$

which obey the once-subtracted dispersion relations. Note that both parts $M^{(-)}$ and $M^{(+)}$ are due to crossing symmetry (3.12) symmetric under interchange of ω with $-\omega'$,

$$M_{ba}^{\beta\alpha(\pm)}(\omega) = M_{ba}^{\beta\alpha(\pm)}(-\omega'), \tag{3.15}$$

and give the total scattering amplitude in the form:

$$M_{ba}^{\beta\alpha}(\omega) = M_{ba}^{\beta\alpha(+)}(\omega) + \tfrac{1}{2}E(\omega+\omega')M_{ba}^{\beta\alpha(-)}(\omega). \tag{3.16}$$

IV. SOFT-MESON THEOREMS AND SUBTRACTED DISPERSION RELATIONS

Our first step in this section is to write down the general soft-meson theorems, i.e., the values of $M^{(-)}$ and $M^{(+)}$ at $\omega=0$ (or $\omega'=0$). Assume without loss of generality that $m_a \geq m_b$. To compute $M_{ba}^{\beta\alpha(-)}(0)$ we antisymmetrize $M_{ba}^{\beta\alpha}(\omega)$ from (3.10) in α and β, divide by the factor $E(\omega+\omega')$, take the limit $\omega\to 0$, and get the following result:

$$M_{ba}^{\beta\alpha(-)}(0) = 2(f_\beta f_\alpha)^{-1}\Big[if^{\beta\alpha\gamma}(F^\gamma)_{ba} + \sum_n{}^{(b)}(X^\alpha)_{bn}(X^\beta)_{na} - \sum_n{}^{(a)}(X^\beta)_{bn}(X^\alpha)_{na}\Big], \tag{4.1}$$

where the sums with the superscripts (b) and (a) run over *single*-particle states with $m_n=m_b$ and $m_n=m_a$, respectively. Similarly, $M_{ba}^{\beta\alpha(+)}(0)$ is calculated by symmetrizing $M_{ba}^{\beta\alpha}(\omega)$ in α and β, dividing by two, and taking the limit $\omega\to 0$ which yields the result:

$$M_{ba}^{\beta\alpha(+)}(0) = -\tfrac{1}{2}(f_\beta f_\alpha)^{-1}\Big[\sum_n{}^{(a)}(2m_n^2 - m_a^2 - m_b^2)(X^\beta)_{bn}(X^\alpha)_{na} + \sum_n{}^{(b)}(2m_n^2 - m_a^2 - m_b^2)(X^\alpha)_{bn}(X^\beta)_{na} + 2(\Sigma^{\beta\alpha})_{ba}\Big]\,. \tag{4.2}$$

It should be noted that if $m_a < m_b$ then the formulas (4.1) and (4.2) hold for $\omega' = 0$. The equations (4.1) and (4.2) represent the soft-meson theorems.

In order to proceed further we emphasize that $M_{ba}^{\beta\alpha}(\omega)$ from (3.10) is an analytic function in the complex ω plane (except for poles and cuts). It obeys the dispersion relation

$$M_{ba}^{\beta\alpha}(\omega) = (f_\beta f_\alpha)^{-1}\Big[\sum_n{}_{(a)} \frac{(X^\beta)_{bn}(X^\alpha)_{na}(m_n^2 - m_b^2)(m_n^2 - m_a^2)}{m_n^2 - m_a^2 - 2E\omega} + \sum_n{}_{(b)} \frac{(X^\alpha)_{bn}(X^\beta)_{na}(m_n^2 - m_b^2)(m_n^2 - m_a^2)}{m_n^2 - m_a^2 + 2E\omega'}\Big]$$
$$+\frac{1}{\pi}\int \frac{A_{ba}^{\beta\alpha}(v)dv}{v - m_a^2 - 2E\omega} + \frac{1}{\pi}\int \frac{A_{ba}^{\alpha\beta}(v)dv}{v - m_a^2 + 2E\omega'}, \tag{4.3}$$

where $A_{ba}^{\beta\alpha}(v)$ is the imaginary part of $M_{ba}^{\beta\alpha}(\omega)$ evaluated at $s = v$, i.e., at $\omega = (v - m_a^2)/2E$. Here the sums with subscripts (a) and (b) run over single-particle states with $m_n \neq m_a$ and $m_n \neq m_b$, respectively, for if $m_n = m_a$ and $m_n = m_b$, respectively, the expression (3.10) does not produce poles. It should be pointed out that (4.3) without subtractions may be a meaningless formal expression exhibiting only the analytic structure of the forward-scattering amplitude $M_{ba}^{\beta\alpha}(\omega)$. The same formal dispersion relations can be easily obtained for $M^{(-)}$ and $M^{(+)}$ by inserting (4.3) in (3.13) and (3.14), respectively. To save space we do not write these rather lengthy relations explicitly.

Next we pass to the soft-meson theorems (4.1) and (4.2) which fix the values of $M^{(-)}$ and $M^{(+)}$ at $\omega = 0$ and use them as the known subtraction constants in the once-subtracted dispersion relations for $M^{(-)}$ and $M^{(+)}$, respectively. After some simple algebra we get

$$M_{ba}^{\beta\alpha(-)}(\omega) = 2(f_\beta f_\alpha)^{-1}\{if^{\beta\alpha\gamma}(F^\gamma)_{ba} - [X^\beta, X^\alpha]_{ba}\}$$
$$+2(f_\beta f_\alpha)^{-1}\sum_n \frac{(m_n^2 - m_a^2)(m_n^2 - m_b^2)}{(m_n^2 - m_a^2 - 2E\omega)(m_n^2 - m_b^2 + 2E\omega)}[(X^\beta)_{bn}(X^\alpha)_{na} - (X^\alpha)_{bn}(X^\beta)_{na}]$$
$$+\frac{2E\omega}{\pi}\int dv\Big[\frac{1}{(v - m_a^2)(v - m_a^2 - 2E\omega)} - \frac{1}{(v - m_b^2)(v - m_b^2 + 2E\omega)}\Big][A_{ba}^{\beta\alpha}(v) - A_{ba}^{\alpha\beta}(v)]\frac{2}{2v - m_a^2 - m_b^2}, \tag{4.4}$$

and

$$M_{ba}^{\beta\alpha(+)}(\omega) = \tfrac{1}{2}(f_\beta f_\alpha)^{-1}\{[X^\beta, [X^\alpha, m^2]]_{ba} + [X^\alpha, [X^\beta, m^2]]_{ba} - 2(\Sigma^{\beta\alpha})_{ba}\}$$
$$+\tfrac{1}{2}(f_\beta f_\alpha)^{-1}\sum_n \frac{(2m_n^2 - m_a^2 - m_b^2)(m_n^2 - m_a^2)(m_n^2 - m_b^2)}{(m_n^2 - m_a^2 - 2E\omega)(m_n^2 - m_b^2 + 2E\omega)}[(X^\beta)_{bn}(X^\alpha)_{na} + (X^\alpha)_{bn}(X^\beta)_{na}]$$
$$+\frac{E\omega}{\pi}\int dv\Big[\frac{1}{(v - m_a^2)(v - m_a^2 - 2E\omega)} - \frac{1}{(v - m_b^2)(v - m_b^2 + 2E\omega)}\Big][A_{ba}^{\beta\alpha}(v) + A_{ba}^{\alpha\beta}(v)]. \tag{4.5}$$

Perhaps it should be noted that the last factor in (4.4) is due to $E(\omega + \omega')$ entering the definition of $M^{(-)}$. Here we have adopted the abbreviations

$$[X^\beta, X^\alpha]_{ba} \equiv \sum_n [(X^\beta)_{bn}(X^\alpha)_{na} - (X^\alpha)_{bn}(X^\beta)_{na}], \tag{4.6}$$

$$[X^\beta, [X^\alpha, m^2]]_{ba} + [X^\alpha, [X^\beta, m^2]]_{ba} \equiv -\sum_n (2m_n^2 - m_a^2 - m_b^2)[(X^\beta)_{bn}(X^\alpha)_{na} + (X^\alpha)_{bn}(X^\beta)_{na}], \tag{4.7}$$

and defined the diagonal mass-squared matrix m^2 by

$$(m^2)_{bn} = m_b^2\delta_{bn}\,. \tag{4.8}$$

The sums (4.6) and (4.7) now run over *all single*-particle states, since the states n which are missing in (4.1) and (4.2) are just those which come from the pole terms in (4.3) at $\omega = 0$.

The dispersion relations (4.4) and (4.5) will be used in Sec. V to derive constraints on the axial-vector coupling matrices X^α and mass spectrum m^2.

V. HIGH-ENERGY THEOREMS

Consider a gedanken experiment studying the behavior of the forward-scattering amplitude of massless mesons by baryons as the function of the initial meson energy ω, while the quantity E is fixed. Suppose that this scattering process takes place in two intersecting storage rings, where the baryon beam can have as high energy as one pleases. This means that the fixed quantity E is, in principle, as large as we like. If this is so, the Mandelstam variable $s=m_a^2+2E\omega$ is in the asymptotic region for any ω from the interval (ϵ,∞), where ϵ is positive. Consequently the scattering amplitude in question should exhibit the proper high-energy behavior, $(E\omega)^{\alpha(0)}$, prescribed by Regge pole theory, for any ω from the aforementioned interval. Here $\alpha(0)$ denotes an intercept of a dominant Regge trajectory exchanged in the t channel. Assume that this scattering process may be described by the scattering amplitude $M_{ba}^{\beta\alpha}(\omega)$ given by (4.4) and (4.5) when inserted in (3.16). In view of what was said, the pole-term contributions from (4.4) and (4.5) exhibiting the behavior $(E\omega)^{-2}$ are, in principle, as small as one pleases for $E\to\infty$ and $\omega\neq 0$. Consequently we are allowed to rule them out from the consideration concerning the description of the above-mentioned gedanken experiment. Then the dispersion relations (4.4) and (4.5) reduce to the forms:

$$M_{ba}^{\beta\alpha(-)}(\omega)=2(f_\beta f_\alpha)^{-1}\{if^{\beta\alpha\gamma}(F^\gamma)_{ba}-[X^\beta,X^\alpha]_{ba}\}+\frac{2E\omega}{\pi}\int dv\left[\frac{1}{(v-m_a^2)(v-m_a^2-2E\omega)}-\frac{1}{(v-m_b^2)(v-m_b^2+2E\omega)}\right][A_{ba}^{\beta\alpha}(v)-A_{ba}^{\alpha\beta}(v)]\frac{2}{2v-m_a^2-m_b^2}+O\left(\frac{1}{E^2\omega^2}\right) \tag{5.1}$$

and

$$M_{ba}^{\beta\alpha(+)}(\omega)=\tfrac{1}{2}(f_\beta f_\alpha)^{-1}\{[X^\beta,[X^\alpha,m^2]]_{ba}+[X^\alpha,[X^\beta,m^2]]_{ba}-2(\Sigma^{\beta\alpha})_{ba}\}+\frac{E\omega}{\pi}\int dv\left[\frac{1}{(v-m_a^2)(v-m_a^2-2E\omega)}-\frac{1}{(v-m_b^2)(v-m_b^2+2E\omega)}\right][A_{ba}^{\beta\alpha}(v)+A_{ba}^{\alpha\beta}(v)]+O\left(\frac{1}{E^2\omega^2}\right). \tag{5.2}$$

As was pointed out above, the scattering amplitudes (5.1) and (5.2) must possess the proper high-energy behavior for $E\to\infty$ and $\omega\neq 0$.[6,7]

The amplitude $M^{(-)}$ has the Regge behavior

$$M_{ba}^{\beta\alpha(-)}(\omega)\approx(E\omega)^{\alpha_1(0)-1}, \tag{5.3}$$

where $\alpha_1(0)$ is the intercept of the dominant Regge trajectory (exchanged in the t channel) which is allowed to contribute to this part of the scattering amplitude and the extra term -1 is due to our normalization of $M^{(-)}$. As is well known, all Regge trajectories have $\alpha(0)<1$ except for the Pomeranchuk trajectory. Since the Pomeranchuk trajectory is not allowed to contribute to $M^{(-)}$, $\alpha_1(0)$ is less than one, and the amplitude $M^{(-)}$ vanishes for any $\omega\neq 0$ and $E\to\infty$. Next we shall apply Weinberg's postulate to (5.1), which requires that both pole terms and the continuum contributions from (5.1) must not violate the Regge behavior (5.3). Hence we must demand that the constant term in (5.1) must be zero, i.e.,

$$[X^\beta,X^\alpha]=if^{\beta\alpha\gamma}F^\gamma, \tag{5.4}$$

and therefore $M^{(-)}$ is represented by

$$M_{ba}^{\beta\alpha(-)}(\omega)=\frac{2E\omega}{\pi}\int dv\left[\frac{1}{(v-m_a^2)(v-m_a^2-2E\omega)}-\frac{1}{(v-m_b^2)(v-m_b^2+2E\omega)}\right][A_{ba}^{\beta\alpha}(v)-A_{ba}^{\alpha\beta}(v)]\frac{2}{2v-m_a^2-m_b^2}+O\left(\frac{1}{E^2\omega^2}\right). \tag{5.5}$$

In (5.4) we have suppressed the hadron indices a and b and that equation is meant as the matrix relation among the axial-vector coupling matrices X^α and the SU(3) generator matrices F^γ. The application of the SU(3) conservation to the defining formula (3.8) for X^α gives us further the matrix relation

$$[F^\alpha,X^\beta]=if^{\alpha\beta\gamma}X^\gamma. \tag{5.6}$$

The SU(3) generator matrices F^α fulfill, of course, the standard commutation relations

$$[F^\alpha,F^\beta]=if^{\alpha\beta\gamma}F^\gamma. \tag{5.7}$$

Thus the matrix equations (5.4), (5.6), and (5.7) represent exactly the Lie algebra of the SU(3)⊗SU(3) group and imply that the *single* particle states we have included in pole terms must for each helicity form a basis for an irreducible or reducible representation of the group in question.

Next we turn out to consider the application of Weinberg's postulate for $M^{(+)}$. The amplitude $M^{(+)}$ has a singlet, octet, and 27-plet exchanged in the t channel. The 27-plet part may be isolated as

$$M_{ba}^{\beta\alpha(27)}(\omega)=M_{ba}^{\beta\alpha(+)}(\omega)-\tfrac{1}{8}\delta^{\beta\alpha}M_{ba}^{\gamma\gamma(+)}(\omega)-\tfrac{3}{5}d^{\beta\alpha\gamma}d^{\gamma\tau\sigma}M_{ba}^{\tau\sigma(+)}(\omega), \tag{5.8}$$

where $d^{\beta\alpha\gamma}$ are the total symmetric structure constants of the SU(3) group. Accordingly $M^{(27)}$ obeys the following dispersion relation:

$$\begin{aligned}M_{ba}^{\beta\alpha(27)}(\omega)=&(f_\beta f_\alpha)^{-1}\{[X^\beta,[X^\alpha,m^2]]_{ba}-\tfrac{1}{8}\delta^{\beta\alpha}[X^\gamma,[X^\gamma,m^2]]_{ba}-\tfrac{3}{5}d^{\beta\alpha\gamma}d^{\gamma\tau\sigma}[X^\tau,[X^\sigma,m^2]]_{ba}\}\\&+\frac{2E\omega}{\pi}\int dv\left[\frac{1}{(v-m_a{}^2)(v-m_a{}^2-2E\omega)}-\frac{1}{(v-m_b{}^2)(v-m_b{}^2)+2E\omega)}\right]\\&\times[A_{ba}^{\beta\alpha(+)}(v)-\tfrac{1}{8}\delta^{\beta\alpha}A_{ba}^{\gamma\gamma(+)}(v)-\tfrac{3}{5}d^{\beta\alpha\gamma}d^{\gamma\tau\sigma}A_{ba}^{\tau\sigma(+)}(v)]+O\left(\frac{1}{E^2\omega^2}\right),\end{aligned} \tag{5.9}$$

where we have used the notation

$$A_{ba}^{\beta\alpha(+)}(v)=\tfrac{1}{2}[A_{ba}^{\beta\alpha}(v)+A_{ba}^{\alpha\beta}(v)] \tag{5.10}$$

and the equality

$$[X^\alpha,[X^\beta,m^2]]_{ba}=[X^\beta,[X^\alpha,m^2]]_{ba}, \tag{5.11}$$

which follows from the Jacobi identity applied for X^α, X^β, and m^2 exploiting (5.4) along with the commutativity of m^2 with F^α. Perhaps it should be emphasized at this point that (5.9) is independent of $\Sigma^{\beta\alpha}$, since, according to our assumption, $\Sigma^{\beta\alpha}$ does not contain any part transforming like the 27-representation of SU(3). The amplitude $M^{(27)}$ given by (5.9) must exhibit the Regge behavior

$$M_{ba}^{\beta\alpha(27)}(\omega)\approx(E\omega)^{\alpha_2(0)} \tag{5.12}$$

for $E\to\infty$ and $\omega\neq 0$. $\alpha_2(0)$ is the value of the dominant 27-plet Regge trajectory at $t=0$. There are several reasons to believe that

$$\alpha_2(0)<0, \tag{5.13}$$

mainly because of the absence of the so-called exotic states in the nature. Granting this to be so then $M^{(27)}$ given by (5.9) must vanish for any $\omega\neq 0$ if $E\to\infty$. We apply once again Weinberg's postulate for (5.9). Consequently we require that neither the pole-term contributions nor the continuum can violate the Regge behavior of $M_{ba}^{\beta\alpha(27)}(\omega)$ given by (5.12). Hence we demand that the constant term in (5.9) be zero, i.e.,

$$[X^\beta,[X^\alpha,m^2]]=\tfrac{1}{8}\delta^{\beta\alpha}[X^\gamma,[X^\gamma,m^2]]+\tfrac{3}{5}d^{\beta\alpha\gamma}d^{\gamma\tau\sigma}[X^\tau,[X^\sigma,m^2]], \tag{5.14}$$

where we have suppressed the hadron indices a and b.

The last equation is a matrix relation among eight matrices X^α and one diagonal mass-squared matrix m^2. The matrices X^α are determined by the algebraic structures (5.4), (5.6), and (5.7) and therefore Eq. (5.14) is the relation for an unknown matrix m^2. In the practical application of (5.14), one might proceed as follows. Each single-particle state may be written, in general, as the sum of unitary irreducible representations of the SU(3)⊗SU(3) group. The matrices X^α are then entirely determined by the unitary irreducible representations of the group in question and by the mixing angles which are defined by the unknown coefficients of representations in the aforementioned sum. The known matrices X^α are then inserted in (5.14) and a nontrivial equation for the unknown matrix m^2 is obtained. A solution to this equation will represent the mass spectrum of hadrons associated with the chosen reducible representation of SU(3)⊗SU(3). The physical masses will, of course, depend on the unknown mixing angles and on internal quantum numbers. It is very easy to imagine that such calculation of the hadron mass spectrum is extremely tedious and should be performed for each specific representation separately without any of the elegance normally associated with group theory. However a general group-theoretical solution to (5.14) applicable to any representation will be found in Sec. VI.

VI. NATURE OF SU(3)⊗SU(3)-SYMMETRY BREAKING

The purpose of this section is to find the tensorial character of the mass-squared matrix m^2 with respect to commutation relations with X^α and F^α. The known tensorial character of this matrix will provide us information on how the SU(3)⊗SU(3) symmetry is broken down. To do this use is made of the procedure developed by Weinberg[1] for the chiral group SU(2)⊗SU(2).

We put Eq. (5.14) in group-theoretic terms by defining SU(3) vectors v^α and u^α and an SU(3) scalar u^0 given by

$$v^{\alpha}=-i[X^{\alpha},m^2], \tag{6.1a}$$

$$u^{\alpha}=-\tfrac{3}{5}id^{\alpha\beta\gamma}[X^{\beta},v^{\gamma}], \tag{6.1b}$$

$$u^0=-\frac{\sqrt{3}}{\sqrt{2}}\frac{i}{8}[X^{\alpha},v^{\alpha}]. \tag{6.1c}$$

Then Eq. (5.14) gets the form

$$[X^{\beta},v^{\alpha}]=i\frac{\sqrt{2}}{\sqrt{3}}\delta^{\beta\alpha}u^0+id^{\beta\alpha\gamma}u^{\gamma}. \tag{6.2}$$

By using merely Jacobi identities and identities[8] relating the structure constants $f^{\beta\alpha\gamma}$ and $d^{\beta\alpha\gamma}$ one can show that (6.1) and (6.2) imply the following commutation relations:

$$[X^{\beta},u^0]=-i\frac{\sqrt{2}}{\sqrt{3}}v^{\beta}, \tag{6.3}$$

$$[X^{\beta},u^{\alpha}]=-i\frac{\sqrt{2}}{\sqrt{3}}\delta^{\beta\alpha}v^0-id^{\beta\alpha\gamma}v^{\gamma}, \tag{6.4}$$

$$[X^{\beta},v^0]=i\frac{\sqrt{2}}{\sqrt{3}}u^{\beta}, \tag{6.5}$$

where v^0 is another SU(3) scalar defined by

$$v^0=\tfrac{1}{8}i\frac{\sqrt{3}}{\sqrt{2}}[X^{\alpha},u^{\alpha}]. \tag{6.6}$$

Since v^{α} and u^{α} are the SU(3) vectors while v^0 and u^0 are the SU(3) scalars by our definitions, they obey the following algebraic structures:

$$[F^{\alpha},u^{\beta}]=if^{\alpha\beta\gamma}u^{\gamma}, \tag{6.7a}$$

$$[F^{\alpha},v^{\beta}]=if^{\alpha\beta\gamma}v^{\gamma}, \tag{6.7b}$$

$$[F^{\alpha},u^0]=[F^{\alpha},v^0]=0. \tag{6.7c}$$

Equations (6.2)–(6.7) show that u^{α}, v^{α}, u^0, and v^0 are members of the $(3,\overline{3})\oplus(\overline{3},3)$ representation of $SU(3)\otimes SU(3)$.

We now complete our goal by noting from (6.1a) and (6.3) that the sum $m^2+(\frac{3}{2})^{1/2}u^0$ commutes with all generators of $SU(3)\otimes SU(3)$, i.e.,

$$\left[X^{\alpha},m^2+\frac{\sqrt{3}}{\sqrt{2}}u^0\right]=0. \tag{6.8}$$

Thus, (6.8) requires that this sum must behave as an $SU(3)\otimes SU(3)$ scalar m_0^2, which yields

$$m^2=m_0^2-\frac{\sqrt{3}}{\sqrt{2}}u^0. \tag{6.9}$$

This is the group-theoretic solution to the mass spectrum condition (5.14) and implies that the mass-squared matrix m^2 behaves as the sum of a scalar and a component of the $(3,\overline{3})\oplus(\overline{3},3)$ representation under $SU(3)\otimes SU(3)$ group transformations. This result and Eq. (5.4) were previously derived by Sudbery[9] within the framework of chirally invariant Lagrangians, where an SU(3)-invariant subgroup was realized linearly.

Next consider that the SU(3) symmetry is broken down to $SU(2)\otimes U(1)$, the isospin and hypercharge subgroup. Then we confine the range of indices α,β to 1, 2, 3, and 8 in the algebraic structures (6.1)–(6.7) and find that the sums $m^2+(\frac{3}{2})^{1/2}u^0$ and $m^2+\sqrt{3}u^8$ commute with all generators of the $SU(2)\otimes SU(2)\otimes U(1)$ group, i.e., with X^{α}, F^{α}, and F^8, where $\alpha,\beta=1,2,3$. Since both of these sums commute with the generator matrices in question, so does their linear combination and consequently the mass-squared matrix m^2 is represented by

$$m^2=m_0^2+k(u^0+cu^8), \tag{6.10}$$

where k and c are arbitrary constants. Therefore it behaves as the sum of an invariant and two components of the $(3,\overline{3})\oplus(\overline{3},3)$ representation of the $SU(3)\otimes SU(3)$ group. This result and Eq. (5.4) were previously derived by Ogievetsky from tree graphs generated by the $SU(3)\otimes SU(3)$ chiral-invariant Lagrangians, where the $SU(2)\otimes U(1)$-invariant subgroup was realized linearly.[10]

We would like to emphasize that (6.9) and (6.10) are not approximations based on a free assumption of weak chiral-symmetry breaking, but rather they are exact consequences of Weinberg's postulate and the asymptotic behavior of the forward-scattering amplitude at high energy. There is no need to expect that the symmetry-breaking terms in (6.9) and in (6.10) are smaller than m_0^2. The only permitted symmetry breaking consistent with Weinberg's postulate is that proposed by Gell-Mann, Oakes, and Renner[11] and is, in fact, independent on whether Σ term from (2.11) belongs to the $(3,\overline{3})\oplus(\overline{3},3)$ or $(1,8)\oplus(8,1)$ representation of $SU(3)\otimes SU(3)$.

Recently it has become popular to write the strong Hamiltonian as the sum of $SU(3)\otimes SU(3)$-invariant piece H_0, plus a small correction term H' breaking down the $SU(3)\otimes SU(3)$ symmetry and to discuss how this symmetry is broken. In the model of Ref. 11, H' belongs to $(3,\overline{3})\oplus(\overline{3},3)$. Sugawara[12] has proposed that H' may transform like $(1,8)\oplus(8,1)$ and more recently several schemes[13] consider H' as a component of the (8, 8) representation.

Starting from rather different approaches, many authors have attempted to find out which of these models is favored by existing experimental data. The analysis performed by a quite large group of authors[14] supports strongly the $(3,\overline{3})\oplus(\overline{3},3)$-symmetry breaking scheme. This analysis is to be contrasted with estimates of Cheng and Dashen[15] which are in serious disagreement with the $(3,\overline{3})\oplus(\overline{3},3)$ model. Recently a very comprehensive analysis of meson-baryon scattering and electroproduction in the (8, 8) model was done by Genz, Katz, and Steiner.[16] They have found good agreement with experimental data, except for cases where experimental values are not reliable.

The purpose of this short discussion is to indicate an existing controversial situation in experimental evidence for supporting or abandoning the chiral-symmetry-breaking models[11-13] which had been proposed on the basis of intuition. The value of the results concerning chiral symmetry breaking derived by Ogievetsky,[10] Sudbery,[9] and in this paper lies mainly in that they are exact consequences of a simple assumption about the high-energy behavior of tree-graph contributions to the forward-scattering amplitude.

VII. CONCLUSIONS

We have presented evidence here that Weinberg's postulate demanding the cancellation of rapidly growing pole terms among themselves and not with the continuum in order to preserve the high-energy behavior of the scattering amplitude is a useful one. In its consequences it predicts the transformation properties of the $SU(3)\otimes SU(3)$-symmetry-breaking term in the mass spectrum of hadrons. The representation $(3,\bar{3})+(\bar{3},3)$ is the only one which is permitted to break the symmetry.

Note added in proof. It has been pointed out to me that the subtracted dispersion relation (5.1) is not consistent with the corresponding unsubtracted dispersion relation given by Weinberg.[1] It should be noted that Eq. (A7) of Ref. 1 contains a printing error. If this is corrected, one obtains the result given by (5.1).

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