amplitude had a very special fixed-angle behavior, a new type of Regge singularity (linked singularities) was needed. Alternatively, it has been observed that particle poles can be accommodated without any complications by splitting the amplitude into different terms ("tree expansion"), and making the right choice for the triple O(2, 1) expansions.[12] This method would not help to accommodate anomalous singularities, and Regge singularities more complicated than factorizable poles will be needed.

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Electromagnetic Radius of the Neutrino

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The electromagnetic radius of the neutrino is investigated in the intermediate-boson theories of Haller, Landovitz, and Goldberg (HLG) and of Lee and Wick. In the Lee-Wick theory, the radius depends on the ratio of the vector-boson mass to the scalar-boson mass. For the case where they are equal, the radius is approximately the same as that of the HLG theory.

I. GENERAL DISCUSSION

The vertex function for the interaction of the neutrino with an electromagnetic field is expressible as

$$\Lambda_\mu = \bar{u}(p_\nu')\tfrac{1}{2}(1-\gamma_5)\gamma_\mu \tfrac{1}{2}(1+\gamma_5)u(p_\nu)F(q^2), \qquad (1.1)$$

where

$$F(q^2) = F(0) + q^2 F'(0) + \cdots . \qquad (1.2)$$

Because of gauge invariance, $F(0)$ is zero. The electromagnetic radius is

$$\langle x^2 \rangle = 6F'(0) . \qquad (1.3)$$

The graphs for the vertex function are shown in Fig. 1. For the intermediate-vector-boson theory of weak interactions, the neutrino electromagnetic radius has been calculated.[1] The result obtained for the radius is divergent unless one assumes the over-all convergence of a function

$$G(\alpha\Lambda^2/M_W^2), \text{ as } \Lambda\to\infty.$$

In this paper, we investigate the electromagnetic radius of the neutrino in the Haller-Landovitz-Goldberg (HLG) intermediate-boson theory,[2] and in the Lee-Wick (LW) intermediate-boson theory.[3] Both of these theories are renormalizable and involve the inclusion of charged scalar bosons as well as charged vector bosons. In the HLG

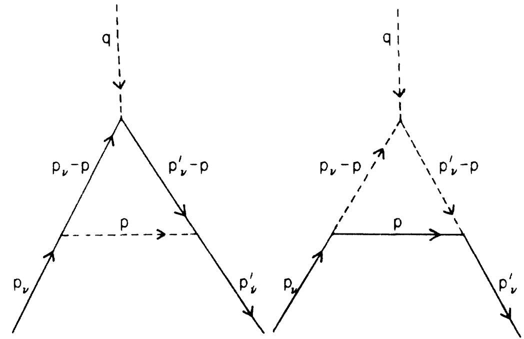

FIG. 1. Graphs for neutrino vertex function.

theory, the mass of the scalar boson is equal to the mass of the vector boson. In the Lee-Wick theory, the mass M_0 of the scalar boson can differ from the mass M_1 of the vector boson. The two theories bear similarities to each other, i.e., the propagators are the same in both theories for M_0 equal to M_1 in LW. However, even for this case, the respective vertex functions differ.[4]

II. HLG THEORY

The Lagrangian for the W boson, including electromagnetic interactions, is

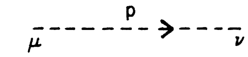

FIG. 2. W-boson propagator.

$$L = -(\partial_\nu W_\mu)^\dagger(\partial_\nu W_\mu) - M_W^2 W_\mu^\dagger W_\mu - ie\kappa F_{\mu\nu} W_\mu^\dagger W_\nu , \tag{2.1}$$

where

$$\partial_\nu = \frac{\partial}{\partial x_\nu} - ieA_\nu \tag{2.2}$$

and κ is the anomalous magnetic moment of the W boson. The expressions for the W boson propagator (Fig. 2), the three-point vertex (Fig. 3), and the four-point vertex (Fig. 4) are, respectively,

$$S = \frac{-i\delta_{\mu\nu}}{p^2 + M_W^2} , \tag{2.3}$$

$$V = ie[\delta_{\alpha\beta}(p+p')_\mu - \kappa\delta_{\alpha\mu}(p-p')_\beta - \kappa\delta_{\beta\mu}(p'-p)_\alpha] , \tag{2.4}$$

$$U = -2ie^2\delta_{\mu\nu}\delta_{\alpha\beta} . \tag{2.5}$$

The expression for the neutrino vertex function is

$$\Lambda_\mu = \frac{ig^2}{(2\pi)^4}\bar{u}(p'_\nu)(1-\gamma_5)\int d^4p\left(\frac{\gamma_\alpha[i(\not{p}'_\nu - \not{p}) - m]\gamma_\mu[i(\not{p}_\nu - \not{p}) - m]\gamma_\alpha}{[(p_\nu - p)^2 + m^2][(p'_\nu - p)^2 + m^2][p^2 + M_W^2]} + \frac{2\not{p}(p_\nu + p'_\nu - 2p)_\mu + \kappa(\gamma_\mu\not{p}\not{q} - \not{q}\not{p}\gamma_\mu)}{[(p_\nu - p)^2 + M_W^2][(p'_\nu - p)^2 + M_W^2][p^2 + m^2]}\right)(1+\gamma_5)u(p_\nu) . \tag{2.6}$$

Upon performing the integration and regularizing Λ_μ, i.e., setting $\Lambda_\mu(p_\nu, p'_\nu, 0)$ equal to zero, one obtains a finite result. The radius obtained is[5]

$$\langle x^2\rangle = \frac{1}{\pi^2}\frac{g^2}{M_W^2}\left(-\ln\frac{M_W^2}{m^2} - \tfrac{1}{3} + \tfrac{3}{4}\kappa\right) , \tag{2.7}$$

where $m = m_e$ for electron-type neutrinos and $m = m_\mu$ for muon-type neutrinos. The coupling constant g is related to the Fermi coupling constant $G = 10^{-5}/M_p^2$ by

$$\frac{g^2}{M_W^2} = \frac{G}{\sqrt{2}} . \tag{2.8}$$

III. LEE-WICK THEORY

The Lagrangian for the W boson is

$$L = -\zeta(\partial_\mu W_\mu)^\dagger(\partial_\nu W_\nu) - (\partial_\nu W_\mu)^\dagger(\partial_\nu W_\mu) + (\partial_\nu W_\mu)^\dagger(\partial_\mu W_\nu) - M_1^2 W_\mu^\dagger W_\mu - ie\kappa F_{\mu\nu} W_\mu^\dagger W_\nu , \tag{3.1}$$

where

$$\zeta = M_1^2/M_0^2 . \tag{3.2}$$

The respective expressions for the W-boson propagator and the vertices are

$$S = \frac{-i}{p^2 + M_1^2}\left[\delta_{\mu\nu} - \frac{p_\mu p_\nu(1-\zeta^{-1})}{p^2 + M_0^2}\right] , \tag{3.3}$$

$$V = ie[\delta_{\alpha\beta}(p+p')_\mu - \delta_{\alpha\mu}(p - \zeta p' + \kappa p - \kappa p')_\beta - \delta_{\beta\mu}(p' - \zeta p + \kappa p' - \kappa p)_\alpha], \tag{3.4}$$

$$U = -ie^2[2\delta_{\mu\nu}\delta_{\alpha\beta} - (1-\zeta)\delta_{\alpha\mu}\delta_{\beta\nu} - (1-\zeta)\delta_{\alpha\nu}\delta_{\beta\mu}]\,. \tag{3.5}$$

Note that for $\zeta = 1$ the expressions for the propagator and the four-point vertex are the same as in the HLG theory. However, the respective three-point vertex expressions are different.

The neutrino vertex function is

$$\begin{aligned}
\Lambda_\mu = {} & \frac{ig^2}{(2\pi)^4}\,\bar{u}(p_\nu')(1-\gamma_5) \\
& \times \int d^4p \left\{ \frac{\gamma_\alpha[i(\not p_\nu' - \not p) - m]\gamma_\mu[i(\not p_\nu - \not p) - m]\gamma_\alpha}{[(p_\nu - p)^2 + m^2][(p_\nu' - p)^2 + m^2][p^2 + M_1^2]} - \frac{(1-\zeta^{-1})\not p[i(\not p_\nu' - \not p) - m]\gamma_\mu[i(\not p_\nu - \not p) - m]\not p}{[(p_\nu - p)^2 + m^2][(p_\nu' - p)^2 + m^2][p^2 + M_1^2][p^2 + M_0^2]} \right. \\
& + \frac{2\not p(p_\nu + p_\nu' - 2p)_\mu - (1+\kappa)\not q\not p\gamma_\mu + (1+\kappa)\gamma_\mu\not p\not q - 2(1-\zeta)p^2\gamma_\mu}{[(p_\nu - p)^2 + M_1^2][(p_\nu' - p)^2 + M_1^2][p^2 + m^2]} \\
& + \frac{(1-\zeta^{-1})[p^2\not p(p_\nu + p_\nu' - 2p)_\mu - (1+\kappa)p^2(p_\nu' - p)\circ q\gamma_\mu + (1-\zeta)p^2(p_\nu' - p)^2\gamma_\mu - (1-\zeta)p^2\not p(p_\nu' - p)_\mu]}{[(p_\nu - p)^2 + M_1^2][(p_\nu' - p)^2 + M_1^2][(p_\nu' - p)^2 + M_0^2][p^2 + m^2]} \\
& + \frac{(1-\zeta^{-1})[p^2\not p(p_\nu + p_\nu' - 2p)_\mu + (1+\kappa)p^2(p_\nu - p)\cdot q\gamma_\mu - (1-\zeta)p^2\not p(p_\nu - p)_\mu + (1-\zeta)p^2(p_\nu - p)^2\gamma_\mu]}{[(p_\nu - p)^2 + M_1^2][(p_\nu - p)^2 + M_0^2][(p_\nu' - p)^2 + M_1^2][p^2 + m^2]} \\
& \left. + \frac{(1-\zeta^{-\prime})^2p^2\not p[-\zeta(p_\nu' - p)^2(p_\nu - p)_\mu - \zeta(p_\nu - p)^2(p_\nu' - p)_\mu + \kappa(p_\nu\cdot p_\nu')(p_\nu + p_\nu')_\mu - 2\kappa(p_\nu\cdot p_\nu')p_\mu - \kappa(p\cdot q)(p_\nu' - p_\nu)_\mu]}{[(p_\nu - p)^2 + M_1^2][(p_\nu - p)^2 + M_0^2][(p_\nu' - p)^2 + M_1^2][(p_\nu' - p)^2 + M_0^2][p^2 + m^2]} \right\} \\
& \times (1+\gamma_5)u(p_\nu)\,.
\end{aligned} \tag{3.6}$$

After regularization, a finite result is obtained for Λ_μ. The resulting expression for the radius is[6]

$$\begin{aligned}
\langle x^2\rangle = \frac{1}{\pi^2}\frac{g^2}{M_1^2}\Big[& -\ln\frac{M_1^2}{m^2} - \tfrac{3}{2}\ln(\zeta) + \tfrac{5}{12} - \tfrac{1}{2}(1-\zeta) - 3\,\frac{1-\zeta}{\zeta}(\tfrac{5}{4} - \zeta)I_1 + 3\,\frac{1-\zeta}{\zeta}(\tfrac{5}{6} - \zeta)I_2 \\
& + \frac{3}{4}\frac{1-\zeta}{\zeta}I_3 - \frac{1-\zeta}{2\zeta}I_4 + \tfrac{3}{4}\kappa - \tfrac{3}{2}\kappa\,\frac{1-\zeta}{\zeta}I_1 - \tfrac{3}{4}\kappa\,\frac{1-\zeta}{\zeta}I_5 \Big],
\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}
I_1 &= M_1^2\int_0^1 \frac{x^2\,dx}{(M_1^2 - M_0^2)x + M_0^2} \\
&= -\frac{\zeta}{1-\zeta}\left[\frac{1}{2} + \frac{1}{1-\zeta} + \frac{1}{(1-\zeta)^2}\ln(\zeta)\right],
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
I_2 &= M_1^2\int_0^1 \frac{x^3\,dx}{(M_1^2 - M_0^2)x + M_0^2} \\
&= -\frac{\zeta}{1-\zeta}\left[\frac{1}{3} + \frac{1}{2(1-\zeta)} + \frac{1}{(1-\zeta)^2} + \frac{1}{(1-\zeta)^3}\ln(\zeta)\right],
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
I_3 &= M_1^2\int_0^1 \frac{x^2\,dx}{(M_0^2 - M_1^2)x + M_1^2} \\
&= \frac{\zeta}{1-\zeta}\left[\frac{1}{2} - \frac{\zeta}{1-\zeta} - \frac{\zeta^2}{(1-\zeta)^2}\ln(\zeta)\right],
\end{aligned} \tag{3.10}$$

$$\begin{aligned}
I_4 &= M_1^2\int_0^1 \frac{x^3\,dx}{(M_0^2 - M_1^2)x + M_1^2} \\
&= \frac{\zeta}{1-\zeta}\left[\frac{1}{3} - \frac{\zeta}{2(1-\zeta)} + \frac{\zeta^2}{(1-\zeta)^2} + \frac{\zeta^3}{(1-\zeta)^3}\ln(\zeta)\right],
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
I_5 &= \int_0^1 dx\,x\ln\frac{(M_0^2 - M_1^2)x + M_1^2}{(M_1^2 - M_0^2)x + M_0^2} \\
&= \frac{1+\zeta}{2(1-\zeta)} + \frac{\zeta}{(1-\zeta)^2}\ln(\zeta)\,.
\end{aligned} \tag{3.12}$$

In expression (3.7),

$$\frac{g^2}{M_1^2} = \frac{G}{\sqrt{2}}\,. \tag{3.13}$$

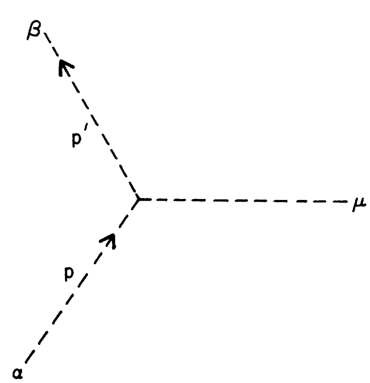

FIG. 3. Three-point vertex.

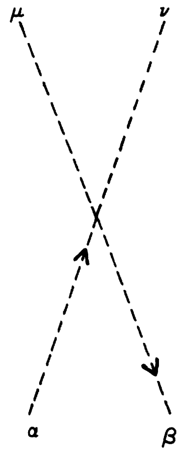

FIG. 4. Four-point vertex.

IV. CONCLUSIONS

The expression (3.7) is dependent on the value of ζ. It is of interest to compare the HLG expression (2.7) with the LW expression (3.7), for the case $\zeta = 1$. For $\zeta = 1$, expression (3.7) becomes

$$\langle x^2 \rangle = \frac{1}{\pi^2} \frac{g^2}{M_W^2} \left(-\ln \frac{M_W^2}{m^2} + \tfrac{5}{12} + \tfrac{3}{4}\kappa \right). \tag{4.1}$$

In both Eqs. (2.7) and (4.1), the $\ln(M_W^2/m^2)$ terms are the dominant ones.[7] The respective expressions involving these terms in Eqs. (2.7) and (4.1) are the same. Thus, for the case $\zeta = 1$, the two theories yield radii which are approximately the same.

In expressions (2.7) and (4.1), the terms involving κ do not have $\ln(M_W^2/m^2)$ factors. Hence a nonzero κ would not alter the respective expressions very significantly. However, this is not the situation for arbitrary ζ in Eq. (3.7).

Since $g^2/M_W^2 = G/\sqrt{2}$, an experimental measurement of the neutrino radius would yield information on the mass of the intermediate boson. Furthermore, both expressions (2.7) and (3.7) yield

$$\langle x^2 \rangle_{\nu_\mu} = \langle x^2 \rangle_{\nu_e} + \frac{G}{\sqrt{2}\,\pi^2} \ln \frac{m_\mu^2}{m_e^2} . \tag{4.2}$$

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[3]T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969); T. D. Lee, Phys. Rev. Letters 25, 1144 (1970).

[4]The indefinite-metric properties of these theories do not present difficulties (see Ref. 3).

[5]Terms of order m^2/m_W^2 have been omitted.

[6]Terms of order m^2/M_1^2 and m^2/M_0^2 have been omitted.

[7]For example, for $M_W = 10$ GeV, we have $\ln(M_W^2/m_e^2) \cong 20$ and $\ln(M_W^2/m_\mu^2) \cong 9$.