*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]M. Bolsterli, Ann. Phys. (N. Y.) 62, 569 (1971), referred to as I.

[2]M. Bolsterli, Phys. Rev. D 6, 3501 (1972), referred to as II.

[3]S. Pekar, J. Phys. (USSR) 10, 341 (1946); G. Höhler, Z. Phys. 140, 192 (1955).

[4]A. S. Krass, Phys. Rev. D 2, 2464 (1970).

[5]E. P. Gross, in *Mathematical Methods of Solid State and Superfluid Theory*, Proceedings of the 1967 Scottish Universities' Summer School, edited by R. C. Clark and G. H. Derrick (Plenum, New York, 1968), p. 94.

[6]This was pointed out to me by D. Liberman.

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Massive Quarks and Deep-Inelastic Phenomena*

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A possible theory of hadrons in terms of a massive-quark field theory is proposed. Some consequences for deep-inelastic phenomena are worked out.

I. INTRODUCTION

The problematic aspects indicated by SLAC experiments[1] and their scaling behavior have insistently hinted at the relevance of a picture of hadrons in terms of constituents, the most popular ones being Feynman's partons.[2] However, these objects seem to have puzzling and disturbing characteristics, such as their small mass and their invisibility.[3]

Attempts to construct a quark field theory of the hadrons have recently appeared in the literature, most notably by the Cambridge school[4]; unfortunately these approaches do not provide any answer to the aforementioned problems. In fact they look all the more unsatisfactory if one tries to think about the alleged constituents of the hadrons in realistic terms, and to use them to build up a coherent picture which explains the basic facts of scaling without running into serious contradictions with experiments.

A possible theory of hadrons in terms of a massive-quark field theory was proposed in 1969.[5] The preliminary results of that study exhibited some attractive features, but the details of the particular model considered were somehow unrealistic.

The motivation for looking at a field theory of quarks is the standard set of arguments in favor of the quark model (e.g., SU_3, current algebra, etc.), which suggest that the quark is a relevant degree of freedom of hadronic matter. On the other hand, the impressive list of negative results in quark searches[6] requires the quarks, if they exist at all as particles, to have a very large mass [>25 GeV according to CERN Intersecting Storage Rings (ISR) experiments[6]], or else the forces have to be so singular as to prevent the quarks from coming out of hadrons even though they can be kinematically produced.[7] We do not think that the latter possibility is particularly appealing in view of the linearity of Regge trajectories.[8]

If we then subscribe to a massive-quark field theory (one can eventually take the limit $M_q \to \infty$ and get rid of "real" quarks) we are led to consider dynamical field-theoretical equations, such as the Bethe-Salpeter (BS) equation, to describe the binding of quarks to form physical particles.[9] In order to solve a BS equation we need as an input a BS kernel, and it is here that dynamics creeps in in a crucial way. We feel, however, that the dynamical situation in the case of superstrong (in the limit infinite) binding may well be considerably simpler than in the intermediate-binding case. This fact appears to be suggested by the regularities of the spectrum of hadrons, the smallness of their widths as compared with their masses, exchange degeneracy, linearity of Regge trajectories, the nonexistence of exotic states, etc.

Thus one can try to guess a simple form for the kernel in the BS equation for the $q\bar{q}$ scattering amplitude which embodies these basic features, and this was indeed taken up in Ref. 5.

From these attempts we are now going to abstract some properties which, in our opinion, should be at the basis of any realistic quark field theory of hadronic matter.

(i) Hadronic wave functions. We can associate

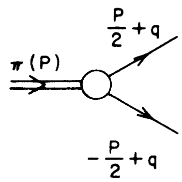

FIG. 1. Description of a π meson in terms of two off-shell quark legs.

to any hadron a Bethe-Salpeter wave function (w.f.) $\psi(\cdots)$ in terms of the minimum number of off-shell quark legs. For instance a π meson is adequately described by a function $\psi_\pi(q)$ (Fig. 1) which decreases rapidly when $|q| \to \infty$. This is obviously a very strong statement, which, if correct, would support the previous contention about the inherent simplicity of superstrong binding. In fact in conventional field theories the description of a particle would require the knowledge of all n-leg w.f.'s (Fig. 2) which are allowed by conservation laws. Thus one is maintaining that a π is a quark pair rather than a "quark soup."

(ii) Absence of exotic states. The poles in the appropriate channels of the $q\bar{q}$ and qqq amplitudes exhaust the particle spectrum. The residue of these poles define the hadronic w.f.'s. The requirement of no pole in the qq channel leads immediately to exchange degeneracy for the meson spectrum.

(iii) Regge behavior. Quark scattering amplitudes have Regge behavior in the appropriate kinematic limits. This is necessary in order for the hadron scattering amplitudes to have such a behavior.

(iv) Mass behavior of off-shell quark legs. We assume the quark scattering amplitudes to be strongly damped by increasing the mass of any external off-shell quark leg. This assumption is a natural one in the Regge region, where the Regge diagram (Fig. 3) provides for this behavior through the damping of the Reggeon's wave function. One should also notice that in the regime of energies and transverse momenta much smaller than the quark mass these wave functions are well approximated by entire functions of the external masses; their rapid decrease, therefore, implies that they have an essential singularity as $|q| \to \infty$.[10]

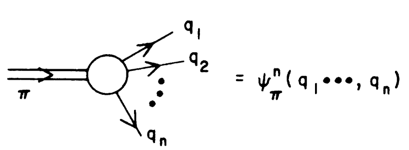

FIG. 2. Conventional field-theoretic description of a π meson in terms of n quark legs.

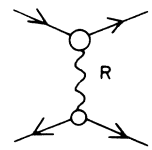

FIG. 3. Regge diagram for quark scattering.

II. CONSTRUCTING THE THEORY

From the previous four abstractions we can set up an iterative framework to compute hadronic amplitudes for both strong and weak processes.

The first step of approximation yields, through the solution of appropriate BS equations, $q\bar{q}$ and qqq scattering amplitudes in terms of meson and baryon bound-state spectra and wave functions.

Once equipped with $q\bar{q}$ and qqq scattering amplitudes, in order to compute particle scattering amplitudes we need suitable n-point kernels to be folded either with the wave functions or with the elementary $q\bar{q}$ and qqq scattering amplitudes. These n-point kernels are so defined as to be irreducible in their various $q\bar{q}$ or qqq channels.

The next stage of approximation then consists in picking the n-point kernels which have the minimal number of quark legs relevant to a given process. To illustrate this approximation, consider the amplitude for producing a given resonance R_n in $\pi\pi$ scattering. Thus we have only to consider the K_6 kernel (Fig. 4) and the total cross section of the resonances for $\pi\pi$ scattering (Fig. 5), where the $q\bar{q}$ amplitude is the one given in the first stage of the approximation in terms of the simple poles of the hadrons.

In addition to resonance formation we must also take into account diffraction scattering. This is obviously given by the 8-point kernel K_8 (Fig. 6), and analogously for baryons. The successive stages involve many-particle production, as well as wave-function renormalization, but do not present any further conceptual difficulty.

At this point it would be desirable to have a way of computing the various kernels. However, it seems at this time very hard to have descriptions of the K_n's valid over wide energy regions. On the other hand one can try more modestly to check the correctness of the whole approach by looking at the high-energy regions where simple Reggeiza-

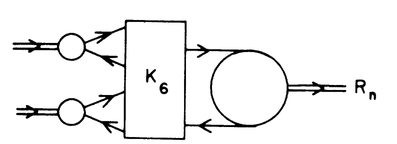

FIG. 4. Resonance formation in $\pi\pi$ scattering described by the K_6 kernel.

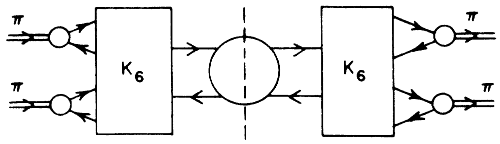

FIG. 5. Total cross section of the resonance for $\pi\pi$ scattering.

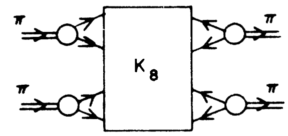

FIG. 6. Diffraction scattering described by the K_8 kernel.

tion is postulated and, according to the preceding abstractions, one can write fairly simple expression for kernels and amplitudes.

Fortunately, this is all that is needed for describing the electromagnetic and weak phenomena in the deep-inelastic region. Thus deep-inelastic phenomena are a very suitable test ground for checking this picture of hadrons and their interactions.

III. POMERANCHUKON DOMINANCE AND SCALING IN HIGH-MASS ELECTROMAGNETIC PROCESSES

We shall now consider various deep-inelastic amplitudes and compute them according to the approximation scheme and the rules described in the preceding section. For the purpose of illustration we shall only consider scalar quarks and scalar currents; the extension to spin-$\frac{1}{2}$ quarks as well as to a richer class of processes shall be the subject of another more detailed paper.

We should remark that much of what is going to be discussed overlaps with much of the recent work in parton models.[4] However, we think it worthwhile to go through these calculations again, especially in view of their significance as a test of what aims to be a possibly comprehensive picture of hadrodynamics.

This is a list of the basic ingredients of the calculation:

(a) Electromagnetic vertex. The basic irreducible electromagnetic vertex (Fig. 7),[11]

$$\gamma(q^2; (\tfrac{1}{2}q+l)^2, (\tfrac{1}{2}q-l)^2)(Q)_a{}^b \,,$$

is given by a function $\gamma=\gamma(q^2; m_1{}^2, m_2{}^2)$ which, according to point (iv) of Sec. I, we need only consider for zero values of the squared 4-momentum of the quarks, i.e., $m_1{}^2=m_2{}^2=0$. This function shall be denoted by $\gamma(q^2)$.

(b) $q\bar{q}$ scattering amplitude. This and all other quark kernels have a fast decrease in the masses of the off-shell quark fields. For large q^2, the imaginary part of this amplitude is given by Regge exchange (Fig. 8),

$$T_{ad}^{bc}(\cdots)\simeq\sum_i (q^2)^{\alpha_i(t)}\lambda_i(t)\beta_i(m_1{}^2, m_3{}^2) \times\bar{\beta}_i(m_2{}^2, m_4{}^2)t_{ad}^{bc} \,. \tag{1}$$

Setting $m^2=a=o(q^2)$, we parametrize

$$\beta(m_1{}^2, m_3{}^2)\sim\beta_i\frac{1}{\sqrt{\pi a}}\exp\left[-\left(\frac{m_1{}^2}{a}\right)^2\right]\times\frac{1}{\sqrt{\pi a}}\exp\left[-\left(\frac{m_3{}^2}{a}\right)^2\right] \,, \tag{2}$$

and assuming a flat Pomeranchukon $\alpha_P(t)\simeq\alpha_P(0)$ we can also write

$$\lambda_P(t)\sim b\exp[b(l_1-l_2)^2] \,. \tag{3}$$

Most of the time we are going to integrate these expressions over the relevant phase spaces; thus we can simplify the calculations considerably by writing for the P term

$$\beta_P(m_1{}^2, m_3{}^2)\bar{\beta}_P(m_2{}^2, m_4{}^2)\sim\beta\bar{\beta}\delta(\tfrac{1}{4}q^2+l_1{}^2)\delta(q\cdot l_1)\delta(\tfrac{1}{4}q^2+l_2{}^2)\delta(q\cdot l_2) \tag{4}$$

and

$$\lambda_P(t)\sim\delta((l_1-l_2)^2) \,. \tag{5}$$

The SU_3 tensor t_{ad}^{bc} shall be taken as SU_3-invariant and a singlet in the t channel. By putting all this together, we write for the $q\bar{q}$ amplitude at large q^2

$$T_{ad}^{bc}(\cdots)\underset{q^2\text{ large}}{\sim}\delta_a^c\delta_d^b(q^2)^{\alpha_P(0)}\beta\bar{\beta}\delta(\tfrac{1}{4}q^2+l_1{}^2)\delta(q\cdot l_1)\delta(\tfrac{1}{4}q^2+l_2{}^2)\delta(q\cdot l_2)\delta((l_1-l_2)^2)+O((q^2)^{1/2}) \,. \tag{6}$$

This parametrization will be carried out, along the same lines, for more complicated scattering amplitudes.

We are now ready to make computations for various deep-inelastic phenomena.

(1) $e^+e^- \to$ hadrons. The relevant diagram is in Fig. 9. Its contribution to the vacuum-polarization function is

$$\Pi(q^2) = \mathrm{Tr}(Q^2)\beta\bar{\beta}\,[\gamma(q^2)]^2[(q^2)^{\alpha_P(0)} + O((q^2)^{1/2})]$$

$$\times \int \frac{d^4 l_1}{(2\pi)^4} \int \frac{d^4 l_2}{(2\pi)^4}\, \delta((l_1 - l_2)^2)\,\delta(\tfrac{1}{4}q^2 + l_1^2)\,\delta(\tfrac{1}{4}q^2 + l_2^2)\,\delta(q\cdot l_1)\,\delta(q\cdot l_2)$$

$$= \frac{\beta}{(2\pi)^3}\frac{\bar{\beta}}{(2\pi)^3}\,\mathrm{Tr}(Q^2)[\gamma(q^2)]^2[(q^2)^{\alpha_P - 1} + O((q^2)^{-1/2})] \qquad (7)$$

Setting $\alpha_P(0) = 1$, we get a free-field behavior provided $\gamma(q^2) \to \mathrm{const} = \gamma$ as $q^2 \to \infty$. This behavior would actually arise if we postulated a pointlike coupling between the electromagnetic current and the quark fields. Had we made this very natural assumption (suggested by current algebra and light-cone physics) we would have gotten a "canonical free field" e^+e^- annihilation cross section, as given by the diagram in Fig. 10. This diagram is a typical "parton-model" diagram, but obviously its physical significance is quite remote. What Eq. (7) shows is that this behavior is reproduced by the assumption that the high-energy contribution of the $q\bar{q}$ amplitude is dominated by P exchange. By no means, then, does our calculation imply the existence of a $q\bar{q}$ pair in the intermediate state.

Actually the free-field singularity is generated by triality-zero contributions to the imaginary part of the $q\bar{q}$ amplitude. Thus, in our approach the diagram in Fig. 10 is to be taken only as a mnemonic aid. The satisfactory aspect of this result is that it overcomes in a physical way the difficulties of interpretation of the parton model. To the question "What is a parton?" our concise answer is that it is an off-shell quark field.

Returning to (7), we can conclude that if we *assume* free-field behavior for $e^+e^- \to$ hadrons, we get the constraint

$$\gamma(q^2) \xrightarrow[|q^2| \to \infty]{} \gamma = \mathrm{const}\,. \qquad (8)$$

Once we have determined the pointlike behavior of the irreducible electromagnetic vertex at high q^2, we can use this as an input in all the following calculations.

A final remark: Equation (7) predicts corrections of the order $(q^2)^{\alpha_P - 1}$ to the scaling result, which also imply that the asymptotic limit will be reached from above.

(2) Deep-inelastic scattering. The diagrams to consider are displayed in Fig. 11. Using the rules expounded in Sec. II, we can immediately write

$$A_\alpha = \gamma^2 \frac{\beta}{(2\pi)^3}(q^2)^{\alpha - 1} \int \frac{d^4 l}{(2\pi)^4}\,\delta(\tfrac{1}{4}q^2 + l^2) \times \delta(q\cdot l)\,\mathrm{Tr}(Q^2 f_p)\,, \qquad (9)$$

$$A_\beta = \gamma^2 \frac{\bar{\beta}}{(2\pi)^3}(q^2)^{\alpha - 1} \int \frac{d^4 l}{(2\pi)^4}\,\delta(\tfrac{1}{4}q^2 + l^2) \times \delta(q\cdot l)\,\mathrm{Tr}(Q^2 f_p^*)\,, \qquad (10)$$

where use has been made of the δ functions in the l' variable to do trivially the l' integration. The l integration involves only the forward 6-point scattering amplitude $q\bar{q}p$. According to Regge theory,[12] we have written ($|q^2| \to \infty$)

$$(T_\alpha)^{a'b}_{ab'} \to (2\pi)^3 \beta \delta^{a'}_a (q^2)^\alpha f^b_{p'b'}\left(\frac{-q^2 + \nu - 2p\cdot l}{\nu + 2p\cdot l}\right) \times c\exp\left[-c\,\frac{-q^2(2p\cdot l - \nu - m^2\omega)}{\nu + 2p\cdot l}\right] \times (\delta \text{ functions})\,, \qquad (11)$$

and analogously for T_β. In (11) we have parametrized the cutoff function in the quark transverse momentum by a simple exponential with slope c. It is easy to see that because of the strong damping in the transverse momentum we must have $2p\cdot l \simeq \nu$, and therefore ($\omega = -q^2/2\nu$) in the Bjorken limit

$$(T_\alpha)^{a'b}_{ab'} \underset{\mathrm{Bj}}{\sim} f^b_{p'b'}(\omega)\exp[-c\omega(2p\cdot l - \nu - \omega m^2)]\,. \qquad (12)$$

Therefore the Bjorken variable ω coincides with the Feynman x variable for the $q\bar{q}p$ 6-point function-

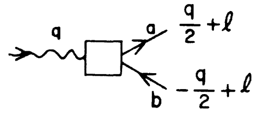

FIG. 7. The basic irreducible electromagnetic vertex.

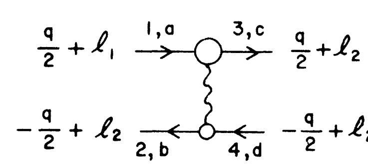

FIG. 8. Imaginary part of the $q\bar{q}$ scattering amplitude given by Regge exchange.

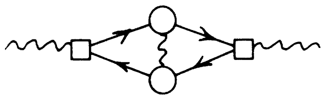

FIG. 9. The relevant diagram contributing to the vacuum-polarization function for $e^+e^- \to$ hadrons.

tion. We believe this to be a nice connection between the variables which describe scaling in weak and strong interactions, respectively.

It is a simple exercise to evaluate (9) and (10) in the Bjorken limit; one gets

$$A_\alpha \underset{\text{Bj}}{\sim} \frac{\beta}{(2\pi)^3}\gamma^2(q^2)^{\alpha-1}\frac{1}{2\nu}\frac{1}{\omega}\,\text{Tr}[Q^2 f_p(\omega)]\,, \qquad (13)$$

$$A_\beta \underset{\text{Bj}}{\sim} \frac{\bar\beta}{(2\pi)^3}\gamma^2(q^2)^{\alpha-1}\frac{1}{2\nu}\frac{1}{\omega}\,\text{Tr}[Q^2 f_p^*(\omega)]\,, \qquad (14)$$

and for the correlation function this implies ($\alpha = 1$)

$$W(q^2,\nu) \sim \frac{1}{2\nu}F(\omega)\,, \qquad (15)$$

where

$$F(\omega) = \frac{\beta}{(2\pi)^3}\gamma^2\frac{1}{\omega}\,\text{Tr}\{Q^2[f_p(\omega)+f_p^*(\omega)]\}\,. \qquad (16)$$

Equation (15) is the canonical scaling law appropriate to scalar currents.[12] So, as a result of this calculation, we have again obtained a connection between scaling and P exchange; we also expect a violation of scaling by terms of order $(q^2)^{-1/2}$, associated with normal Regge exchanges. One should notice that, in contrast with other parton calculations, we do not have the diagrams of Fig. 12, because they are clearly absent in a massive-quark theory (the quark propagator is a constant, and does not have any pole).[10] However, the structure which has emerged can again be put in one-to-one correspondence with these diagrams, whose usefulness is only mnemonic. The connection of this calculation with canonical light-cone expansions is also quite clear, once we realize that in order to obtain the nth moment of the scaling function $F(\omega)$ we must evaluate the diagram in Fig. 13, where O_n is an appropriate kernel. In the approximation stage at which we are working, this diagram corresponds to the evaluation of the matrix element of a local operator $O_{\alpha_1 \cdots \alpha_n}(0)$ defined by its kernel O_n, provided the $P_\perp$ cutoff of the quark lines does not depend on the particular particle the quarks are scattering off. Thus the light-cone operator expansions[13] provide useful constraints on the analysis which led to (13) and (14), and can be regarded as a complementary approach to the one pursued here.

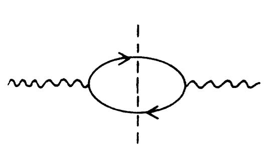

FIG. 10. "Canonical free field" e^+e^- annihilation cross section.

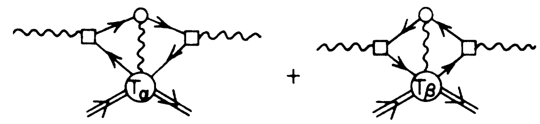

FIG. 11. Relevant diagrams for deep-inelastic scattering.

For $\omega \cong x \to 0$ we are in the central region of the $q\bar{q}p$ scattering amplitude, and expect the double-Regge limit to hold (Fig. 14).

$$T \underset{\omega\to 0}{\sim} \sum_i \beta_{Pi}\gamma_i \frac{(d)^{\alpha_i+1}}{\Gamma(\alpha_i+1)}(-\nu+2p\cdot l)^{\alpha_i} \times \exp[-d\omega(2p\cdot l - \nu - \omega m^2)]\,, \qquad (17)$$

which gives for $F(\omega)$ the Regge behavior

$$F(\omega) \underset{\omega\to 0}{\sim} \sum_i \beta_{Pi}\frac{\gamma_i}{(2\pi)^3}\omega^{-\alpha_i-1}\,. \qquad (18)$$

This is a well-known result, and is consistent with present data.[2,13] In the context of this calculation we can also have an idea for what values of ω we should expect the Regge expression (18) for the scaling function $F(\omega)$ to hold. In fact they should coincide with the values of x which characterize the central region in hadronic scaling. A glance at strong-interaction data seems to suggest that central-region behavior sets in at $x \simeq 0.1$,[14] which fits nicely with the observed onset of the diffractive region in deep-inelastic scattering.[2]

(3) Deep-inelastic annihilation. The process is

$$e^+e^-(q) \to h(p) + \text{anything}\,,$$

and the relevant diagram is given in Fig. 15. This calculation is identical to the previous one. In fact for large q^2 we have the Regge diagrams of Fig. 16, which give

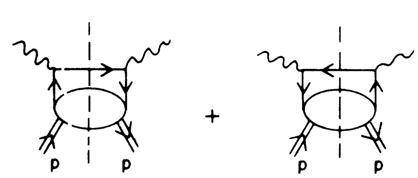

FIG. 12. Diagrams absent in a massive-quark theory.

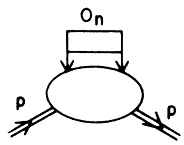

FIG. 13. The matrix element of a local operator defined by the kernel O_n.

$$\overline{T}_\alpha \sim (2\pi)^{-3}\beta\delta_a^{a'}(2\nu)^\alpha \overline{f}^{\,b}_{p,b}\left(\frac{4p\cdot l}{q^2}\right)$$
$$\times\, g\, \exp\left[\frac{-g(\nu+2p\cdot l)(\nu-2p\cdot l)}{q^2}-m^2\right]$$
$$\times(\delta \text{ functions})\,, \qquad (19)$$

and analogously for $\overline{T}_\beta$. From these we easily get for the correlation function of deep-inelastic annihilation the canonical scaling law

$$\overline{W}(q^2\nu)\underset{\text{Bj}}{\sim}\frac{1}{2\nu}\overline{F}(\omega)\,, \qquad (20)$$

where

$$\overline{F}(\omega)=\frac{\beta}{(2\pi)^3}\gamma^2\,\text{Tr}\{Q^2[f_p(1/\omega)+\overline{f}_p^{\,*}(1/\omega)]\}\;. \qquad (21)$$

It is worthwhile noting that if the target and the current's constituents were the same objects (as they usually are in field-theoretic calculations) then we would have gotten

$$\overline{f}(x)=f(x)$$
and
$$c=g \qquad (22)$$

and consequently the reciprocity relation

$$\overline{F}(\omega)=\frac{1}{\omega}F\left(\frac{1}{\omega}\right) \qquad (23)$$

which was first proposed by Gribov and Lipatov.[15] We do not expect this "reciprocity" to hold true in general. However, for ω large, we again enter the central region and we expect double-Regge behavior to set in. The amplitude $\overline{T}_\alpha$ becomes

$$\overline{T}_\alpha \underset{\omega\text{ large}}{\sim}\sum_i \frac{\overline{\beta}}{(2\pi)^3}(-\nu+2p\cdot l)^{\alpha_i}\beta^P_{iP}\frac{(h)^{\alpha_i+1}}{\Gamma(\alpha_i+1)}$$
$$\times\exp\left[-h\left(\frac{\nu-2p\cdot l}{\omega}-m^2\right)\right], \qquad (24)$$

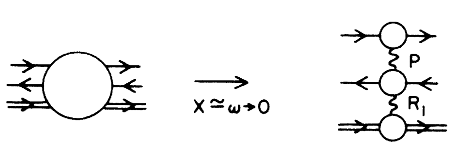

FIG. 14. The double-Regge limit for the $q\overline{q}p$ scattering amplitude.

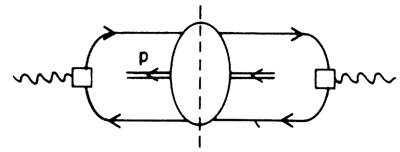

FIG. 15. The relevant diagram for deep-inelastic annihilation.

where h and β^P_{iP} are directly given by one-particle spectra in purely hadronic processes. By integrating (23) we obtain

$$\overline{F}(\omega)\underset{\omega\text{ large}}{\sim}\frac{\beta}{(2\pi)^3}\frac{\overline{\beta}}{(2\pi)^3}\gamma^2\,\text{Tr}(Q^2)\sum_i \beta^P_{iP}\omega^{\alpha_i}\;. \qquad (25)$$

We see immediately that for $i=P$, this leads to a logarithmic growth of the multiplicity of particle p.[16] In addition (24) contains the prediction that the multiplicities will follow the same pattern as in purely strong-interaction physics; i.e., π's will be produced more abundantly than K's and protons, by about the same factor ($\sim 10:1:\frac{1}{2}$). This seems quite reasonable.

Our calculations end here. With only a little more effort one could compute the processes

$$pp\to\mu^+\mu^-+\text{anything}\,,$$
$$ep\to eh+\text{anything}\,,$$

which are of high current interest. There is, however, little merit in doing so here, without taking into account spin and gauge invariance.

In fact the calculations presented in this section serve only the purpose of illustrating the main qualitative features of this approach.

IV. CONCLUSIONS AND OUTLOOK

We have shown that it is possible to obtain the prominent experimental features of scaling in deep-inelastic scattering from a simple "quark" picture, which is however immune from the serious difficulties of interpretation of the popular parton models. Let us summarize the ideas which form the basis of this picture.

(α) The hadronic world can be described by

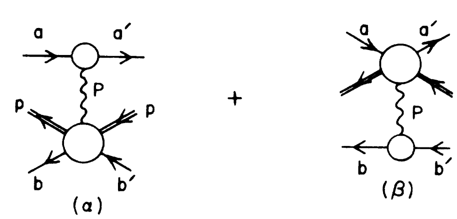

FIG. 16. Regge diagrams for large q^2 contributing to deep-inelastic annihilation.

fields which carry quark quantum numbers.

(β) The quanta associated with quark fields have a very large mass M_q ($M_q \to \infty$ is also conceivable).

(γ) Hadrons are bound states of conglomerates of strongly interacting quarks, carrying zero triality. The interaction of quarks shows saturation properties, and therefore single hadrons can be well described by the conglomerates of the smallest number of quarks ($q\bar{q}$ for mesons, qqq for baryons).

(δ) The quark scattering amplitudes (n-point kernels) display Regge behavior in the appropriate high-energy limits.

The motivation of (α) is evidently the success of SU_3 and the various quark-model descriptions. The reason for (β) is also obvious; it is meant to exorcise strongly interacting light quarks from appearing in ordinary high-energy experiments. The possibility of taking $M_q \to \infty$ is quite attractive, and would dispose forever of the necessity of having real quarks in order to understand the relevance of the degrees of freedom of the quarks.

As for (γ), it associates to any hadron a bound-state wave function in terms of the minimum of quark legs. We believe that this is the most crucial idea of this whole picture. In fact not only does it allow us to account for important features of the hadron spectrum (absence of exotic states, exchange degeneracy, etc.), but it lends support to the idea that hadronic amplitudes exhibit a perturbative structure of some sort. As discussed in Sec. II, we identify the small expansion parameter with the number of quark legs needed for describing a given process.

Finally, (δ) relates Regge behavior as seen in hadron physics to the behavior of quark amplitudes.

Quite obviously this picture is not detailed enough to allow us to compute explicitly the various properties of hadrons and their interactions. However, we have shown that it can explain many facts of deep-inelastic phenomena, and for one thing it provides an intriguing connection between Feynman and Bjorken scaling, Pomeranchukon dominance, and canonical dimensions in deep-inelastic scattering. Deep-inelastic scattering in this picture is in agreement with light-cone operator-product expansions[13]; however, unlike the light cone, it gives us correlations among different processes, e.g., the interesting connection between deep-inelastic annihilation and hadronic inclusive physics. On the other hand, the light-cone approach yields various sum rules which are not identically satisfied in our calculation. In view of this it seems to us that these are two complementary ways to approach the problems of deep-inelastic weak and electromagnetic phenomena.

Another promising aspect of this approach is that it seems to put weak and electromagnetic processes at large mass on the same footing as hadronic processes at large transverse momenta.[16]

We can also systematically investigate scattering processes at high $P_\perp$, such as "deep-inelastic" pp scattering, one-particle distributions at large $P_\perp$, etc., which begin now to be experimentally accessible.

Once this field has been explored, we think we should go back and try to understand the intermediate- and low-energy structure of the quark scattering amplitudes, i.e., the low-energy spectrum, the form factors, and so on. This is admittedly a much harder task, but perhaps the ideas described in Ref. 5 can, suitably modified, be a good start in this direction.[17]

ACKNOWLEDGMENTS

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[1]For the most recent account see H. Kendall, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).

[2]R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969).

[3]An interesting attempt to overcome these difficulties is in S. D. Drell and T. D. Lee, Phys. Rev. D 5, 1738 (1972).

[4]P. V. Landshoff, J. C. Polkinghorne, and R. D. Short, Nucl. Phys. B28, 255 (1971); P. V. Landshoff and J. C. Polkinghorne, *ibid.* B32, 541 (1971); B33, 221 (1971).

[5]These results were obtained in collaboration with S. Coleman and were presented at the 1969 Erice Summer School by the author: G. Preparata, in *Subnuclear Phenomena*, edited by A. Zichichi (Academic, New York, 1970).

[6]The most recent assessment of the situation is in R. K. Adair, Rapporteur's talk at the Sixteenth International Conference in High Energy Physics, Chicago, Ill., 1972 (unpublished). For the recent ISR experiments see M. Bott-Bodenhausen *et al.*, Phys. Letters 40B, 693 (1972).

[7]This view is advocated in a very interesting paper by K. Johnson, Phys. Rev. D 6, 1101 (1972).

[8]In Johnson's paper (Ref. 7) one obtains approximately linear Regge trajectories only for very high t.

[9]The BS equation has a quite long history; for a list of references relevant to the problem at hand see Ref. 3.

[10]This property distinguishes crucially our approach from the one advocated by the Cambridge group (Ref. 4). In fact, owing to the singularity at infinity of our amplitudes, it is not possible to perform the rotation of contours in the complex plane, by which these authors show the vanishing of the diffractive contribution in several cases.

[11]Any amplitude is so defined that every quark leg carries only the square root of the quark propagator. In this way we need not take explicitly into account quark propagators in both formulas and figures. No claim is made as to the validity of the unsubtracted Lehmann-Källén representation for the quark two-point function.

[12]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[13]See, for example, R. Brandt and G. Preparata, lectures given at the 1971 Hamburg Summer School (unpublished) and references therein.

[14]We thank Frank Paige for help in this estimate.

[15]V. N. Gribov and L. N. Lipatov, Phys. Letters 37B, 78 (1971), and report (unpublished); P. M. Fishbane and J. D. Sullivan, Phys. Rev. D 6, 645 (1972).

[16]This has been also noticed, with the mentioned differences, in parton models by P. Landshoff and J. C. Polkinghorne [Nucl. Phys. B33, 221 (1971)] and in an interesting series of papers by R. Blankenbecler, S. J. Brodsky, and J. F. Gunion [Phys. Letters 39B, 649 (1972) and recent SLAC reports (unpublished)].

[17]Recent work has been carried out along similar lines by M. Bohm, H. Joos, and M. Krammer, Nucl. Phys. B51, 397 (1973).

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Multiparticle Partial-Wave Amplitudes and Inelastic Unitarity. II. Analysis of Three-Body Unitarity

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The three-body unitarity equations are analyzed and it is shown that the partial-wave amplitude for three reacting particles almost fixes the form of the partial-wave amplitude for two reacting particles and the production partial-wave amplitude. Further, it is shown that unitarity imposes constraints on the form taken by the three-body partial-wave amplitude. A program is suggested for utilizing these ideas.

I. INTRODUCTION

Of the various model-independent constraints imposed on relativistically invariant scattering amplitudes, unitarity is, in general, one of the most difficult to satisfy. Only at low energies, where two-body or quasi-two-body reactions predominate, is it reasonably clear how to satisfy unitarity constraints. But in those energy regions where (nonresonant) particle production reactions are important the constraints imposed by unitarity become very complicated.

In this paper we will analyze three-body unitarity involving $2\to 2$, $2\to 3$, and $3\to 3$ reactions. (The numbers designate the numbers of reacting particles.) The motivation for such an analysis arises from the pion-nucleon system, where unitarity relates the reactions $NN\to NN$, $NN\to NN\pi$, and $NN\pi\to NN\pi$; but at this stage the three particles under consideration will be spinless and distinguishable in order to focus on the unitarity equations alone. In a later paper, dealing specifically with the pion-nucleon system, spin and other complications will be taken into account.

Thus, we wish to look at a system consisting of three spinless, distinguishable particles, interacting at energies between the three-body and four-body thresholds. The starting point for the analysis is the observation that in this region the unitarity equations form a closed set of nonlinear equations relating the amplitudes $2\to 2$, $2\to 3$, and $3\to 3$. The main problem is how to deal with the $3\to 3$ amplitude, for the corresponding reaction is not experimentally accessible (unless one of the initial particles is viewed as a bound state of two particles) and very little is known about it theoretically. Further, it will be shown that the $3\to 3$ amplitude dominates the unitarity equation in the sense that knowledge of it almost fixes the $2\to 2$ and $2\to 3$ amplitudes.

A related problem has to do with the analysis of experimental data concerning production processes. While a phase-shift analysis is feasible for two-body scattering processes, not only because