A New Theory of Gravity*

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A new relativistic theory of gravity is presented. This theory agrees with all experiments to date. It is a metric theory; it is Lagrangian-based; and it possesses a preferred frame with conformally flat space slices. With an appropriate choice of certain adjustable functions and parameters and of the cosmological model, this theory possesses precisely the same post-Newtonian limit as general relativity.

I. INTRODUCTION

Since 1970, the gravitation research group at Caltech has been analyzing the experimental foundations of relativistic theories of gravity. Our results to date are summarized in Will.[1] Those results had led us to hope that current experiments were good enough to rule out all theories except (i) general relativity and (ii) theories which reduce to general relativity when their adjustable parameters are appropriately adjusted (e.g., the Brans-Dicke-Jordan theory which reduces to general relativity as $\omega\to\infty$). We also had come to hope that general relativity could be distinguished from all other viable metric theories by the form of its post-Newtonian limit (PPN parameter values $\beta=\gamma=1$, $\alpha_1=\alpha_2=\alpha_3=\zeta_1=\zeta_2=\zeta_3=\zeta_4=0$).

The purpose of this paper is to explode our hopes. More particularly, this paper will formulate a new theory of gravity which (for certain values of its adjustable parameters) has precisely the same post-Newtonian limit as general relativity, but can never reduce to general relativity in the full, nonlinear case.

To distinguish experimentally between this new theory and general relativity, one will have to use non-post-Newtonian experiments. These could include: (i) gravitational-wave experiments, (ii) cosmological observations, and (iii) (in the distant future) post-post-Newtonian experiments. The present paper will not discuss such possibilities. Rather, it will merely present the new theory (Sec. II) and compute its post-Newtonian limit (Appendix).

II. PRESENTATION OF THE THEORY

We present the new theory using the notation and format of the author's recent compendium of gravitation theories.[2] (In particular, note that we set $c=G=1$.)

a. Gravitational fields present. A flat background metric $\underline{\eta}=\eta_{ij}\,\underline{dx}^i\otimes\underline{dx}^j$, scalar fields ϕ and t, a one-form field $\underline{\psi}=\psi_i\,\underline{dx}^i$, and the physical metric $\underline{g}=g_{ij}\underline{dx}^i\otimes\underline{dx}^j$.

b. Arbitrary parameters and functions. Three arbitrary functions $f_1(\phi)$, $f_2(\phi)$, $f_3(\phi)$, and one arbitrary parameter e; in the post-Newtonian limit, with appropriate choice of the cosmological model, there are four arbitrary parameters a, b, d, and e (see below).

c. Prior geometry. The following constraints are imposed, *a priori*, on the geometrical relationships between the gravitational fields:

(i) flatness of the metric $\underline{\eta}$

$$(\text{Riemann tensor constructed from }\underline{\eta})=0\;; \tag{1a}$$

(ii) meshing constraints on t, $\underline{\eta}$, and $\underline{\psi}$

$$t_{|ij}=0\;, \tag{1b}$$

$$t_{,i}t_{,j}\eta^{ij}=1 \tag{1c}$$

(here and below a vertical bar denotes a covariant derivative with respect to $\underline{\eta}$, and η^{ij} is the inverse of η_{ij}),

$$t_{,i}\psi_j\eta^{ij}=0\;; \tag{1d}$$

(iii) algebraic equation for the physical metric in terms of the auxiliary gravitational fields $\underline{\eta}$, ϕ, t, $\underline{\psi}$

$$\underline{g}=f_2(\phi)\underline{\eta}+[f_1(\phi)-f_2(\phi)]\underline{dt}\otimes\underline{dt}+\underline{\psi}\otimes\underline{dt}+\underline{dt}\otimes\underline{\psi}\;. \tag{1e}$$

d. Preferred coordinate system. The prior-geometric constraints (1) guarantee the existence of a preferred coordinate system in which (i) the time coordinate is equal to the scalar field t; (ii) the components of $\underline{\eta}$ are Minkowskiian

$$\eta_{ij}=\text{diagonal }(1,-1,-1,-1)\;; \tag{2a}$$

(iii) $\underline{\psi}$ is purely spatial

$$\psi_0=0\;; \tag{2b}$$

(iv) the physical line element $g_{ij}dx^i dx^j$ is

$$ds^2=f_1(\phi)dt^2-f_2(\phi)(dx^2+dy^2+dz^2)+2\psi_1dxdt+2\psi_2dydt+2\psi_3dzdt\;. \tag{2c}$$

e. Lagrangian. The field equations are determined by an action principle

$$\delta \int \mathcal{L}\, d^4x = 0 \,, \tag{3a}$$

where the Lagrangian density $\mathcal{L}$ is

$$\mathcal{L} = L_I \sqrt{-g} + 2\{(1/e)\psi_{i|k}\psi_{j|l}\eta^{ij}\eta^{kl} - \phi_{,i}\phi_{,j}\eta^{ij} + [f_3(\phi)+1](\phi_{,i}t_{,j}\eta^{ij})^2\}\sqrt{-\eta} \,. \tag{3b}$$

Here L_I is the standard interaction Lagrangian obtained by taking the standard Lagrangian for matter and nongravitational fields in flat spacetime, and replacing the Minkowskii metric by $\underline{g}$ (equivalence principle). The quantities g and η are the determinants of $\| g_{ij}\|$ and $\| \eta_{ij}\|$. In the action principle (3a), one is to vary the standard matter and nongravitational fields that appear in L_I, and the gravitational fields ϕ and $\underline{\psi}$, while maintaining the prior-geometric constraints (1). In the preferred coordinate system (2), the Lagrangian density reduces to

$$\mathcal{L} = L_I \sqrt{-g} + (2/e)(\psi_{\alpha,\beta}\psi_{\alpha,\beta} - \psi_{\alpha,t}\psi_{\alpha,t}) + 2\phi_{,\alpha}\phi_{,\alpha} + 2f_3(\phi)\phi_{,t}\phi_{,t} \tag{4}$$

(summation on repeated Greek indices).

f. Field equations. The nongravitational field equations derived from this action principle take on their standard general relativistic form (equivalance principle and comma-goes-to-semicolon rule). The gravitational field equations derived from the action principle are

$$\begin{aligned}\psi_{i|j}{}^{|j} &= 2\pi e(\sqrt{-g}/\sqrt{-\eta}\,)T^{kl}(\partial g_{kl}/\partial\psi_j)(\eta_{ij} - t_{|i}t_{|j}) \,,\\ \phi_{|j}{}^{|j} - [f_3(\phi)+1]\phi_{|i}{}^{|j}t^{|i}t_{|j} + \tfrac{1}{2}f_3{}'(\phi)(\phi_{|j}t^{|j})^2 &= -2\pi(\sqrt{-g}/\sqrt{-\eta}\,)T^{ij}(\partial g_{ij}/\partial\phi) \,.\end{aligned} \tag{5a}$$

In the preferred coordinate system, these equations reduce to

$$\begin{aligned}&\psi_{\beta,\alpha\alpha} - \psi_{\beta,tt} = 4\pi e\sqrt{-g}\; T^{0\beta} \,,\\ &\phi_{,\alpha\alpha} + f_3(\phi)\phi_{,tt} - \tfrac{1}{2}f_3{}'(\phi)\phi_{,t}\phi_{,t} - 2\pi\sqrt{-g}\; T^{ij}(\partial g_{ij}/\partial\phi) = 0 \,.\end{aligned} \tag{5b}$$

Here the stress-energy tensor is the same as appears in the field equations of general relativity:

$$\begin{aligned}T_{ij} &\equiv -\frac{1}{8\pi}\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\; L_I)}{\partial g^{ij}} \,,\\ T^{kl} &\equiv g^{ik}g^{jl}T_{ij} \,.\end{aligned} \tag{6}$$

g. Post-Newtonian limit. Expand the arbitrary functions $f_1(\phi)$, $f_2(\phi)$, and $f_3(\phi)$ in powers of ϕ. In order that the metric will become flat in the absence of gravity ($\phi = \psi = 0$), require $f_1(0) = f_2(0) = 1$. In order that the theory will reduce to Newton's theory in the weak-field slow-motion limit, require $f_1(\phi) = 1 - 2\phi + \cdots$. Define a, b, d to be the coefficients of the first unconstrained terms (post-Newtonian terms) in the expansions

$$f_1(\phi) = 1 - 2\phi + 2b\phi^2 + \cdots \,, \tag{7a}$$

$$f_2(\phi) = 1 + 2a\phi + \cdots \,, \tag{7b}$$

$$f_3(\phi) = d + \cdots \,. \tag{7c}$$

Impose the cosmological boundary conditions

$\underline{\psi} = \phi = 0$ far from the solar system (or whatever other system is being analyzed). (7d)

[Note: The values of $\underline{\psi}$ and ϕ in interstellar space must actually be determined by the cosmological model. This paper makes no attempt at constructing cosmological models. However, it seems that, in order to exhibit large-scale homogeneity and isotropy as viewed from Earth, the cosmological model will have to set $\underline{\psi} \simeq 0$ and $\phi \simeq \phi(t)$ in the neighborhood of the solar system. By a redefinition of ϕ and renormalization of constants, one can then set $\phi \simeq 0$ far from the solar system in the present epoch.] Then the post-Newtonian limit of the theory reduces to the Nordtvedt-Will[3] PPN formalism with PPN parameter values

$$\begin{aligned}&\gamma = a \,, \quad \beta = b \,,\\ &\alpha_1 = -2e - 4a - 4 \,, \quad \alpha_2 = -d - 1 \,,\\ &\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0 \,.\end{aligned} \tag{8}$$

(The proof is given in the Appendix.)

h. Comparison with experiment. By comparing the PPN parameter values [Eq. (8)] with the list of experimental limits on PPN parameters as given by Ni,[4] one learns that *this theory agrees with all experiments to date* if

$0.96 < a < 1.12$ (time-delay experiments) ,

$0.84 < b < 1.34$ (perihelion-shift plus time-delay experiments) , (9)

$-1.03 < d < -0.97$ (Earth-tide measurements) ,

$-2.2 < e + 2a < -1.8$ (Earth-rotation-rate experiments) .

i. Comparison with general relativity. Notice that if

$$a = b = 1 \,, \quad d = -1 \,, \quad e = -4 \,, \tag{10}$$

then this theory has precisely the same post-Newtonian limit as general relativity. Thus, no post-Newtonian experiment can hope to make a clean distinction between this theory and general relativity.

j. Comparison with other Lagrangian-based theories. Will[5] and Ni[6] have shown that all Lagrangian-based metric theories whose post-Newtonian limits can be put into PPN form must satisfy the PPN parameter constraints

$$\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0 . \tag{11}$$

Notice that the theory presented here has arbitrary values for all the remaining, unconstrained parameters. Thus, this theory possesses a most general post-Newtonian limit permitted for Lagrangian-based metric theories.[7] This means that no post-Newtonian experiment can hope to make a clean distinction between this theory and *any* other Lagrangian-based metric theory which has PPN form post-Newtonian limit.

k. Special cases. When the arbitrary functions $f_1(\phi)$, $f_2(\phi)$, and $f_3(\phi)$ are suitably specialized, one obtains the following theories: Papapetrou II [see Sec. III.D.vi of Ref. 2], and Ni's Lagrangian-based, stratified theory [see Sec. III.D.vii of Ref. 2].

l. Conservation laws and gravitational radiation. Global conservational laws and gravitational radiation for this theory will be discussed in a future paper.

III. CONCLUDING REMARKS

This theory requires considerable further study. Crucial items in testing it will be (i) its success or failure to produce cosmological models that agree with the large-scale features of our universe, and (ii) the properties (polarization, intensity, and propagation speed) of its gravitational waves.

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APPENDIX: COMPUTATION OF THE POST-NEWTONIAN LIMIT

To obtain the post-Newtonian limit of our theory, we proceed as follows. For convenience, we shall work in the preferred coordinate system, and we shall set $\phi \to 0$ and $\underline{\psi} \to 0$ as the field point $|\underline{x}|$ goes to infinity [see remarks following Eq. (7d)]. Let

$$\begin{aligned} \phi &= \phi_1 + \phi_2 + O(6) , \\ \psi_\beta &= \psi_{\beta 2} + O(5) , \end{aligned} \tag{A1}$$

where $\phi_1 = O(2)$, $\phi_2 = O(4)$, and $\psi_{\beta 2} = O(3)$. [Here $O(n)$ means of order c^{-n} in a post-Newtonian expansion.] Correct to post-Newtonian order, the field equations (5b) are

$$\begin{aligned} -d \frac{\partial^2 \phi_1}{\partial t^2} - \nabla^2 \phi_1 - \nabla^2 \phi_2 &= 4\pi\rho[1 + (3a - 1)U] \\ &\quad \times [1 + (2 - 2b)U + v^2(1 + a) + 3ap/\rho + \Pi] , \\ \nabla^2 \psi_{\beta 2} &= 4\pi e \rho v_\beta . \end{aligned} \tag{A2}$$

The $O(2)$ part of the field equations is

$$\nabla^2 \phi_1 = -4\pi\rho , \tag{A3}$$

i.e.,

$$\phi_1 = U , \tag{A4}$$

where U is the Newtonian potential. The $O(3)$ part is

$$\nabla^2 \psi_{\beta 2} = 4\pi e \rho v_\beta , \tag{A5}$$

i.e.,

$$\psi_{\beta 2} = -e V_\beta = -e \int \frac{\rho(\vec{x}', t) v_\beta(\vec{x}', t) d\vec{x}'}{|\vec{x} - \vec{x}'|} . \tag{A6}$$

The $O(4)$ part is

$$\nabla^2 \phi_2 = -dU_{,tt} - 4\pi\rho[(3a + 1 - 2b)U + (1 + a)v^2 + 3ap/\rho + \Pi] . \tag{A7}$$

Let χ be the solution of

$$\nabla^2 \chi = -2U , \tag{A8}$$

i.e.,

$$\chi = -\int \rho |\vec{x} - \vec{x}'| \, d\vec{x}' . \tag{A9}$$

We can transform Eq. (A7) to

$$\nabla^2(\phi_2 - \tfrac{1}{2} d\chi_{,tt}) = -4\pi\rho[(3a + 1 - 2b)U + (1 + a)v^2 + 3ap/\rho + \Pi] . \tag{A10}$$

Therefore

$$\phi_2 = \tfrac{1}{2} d\chi_{,tt} + 2\Phi , \tag{A11}$$

where

$$\nabla^2 \Phi = -4\pi\rho[\tfrac{1}{2}\Pi + \tfrac{1}{2}(3a + 1 - 2b)U + \tfrac{1}{2}(1 + a)U + \tfrac{3}{2}ap/\rho] . \tag{A12}$$

Combining Eqs. (A1), (A4), and (A11), we find

$$\phi = U + 2\Phi + \tfrac{1}{2} d\chi_{,tt} + O(6) . \tag{A13}$$

According to Eqs. (2c), (A6), and (A13), the physical metric is

$$g_{00}=1-2U+2bU^2-4\Phi-d\chi_{,tt}+O(6)\,,$$
$$g_{0\alpha}=-eV_\alpha\,, \qquad \text{(A14)}$$
$$g_{\alpha\beta}=-\delta_{\alpha\beta}(1+2aU)\,.$$

By using the gauge transformation

$$x^{0\dagger}=x^0-\tfrac{1}{2}d\chi_{,t}\,,$$
$$x^{\alpha\dagger}=x^\alpha\,, \qquad \text{(A15)}$$

we can transform the metric into the form

$$g^\dagger_{00}=1-2U+2bU^2-4\Phi+O(6)\,,$$
$$g^\dagger_{0\alpha}=(\tfrac{1}{2}d-e)V_\alpha-\tfrac{1}{2}dW_\alpha+O(5)\,, \qquad \text{(A16)}$$
$$g^\dagger_{\alpha\beta}=-(1+2aU)\,,$$

where

$$W_\alpha(\vec{x},t)=\int\frac{\rho(\vec{x}',t)v_\beta(\vec{x}')(x_\beta-x'_\beta)(x_\alpha-x'_\alpha)}{|\vec{x}-\vec{x}'|^3}d\vec{x}'\,. \qquad \text{(A17)}$$

By comparing this with the PPN metric as given by Will and Nordtvedt,[8] we obtain the PPN parameter values listed in Eq. (8).

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[4]Reference 2.

[5]C. M. Will, Astrophys. J. 169, 125 (1971).

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[7]*Exception*: One could conceive of Lagrangian-based metric theories with post-Newtonian limits that are more complex than the Nordvedt-Will 9-parameter formalism. Our results do not apply to such theories.

[8]Reference 3.