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Dispersion of Gravitational Waves by a Collisionless Gas*

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Gravitational waves in a hot collisionless gas do not propagate on null geodesics. Using the methods of plasma kinetic theory, we derive dispersion relations from a self-consistent, first-order perturbation expansion of Einstein's, Boltzmann's, and the geodesic "force" equations. The wave's interaction with the higher shear moments of the ensemble introduces new length scales into the dispersion relations, which can affect the familiar modes and allow new ones. For an isotropic photon gas, the strongest mode has the dispersion relation $\omega^2 \approx c^2k^2 - 144\pi GP/5c^2$, where P is the gas pressure. Because the length scale $c^2/(GP)^{1/2}$ is very large (10^{30} cm for the 3 °K cosmic radiation), this retarded wave is practically indistinguishable from the vacuum solution. We also find some weak, novel effects, like collisionless amplification and evanescent wave modes, which are very sensitive to the particles' momentum distribution.

I. INTRODUCTION

There are no truly free-space gravitational waves. The universe is full of things such as the cosmic 3 °K photons, so wavelike solutions of Einstein's equations in the absence of sources may be a good approximation, but they may also miss interesting real-world modes. Such enriched behavior is well known for electromagnetic waves in ionized plasmas. We will apply the methods of plasma kinetic theory to solve for gravitational wave modes in a collisionless gas.

Although only a linearized perturbation calculation, this explicitly self-consistent source or field approach is rare in general relativity, where the complexity of the equations tempts one to simply specify either the source or field and then solve for the other. For instance, Zipoy,[1] Sachs and Wolfe,[2] and Dautcourt[3] have each estimated the scattering of photons by specified metric fluctuations. Esposito[4] has examined dissipation in viscous fluids driven by gravitational waves. Issacson[5] has used self-consistent fields to look at the scattering of gravitational waves in a vacuum, with the nonlinear part of the field as an effective source. Weber[6] has made a genuinely self-consistent calculation for the absorption of gravitational waves inside elastic solids, concluding that planets have very small cross sections, as has Dyson.[7]

The covariant treatment of particle ensembles in curved space is well reviewed by Stewart,[8] who is concerned with collisional transfer processes. Collision-dominated fluids, however, do not interact well with the shear in gravitational waves. Finally, McCone[9] has investigated self-consistent, longitudinal gravitational fields associated with condensations in a collisionless gas, using a kinetic-theory approach to follow the field-particle interactions. He recovered a generalized Jeans's instability.

This paper derives the dispersion relations for short, weak, traceless, transverse, plane gravitational waves which are self-consistent with ensembles of collisionless particles following geodesics.

II. GRAVITATIONAL KINETIC THEORY

The basic method of kinetic theory is to use the Liouville or Boltzmann equation to follow the wave-induced perturbations in the source distribution, and then substitute these back into the field or source equations, making everything self-consistent to, say, first order. Following Weber's geometrized ($G=1=c$) notation,[10] the linearized form of Einstein's field or source equations is the classic wave equation

$$\Box h_{mn} = -16\pi T_{mn}(h), \tag{1.1}$$

where the stresses are the second moments of the perturbed particle distribution function $f(h)$, defined by

$$T^{\mu\nu}(h) \equiv \iiint f(h)p^\mu p^\nu \frac{dp^1dp^2dp^3}{p^0}, \tag{1.2}$$

consistent with the weak, transverse, traceless, waves h_{mn} traveling in the x^1 direction (m and n $=2$ or 3) on a locally Minkowski metric:

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{mn}, \quad \text{with } |h_{mn}| \ll 1. \tag{1.3}$$

The gas is represented by a single-particle distribution function $\mathfrak{F}$, evolving along a world tube of phase-space geodesics according to the collisionless Boltzmann equation

$$0=\left(\frac{d\mathfrak{F}}{d\tau}\right)_{\text{coll}}$$
$$=\frac{dx^\alpha}{d\tau}\frac{\partial\mathfrak{F}}{\partial x^\alpha}+\frac{du^j}{d\tau}\frac{\partial\mathfrak{F}}{\partial u^j}$$
$$\approx u^\alpha\frac{\partial f}{\partial x^\alpha}+\dot{u}^j\frac{\partial F}{\partial u^j}\;, \tag{1.4}$$

where $f(h)\propto h$ is the perturbed part of a homogeneous, static background ensemble F. The mass-shell constraint ($m^2=p^\alpha p_\alpha$) allows us to choose the spacelike velocities (u^j) as independent variables. In the absence of other forces, the acceleration of a particle in this flat space is the geodesic "force"

$$\dot{u}^\alpha=-u^\beta u^\gamma\Gamma^\alpha_{\beta\gamma}(h). \tag{1.5}$$

The Boltzmann and geodesic equations (1.4) and (1.5) can be solved for the Fourier transform of the perturbed 4-momentum distribution

$$f(h(\omega,k))\approx\frac{p^\alpha p^\beta}{p^a k_a}\frac{\partial F}{\partial p^j}G^j_{\alpha\beta}(h(\omega,k))\,, \tag{1.6}$$

where the $G_{\alpha\beta\gamma}\neq 0$ only for $G_{amn}=-G_{man}=-G_{mna}$ ($a=0$ or 1, as in $k^0=\omega$ and $k^1=k$), and are defined by

$$G_{amn}=-\tfrac{1}{2}k_a h_{mn}(\omega,k)\;. \tag{1.7}$$

The Fourier transform of the perturbed stresses in Eq. (1.2) is then found from Eqs. (1.6) and (1.7), and substituted into the linearized Einstein equations (1.1) to give the self-consistent wave equation

$$(\omega^2-k^2)h_{mn}=-16\pi h^{rs}\iiint p_m p_n p_r\left(\frac{\partial F}{\partial p^s}-\frac{1}{2}\frac{p_s k^1}{p^a k_a}\frac{\partial F}{\partial p^1}\right)\times\frac{dp^1dp^2dp^3}{p^0}. \tag{1.8}$$

If we call the integrals over the background distribution $J_{mnrs}(F;\omega,k)$, Eq. (1.8) can be rearranged to display the dielectriclike tensor response of particle-inhabited space to a gravitational wave:

$$[(\omega^2-k^2)\delta_{mr}\delta_{ns}+16\pi J_{mnrs}]h^{rs}\equiv D_{mnrs}h^{rs}=0. \tag{1.9}$$

The zeros of the determinant $D(F;\omega,k)$ are poles in the (ω,k) plane, so they are conventionally regarded as the normal modes of the medium's response to the wave. Equation (1.8) displays the important characteristics of this wave-particle coupling: (i) It is to the higher shear moments of the ensemble, so that cold gases cannot interact; (ii) it depends on gradients in the distribution of momentum, so that the sign of the coupling can be influenced by streams; (iii) part of the coupling is independent of the wave's phase velocity, so that it "stirs" the whole ensemble; and (iv) part of the coupling is by resonance between wave-phase velocity and parallel-particle velocity:

$$p^a k_a=p^0(v^1-\omega/k). \tag{1.10}$$

The inability of any particle to resonate with phase velocities faster than light introduces a branch point at $\omega/k=\pm 1$ in D. We will find that the branch cuts must be carefully considered in order to estimate the strength of the zeros of D correctly in relativistic kinetic theory.

III. THE ISOTROPIC CASE

The dielectric equation (1.9) indicates that even a homogeneous particle ensemble can entangle orthogonal linearly polarized waves by momentum anisotropies. In fact, the simplest distribution to consider is isotropic photons, since all normally independent components decouple and momenta are scaled by the energy p^0. Then, the zeros of the dispersion function are solutions of

$$0=D_{\text{isot}}=(\omega^2-k^2)\left[1+3\pi\frac{P}{k^2}\int_{C\,-1}^{1}\left(\frac{1-u^2}{u-\omega/k}\right)^2du\right]+\tfrac{1}{5}\,144\pi P, \tag{2.1}$$

where P is the geometric ($G=1=c$) radiation pressure in units of inverse length or inverse time, $u=p^1/p^0$ is the pitch angle to the wave normal, and C is a u-space contour best chosen to enclose the resonance pole at ω/k above the real u axis between the branch points at ±1. For this choice, the integral is

$$\int_{C\,-1}^{1}\left(\frac{1-u^2}{u-\omega/k}\right)^2du=\frac{8}{3}+4\left(\frac{\omega^2}{k^2}-1\right)\left[2+\ln\left(\frac{\omega-k}{\omega+k}\right)\right]-8\pi i\frac{\omega}{k}\left(\frac{\omega^2}{k^2}-1\right)H_C(\omega/k), \tag{2.2}$$

where $H_C(\omega/k)$ analytically continues the resonance pole, being equal to unity between $\omega/k=\pm1$ for

$\mathrm{Im}(\omega/k) \geqslant 0$, and zero otherwise. This gives D_{isot} vertical branch cuts in the ω/k plane, shown in Fig. 1.

There are two kinds of wave modes associated with zeros of Eq. (2.1). First, there are the near-vacuum dispersion relations, coming when the real and imaginary parts of the pitch-angle integral are small:

$$(\omega/k)_{\mathrm{vac}} \approx \pm(1 - 72\pi P/5k^2) + \tfrac{1}{25}\, i\, 9\pi^4 (24P/k^2)^3 . \tag{2.3}$$

These gravitational wave modes are slightly slower than light and are very slightly damped, depending on the ratio of wavelength to pressure scale. The physical damping mechanism is something like collisionless Landau damping of electrostatic waves in an ionized plasma – the wave resonates with more particles of low projected momentum, transferring energy to them.

The second kind of dispersion relation occurs when the resonant pole in the pitch-angle integral of Eq. (2.2) is large:

$$(\omega/k)_{\mathrm{res}} \approx i(24\pi^2 P/k^2)^{-1/3} \gg i \; . \tag{2.4}$$

This is a nonpropagating, rapidly evanescent mode, novel to hot particle universes. The noncausal damping rate of this transient suggests that the physical mechanism is something like phase mixing – some of a wave's initial coherence is lost because the thermal motions of particles all along the wave train are a source of noise. Indeed, the amplitude of the transient is very low, of the order of $(P/k^2)^{1/3}$ if we ignore the branch cuts, and of (P/k^2) if we include their influence.

The task of evaluating the contribution from the branch cuts in the dispersion function D for relativistic kinetic theory is delicate, because the zeros of D are usually more damped in time than the branch points. In addition, the arbitrary orientation of the cuts can influence the very existence of zeros of D. For example, choosing them to join $\omega/k = \pm 1$ in Fig. 1 defines a D_{isot} with no zeros at all – the physics becomes entirely hidden in the behavior of D_{isot} along the cuts. We chose vertical branch cuts in the (ω/k) plane because their contribution was then small and in a form to be subtracted from the poles' amplitudes.[11]

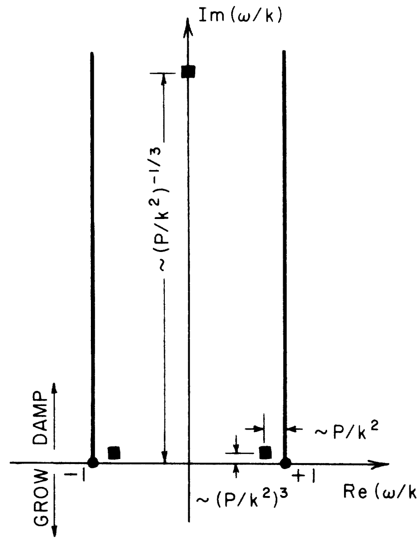

FIG. 1. The branch cuts and poles of $D(F;\omega,k)$ in the ω/k plane for an isotropic photon gas. The dependences on the small pressure-scale/wavelength ratio P/k^2 are characteristic.

IV. AN ANISOTROPIC CASE

In general, anisotropic media entangle the field components of the wave equation (1.8), making it hard to solve. However, if one has gravitational wave propagating along an azimuthally symmetric anisotropy, then the polarizations decouple nicely. An example of cosmological interest is Misner's mixmaster universe,[12] where a once-isotropic radiation field can be expanded or compressed collisionlessly to give a pitch-angle-dependent temperature $T(u)$:

$$F = \{\exp[p^0/T(u)] - 1\}^{-1} , \tag{3.1}$$

where

$$T(u) = \frac{T_\perp}{(1 + Au^2)^{1/2}} , \qquad -1 \leqslant A \equiv \frac{T_\perp{}^2 - T_\parallel{}^2}{T_\parallel{}^2} \leqslant \infty . \tag{3.2}$$

The dispersion function is then structurally similar to the isotropic case in Eq. (2.1):

$$D_{\mathrm{anis}} = (\omega^2 - k^2)\left[1 + 6\pi \frac{P}{k^2} \frac{\mathcal{C}\!\int_{-1}^{1} du\,[(1-u^2)/(u-\omega/k)]^2/(1+Au^2)^2}{\int_{-1}^{1} du/(1+Au^2)^2}\right] + 6\pi P \frac{\int_{-1}^{1} du\,(1-u^2)(u^2+7)/(1+Au^2)^2}{\int_{-1}^{1} du/(1+Au^2)^2} . \tag{3.3}$$

Here, P is the average radiation pressure. The integrals in Eq. (3.3) are complicated functions:

$$I(A)\equiv \tfrac{1}{2}\int_{-1}^{1} du/(1+Au^2)=\begin{cases}\dfrac{\arctan A^{1/2}}{A^{1/2}}\ , \ \text{if } A\geqslant 0\\[2ex] \dfrac{1}{2\,|A|^{1/2}}\ln\left(\dfrac{1+|A|^{1/2}}{1-|A|^{1/2}}\right),\ \text{if } A\leqslant 0\ ,\end{cases} \tag{3.4a}$$

$$\int_{-1}^{1} du/(1+Au^2)^2=I(A)+1/(1+A)\ , \tag{3.4b}$$

$$\int_{-1}^{1} du\,(1-u^2)(7+u^2)/(1+Au^2)^2=[7A-3+(7A^2-6A+3)\,I(A)]/A^2, \tag{3.4c}$$

$$\begin{aligned}\int_{C-1}^{1} du[(1-u^2)/(u-\beta)]^2/(1+Au^2)^2 &= \frac{2(\beta^2-1)}{(1+A\beta^2)^2}+\frac{4(1+A)(\beta^2-1)}{(1+A\beta^2)^2}\left[\beta\ln\frac{\beta-1}{\beta+1}+(1-A\beta^2)I(A)\right]\\ &\quad+\frac{(1+A)^2I(A)}{A(1+A\beta^2)}-\frac{(1+A)(1-A\beta^2)}{A(1+A\beta^2)^2}-\frac{4\pi i(1+A)\beta(\beta^2-1)}{(1+A\beta^2)^3}\,H_C(\beta)\ ,\end{aligned} \tag{3.4d}$$

where $\beta=\omega/k$ is the phase velocity. So, even for this relatively simple anisotropic case, it is hard to find the wave modes represented by the zeros of D_{anis}.

Table I lists the approximate dispersion relations in several limiting situations. For very small anisotropy, all modes are essentially the same as for the strictly isotropic case in Eqs. (2.3) and (2.4). For moderate positive anisotropy ($T_\perp \gtrsim T_\parallel$), the near-vacuum modes are unaffected, but the evanescent mode proceeds at a much slower rate, determined by the anisotropy. The amplitude of this relaxation mode is still weak – it is of the order of P/k^2, when the contributions from the branch cuts are included.

For strong positive anisotropy ($T_\perp \gg T_\parallel$), the vacuum mode is still retarded by a fraction of the order of P/k^2, but the collisionless damping is faded. Physically, this is due to the fact that the retarded wave finds very little parallel momentum to resonate with when $T_\parallel \ll T_\perp$. For $A\to\infty$, the evanescent mode becomes very slow, a fact of obscure physical significance. The amplitude of these transient modes was not calculated for large anisotropy because the contribution from the branch cuts is then difficult to estimate.

For strong negative anisotropy ($T_\perp \ll T_\parallel$), the relaxation mode is very fast, and the interaction terms in the vacuum mode vanish. Physically, this is due to the wave coupling into the transverse momentum in Eq. (1.8), so that all interactions vanish along with $T_\perp$.

We can conclude from our analysis of all these anisotropic cases that, in general, the vacuum modes are fractionally retarded by P/k^2 and collisionlessly damped by $(P/k^2)^3$, while the evanescent modes are sensitive to the details of the distribution, but have low amplitude.

V. DISCUSSION

We found that the imaginary part of the vacuum modes was understandable in terms of a retarded wave resonating with the longitudinal gradient in

TABLE I. There are two kinds of dispersion relations for a gravitational wave traveling along the symmetry axis of an anisotropic photon gas characterized by two temperatures, $T_\perp$ and $T_\parallel$. The near-vacuum modes are only weakly affected by particle anisotropy, while the nonpropagating modes are very sensitive.

Anisotropy $A\equiv(T_\perp{}^2-T_\parallel{}^2)/T_\parallel{}^2$	"Vacuum" mode dispersion relation	"Evanescent" mode dispersion relation
$-1\lesssim A$ or $T_\perp \ll T_\parallel$	$\frac{\omega}{k}\simeq\pm[1-216\pi(1+A)P/k^2]+i2^{12}3^7\pi^4\frac{(P/k^2)^3}{-\ln[\frac{1}{2}(1+A)]}$ $\to\pm1$	$\frac{\omega}{k}\simeq i\frac{-\ln[\frac{1}{2}(1+A)]}{96\pi^2(1+A)(P/k^2)}\to i\infty$
$\lvert A\rvert^{3/2}\ll P/k^2\ll 1$ or $T_\perp\approx T_\parallel$	$\frac{\omega}{k}\simeq\pm\left(1+\frac{72\pi}{5}\frac{P}{k^2}\right)+i\frac{2^9 3^5\pi^4}{5^2}\left(\frac{P}{k^2}\right)^3$	$\frac{\omega}{k}\simeq\frac{i}{(24\pi^2P/k^2)^{1/3}}\gg i$
$(P/k^2)^{2/3}\ll A\ll(P/k^2)^{-1}$ or $T_\perp\gtrsim T_\parallel$	Same as above	$\frac{\omega}{k}\simeq\frac{i}{A^{1/2}}$
$A\gg(P/k^2)^{-1}$ or $T_\perp\gg T_\parallel$	$\frac{\omega}{k}\simeq\pm[1-21\pi P/k^2]+i2^4 3^3 7^2\pi^4\left(\frac{P}{k^2A^{1/2}}\right)^3$	$\frac{\omega}{k}\simeq\frac{i}{24\pi^2A^{3/2}P/k^2}\to 0$

the momentum, $\partial F/\partial v_{\parallel}$ at $v_{\parallel} \equiv p^1/p^0 = \omega/k \simeq \pm(1-P/k^2)$ in Eq. (1.8). So, we get collisionless damping if $\partial F/\partial v_{\parallel}|_{\omega/k} < 0$, and could get collisionless amplification of gravitational waves if $\partial F/\partial v_{\parallel}|_{\omega/k} > 0$. Ordinarily, such amplification should be negligible, since the streaming must be severe to have many particles faster than the slightly $(P/k^2 \ll 1)$ retarded wave, and the secular rate is always slow $[\lesssim (P/k^2)^3]$.

All of the interaction effects are scaled by P/k^2, so one naturally wants to look at long-wave or high-pressure situations in order to speculate on the largest possible effects. For example, the 3°K cosmic radiation has a pressure scale $P = GP/c^4 \approx (10^{30}\text{ cm})^{-2}$, larger than the Hubble scale $c/H \approx 10^{28}$ cm. Likewise, the degeneracy pressure inside of a neutron star gives $P \approx (10^8\text{ cm})^{-2}$, which is larger than the 10^6-cm object. It is, of course, no accident that objects have the scale hierarchy: Schwarzschild radius $\leqslant$ physical radius $\leqslant$ radius of space-time curvature due to density $\leqslant$ radius of space-time curvature due to pressure. Consequently, the term P/k^2 is small when there is more than one wavelength inside of an object.

Indeed, the perturbation analysis developed here has concentrated on that part of the dispersion due to wave-particle coupling. There are two other important sources of deviation from flat-space, null-geodesic motion which have been ignored: the background space-time curvature, and the nonlinear action of the wave. Since some of the background radii of curvature are $\leqslant P^{-1/2}$, these terms in a correct curved-space wave operator on the left-hand side of Eq. (1.1) are at least as large as the interaction terms of the right-hand side. Likewise, terms like k^2h^2 have been dropped. Now, observable waves might carry a strain of $h \approx 10^{-17}$, which is much larger than, say, $P/k^2 \approx 10^{-46}$ for kilohertz waves dispersed by the 3°K radiation. Thus, both deviations can be larger than P/k^2. They are familiar terms,[5] representing both deviations from null geodesics and conceptual errors in the use of a separable, flat background, while the P/k^2 terms represent the error in the use of sources separate from the wave. Although such a general perturbation analysis, mixing all of these scales, is beyond the scope of this paper, we expect that the effects will add independently in the short-wave limit.

The analysis here was for photon ensembles because their simple momentum-energy relationship make the stress integrals easy to do. Dust ensembles have been analyzed and found less effective, since massive particles have less momentum than photons of the same total energy.

Very general considerations indicate that one will find strongly dispersed modes in a medium that has some short characteristic scale that can interact with the wave. For instance, the cyclotron frequency is an important scale in plasma physics. Unfortunately, the magnetic field analog in general relativity is weak in ordinary situations. A high collision rate could be introduced into the Boltzmann equation (1.4) to give a natural short scale, but, since collisions decrease the shearing ability of the ensemble, we expect collisions to decrease the net wave-particle interaction, although they might increase dissipation.

Analogies between light waves in an ionized plasma and gravitational waves in gas cannot be pushed too far. For instance, while the pressure P introduces a gravitational scale analogous to the density-dependent electron plasma frequency, the signs on these quantities are such that the phase velocity of lightlike modes are greater than c, making resonant interactions impossible, while gravitational waves have near-vacuum modes at less than c, allowing collisionless secular effects. This seems due to the fact that electrons are electrically repulsive but gravitationally attractive.

Two of the barriers to considering gravitational waves in more complex media have already been mentioned: (i) The several field components in Eq. (1.8) are generally coupled, making the dispersion function a large determinant of terms like Eq. (1.9); and (ii) the interaction integrals, called J_{mnrs} in Eqs. (1.8) and (1.9), are generally intractable. Another constraint, not previously mentioned, is the need to keep the theory gauge-invariant. For the chosen form of metric fluctuations in Eq. (1.3), we tailored the ensemble to this coordinate system by restricting it to be homogeneous and in the "center of momentum," so that $T^{ab}(h) = 0$ was satisfied in the wave equation (1.1). More general gauge formulations can be made, of course, but the physics then gets lost in the notation.

Naturally, one could apply a multitude of kinetic-theory techniques to the coupled Einstein and Liouville equations to derive the effects of particle correlations, wave-wave coupling, inhomogeneities, etc. However, these are unpromising and difficult calculations, due to the weakness of the coupling and to the many nonlinear terms.

Finally, the retarding pressure term in the dispersion relation for the vacuum modes has the same effect as assigning an effective mass or a long-wave cutoff to the graviton. This probably has no significance for the fundamental problem of formulating a quantum theory of gravity,[13] but it may be a useful concept for heuristic models of the interaction of gravitational waves with, say, phonons inside of a neutron star.

VI. CONCLUSIONS

A gravitational wave in a collisionless gas has the familiar vacuum modes retarded, allowing resonant interactions with the particles. New evanescent modes, sensitive to the details of the ensemble's momentum distribution, are also possible. However, the strengths of the interactions depend on the ratio of wavelength to pressure, GP/c^4k^2. This is so small that the modes are indistinguishable from the vacuum case in practical situations. Once again, the fundamental weakness of the gravitational coupling constant keeps life simple and uninteresting.

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*Based on a Ph.D. thesis done in the Astronomy Program of the Physics Department of the University of Maryland at College Park.

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Correlating Arrows of Time*

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A thought experiment and some model systems are studied in an effort to examine the relation between the cosmological and thermodynamic arrows of time. Some doubt is cast on the hypothesis that the latter must follow the former.

A discussion of the physical problems associated with the arrow of time is given by Gold.[1,2] We here wish to make only the brief remark that a physical "solution" to these problems may take one of the following forms: (1) proof that various arrows are correlated, e.g., that the thermodynamic arrow must follow the expansion of the universe (the cosmological arrow); (2) proof that some arrow arises as a spontaneous symmetry-breaking effect, a catastrophe.[3,4]

This article is confined to analyzing a thought experiment and some models which are supposed to correlate the cosmological arrow with the thermodynamic arrow. The abbreviations AT and AC stand, respectively, for the thermodynamic arrow and the cosmological arrow. AC points in the direction of the expansion of the universe. AT points towards increasing entropy. That AT and AC are well defined is an experimental fact.

The following thought experiment has been used[1] to demonstrate that AT follows AC. A star is put in an insulating box and in due course comes to thermal equilibrium. Motion pictures or other records of occurrences inside the box now show no arrow (AT is lost in equilibrium). A window is then opened for some period of time and radiation is allowed to escape. It escapes because the universe outside the box is cooler than the star. This in turn follows (as in resolutions of Olber's paradox) from AC. After the window is closed, the