Numerical Solutions to Two-Body Problems in Classical Electrodynamics: Straight-Line Motion with Retarded Fields and No Radiation Reaction*

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Trajectories of two charged particles of equal mass have been calculated for straight-line motion using retarded fields without radiation reaction. Both attractive and repulsive interactions are considered. For the attractive case, in which particles are released from rest with various initial separations d, the acceleration is shown to approach $-\sqrt{2}$ (in natural units) as the particles approach each other, and oscillations which are significant for $d \lesssim 1$ are strongly damped for $d \gg 1$. For the repulsive case, in which the particles are thrown directly at one another with high initial velocity, no lower limit is found for the distance of closest approach as initial velocities are increased. Furthermore, the particle energy is found to *increase* during collision. We explain this result in terms of an energy-conservation theorem and discuss its significance with regard to the omission of radiation reaction.

I. INTRODUCTION

Basic difficulties in quantum field theory are related to problems of self-energies and of interactions among elementary particles and between particles and fields. One naturally looks to the simplest classical problems for solutions which may shed light on the quantal difficulties.

We consider here the problem of two charged point particles of equal mass interacting in straight-line motion by means of retarded Liénard-Wiechert potentials when either (i) the particles have opposite charges and attract one another when released from rest at an initial separation d, or (ii) the particles have like charges and are thrown at each other with initial velocities $dx_1(-\infty)/dt_1$ from a large initial separation. Even this relatively simple problem has, till now, not been fully solved. One of the difficulties has been to solve differential equations involving both present and retarded times. We have studied such problems for the scalar field[1] and apply them here to the electromagnetic case.

Synge[2] has considered analytic solutions to the two-body problem and has applied a method of successive approximation, valid when the masses of the interacting particles are greatly different, to the case of nearly circular orbits. He finds a gradual loss of mechanical energy, a loss however much smaller than that predicted when radiation reaction is included.

The existence and uniqueness of solutions to the two-body problem with retarded interactions has been discussed by Driver.[3] When the past history of the particles is specified, as we have done, the problem of future trajectories is well-posed and has a unique solution. For some two-body problems with straight-line motion, Driver[3] has shown that the assumption of retarded electrodynamic interaction at all past times implies a unique solution for each set of *instantaneous* position and velocity values. In our treatment of cases (i) and (ii) [see above], the past histories specified become consistent with the retarded electrodynamic interactions in the limit of large initial separation, and we have increased the initial separations used until no further difference appeared in the qualitative behavior of the collision.

Case (i) has recently been treated numerically by Kasher and Schwebel.[4] They used initial separations $d \leq \frac{10}{3} e^2/m$ (we use units with $c=1$), i.e., they did not consider the problem for initial separations larger than $\frac{10}{3}$ classical charge radii. After applying an initial analytical starting solution, they followed the particles in toward the origin with a rather coarsely meshed numerical integration. We have repeated their calculations obtaining higher accuracy, and have extended the results to much larger, physically more meaningful starting separations. The deviation from their results is significant.

Case (ii) has been solved for half-retarded plus half-advanced fields by Andersen and von Baeyer,[5] who used a method of successive approximation to calculate trajectories for particles with initial velocities up to 0.95. At higher velocities their iteration method no longer converged. They suggested that the distance of closest approach $2x_1(0)$ of the particles might not decrease monotonically with increasing initial velocities, but rather might have a minimum value of about $2x_1(0) \simeq 0.9$. We have found some striking differences for the retarded-field results.

Our calculations include no radiation reaction. Computational difficulties with the Lorentz-Dirac[6] treatment of radiation reaction are well known.[7]

In some formulations of classical electrodynamics[8,9] either radiation reaction does not exist or its existence depends on the choice of "boundary conditions." Our work may be considered a consistent model calculation within such a formulation, or it may also be taken as a calculation in conventional classical electrodynamics, the equations of which have been approximated (as is often done) by ignoring radiation reaction. From the results of our calculations, we will be able to draw some conclusions about the importance of the radiation reaction.

In Sec. II the retarded theory including the equations of motion and a statement of the conservation laws is presented. Section III discusses the method of attack used to compute the numerical solutions which are discussed in Sec. IV. Conclusions are drawn in Sec. V.

II. EQUATIONS OF MOTION AND CONSERVATION THEOREMS

We consider a system of charges which interact with each other via Liénard-Wiechert potentials.[10] We label the potentials and eliminate all self-interaction terms. This is equivalent to the interaction theory of Leiter[9] with the boundary condition $\lambda^{\mathfrak{n}} = \delta^{\mathfrak{n}}$ (in the notation of Ref. 9) and leads to essentially the same equations used by Synge,[2] Driver,[3] and others. The equation of motion may be written

$$m^{(K)}\dot{\gamma}^{(K)} = \sum_{J\neq K} q^{(K)} \frac{d\vec{\mathrm{R}}^{(K)}}{dt} \cdot \vec{\mathrm{E}}^{(J)}(\vec{\mathrm{R}}^{(K)}), \tag{1}$$

where $m^{(K)}$ is the mass and $q^{(K)}$ the charge of the Kth particle which is at $\vec{\mathrm{R}}^{(K)}$ with respect to the observer, $\gamma^{(K)} = [1-(d\vec{\mathrm{R}}^{(K)}/dt)^2]^{-1/2}$, a dot indicates a derivative with respect to the observer's time, and $\vec{\mathrm{E}}^{(J)}(\vec{\mathrm{R}}^{(K)})$ is the electric field of the Jth particle at $\vec{\mathrm{R}}^{(K)}$. The left-hand side (L.H.S.) of Eq. (1) is the time derivative of the rest plus kinetic energy of the Kth particle.

The energy-conservation theorem may be obtained by summing both sides of the equation of motion over all particles and converting the right-hand side (R.H.S.) by application of $\Box A^{(K)\mu} = -4\pi J^{(K)\mu}$ to an expression of interacting fields. The result is

$$\sum_K m^{(K)}\dot{\gamma}^{(K)} = -\sum_{K,J\neq K} \frac{d}{dt}\int d^3R\,[\vec{\mathrm{E}}^{(K)}\cdot\vec{\mathrm{E}}^{(J)} + \vec{\mathrm{B}}^{(K)}\cdot\vec{\mathrm{B}}^{(J)}]/8\pi, \tag{2}$$

where the right-hand side is the integration over all space of the field interaction energy density. Even though we consider no self-interaction whatever and no radiation-reaction terms, an energy-conservation law [Eq. (2)] is still valid. The above equations form a simple and consistent mathematical model; the equation of motion is equivalent to that used by Kasher and Schwebel. We will see below that solutions have some unphysical behavior which can be traced to the lack of any sort of radiation-reaction term, or equivalently to the non-positive definiteness of the field energy.

Specifically, we consider two particles of equal mass $m=1$ at positions $\pm x_1(t_1)$ on the x axis. We let $x_2(t_2)$ be the position at the retarded time t_2:

$$x_1 + x_2 = t_1 - t_2, \tag{3}$$

and find from Eq. (1) for cases (i) (minus sign) and (ii) (plus sign),

$$\frac{d^2}{dt_1^2} x_1 = \pm \gamma_1^{-3}(x_1+x_2)^{-2} \frac{1-dx_2/dt_2}{1+dx_2/dt_2}, \qquad \gamma_1^{-2} \equiv 1-(dx_1/dt_1)^2. \tag{4}$$

The product $q^{(1)}q^{(2)}$ is taken equal to unity so that distances in Eq. (4) are in units of the classical charge radius $q^{(1)}q^{(2)}/mc^2$.

III. NUMERICAL CALCULATION

The numerical approach is in principle that used to solve retarded scalar-field problems[1]: The kinematic information is stored in arrays and retrieved by interpolation when needed later for the retarded quantities. Now, however, an improved Hamming predictor-corrector method[11] is used in which the integration step size is varied in a way to preserve accuracy and stability of the solutions. The kinematic quantities are stored at every step in large arrays which can be recycled to flush old data which are no longer needed. The interpolation of retarded values proceeds now by a stable and accurate double quadratic procedure: To find a value lying between storage points i and $i+1$, for example, Eq. (3) is solved twice, once using x_2, $\dot{x}_2$, and $\ddot{x}_2$ values at i, and once using these values at $i+1$, and the results are averaged.

All calculations were performed in double-precision on IBM 360/50 and 360/65 computers. Accuracies demanded were generally in the range 10^{-7} to 10^{-10} and were achieved by step sizes which ranged from around 1 at large separations to less than 10^{-13} at small separations. Runs made with different specified accuracies were compared to ensure that numerical error was kept negligibly small.

The Hamming-method integration is initiated by three Gill-Runga-Kutta[12] steps. The seven most recently found present and retarded kinematic quantities are generally kept in temporary arrays for use in the integration procedure. This keeps searches and interpolations among the permanent arrays containing past information to a minimum. The step size changes only by factors of two. The normal criterion for doubling the step size was stiffened to prevent doubling more often than once every four steps.

IV. RESULTS

A. Attractive Case

The oppositely charged particles are held fixed at a separation d until $t=0$, when they are released. Runs were made with small d values for comparison with the Kasher and Schwebel result[4] as well as with larger, physically more meaningful d values. In Fig. 1, results for $d=1$ are compared. The apparent discontinuities in the slopes of the Kasher and Schwebel x_1 and dx_1/dt_1 curves (dashed lines) and the corresponding inconsistencies in x_1, dx_1/dt_1, and d^2x_1/dt_1^2 values were not found in our results. Some oscillatory behavior as found by Kasher and Schwebel was present, however.

In Figs. 2 and 3 the velocity and acceleration of the particles are followed to distances $x_1 < 10^{-11}$. The solid line is for $d=10$, the dashed for $d=1$, and the dot-dashed for $d=0.5$. The oscillations are most pronounced in the $d=0.5$ curves, decrease as d is increased, and practically disappear for d as large as 10.

From the unit slope in the log-log plot in Fig. 2, it is evident that (i) the particle velocity dx_1/dt_1 is approaching -1 as the particles approach the origin, and (ii) the acceleration d^2x_1/dt_1^2 is nearly a constant.

From the equation of motion, it is straightforward to show that the acceleration d^2x_1/dt_1^2 must approach a constant as $x_1 \to +0$: Let $s_1 \equiv t_c - t_1$ be the time left until collision, i.e., $x_1(t_c)=0$; and let $d^n x_1/dt_1^n\ (t_1 \to t_c) \equiv -a$ be the lowest-order time derivative of x_1 with $n>1$ which does not vanish as $x_1 \to +0$. For sufficiently small s_1, we can expand x_1 and x_2 in a Taylor series:

$$x_1 = s_1 - a s_1^n/n! + \cdots , \tag{5a}$$

$$x_2 = s_2 - a s_2^n/n! + \cdots , \tag{5b}$$

where $s_2 \equiv t_c - t_2$ is the retarded time corresponding to s_1:

$$s_2 - s_1 = x_1 + x_2 . \tag{5c}$$

Differentiation and combination give the expressions

$$\frac{dx_1}{dt_1} = -1 + \frac{a s_1^{n-1}}{(n-1)!} + \cdots , \tag{6a}$$

$$\frac{d^2x_1}{dt_1^2} = -\frac{a s_1^{n-2}}{(n-2)!} + \cdots , \tag{6b}$$

$$\frac{dx_2}{dt_2} = -1 + \frac{a}{(n-1)!}\left(\frac{2n!}{a} s_1\right)^{(n-1)/n} + \cdots , \tag{6c}$$

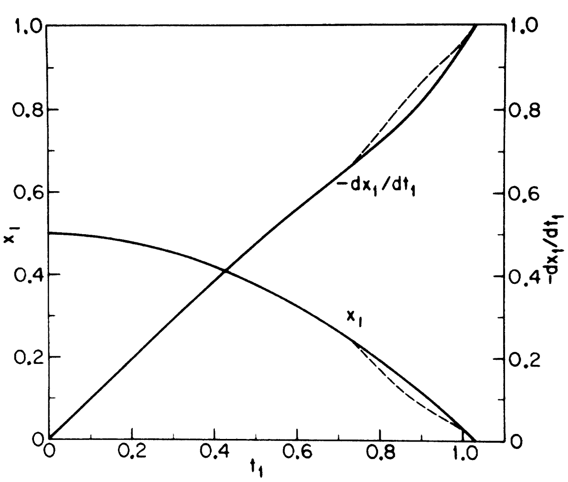

FIG. 1. Position (in units of classical charge radius) and velocity (in units with $c=1$) of particles with opposite charges starting from rest with an initial separation $d=1$. The dashed curves are from the calculation of Ref. 4.

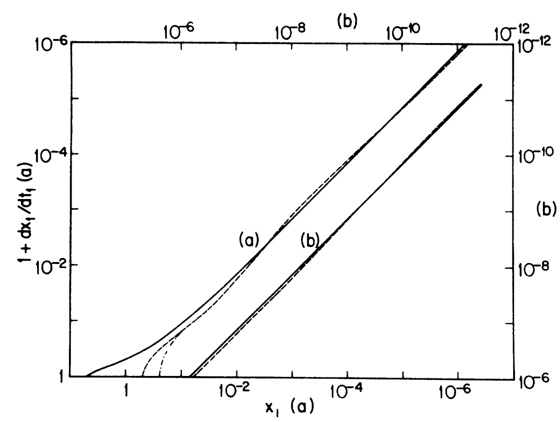

FIG. 2. The velocity in the attractive case approaches -1 as the particles reach the origin. The coordinates for part (b) of the curve are on the upper and right-hand margins. Curves are shown for three initial separations: $d=10$ (solid curve), $d=1$ (dashed curve), and $d=0.5$ (dot-dashed curve, which in order to keep the diagram clear is shown only for $x_1 > 0.1$).

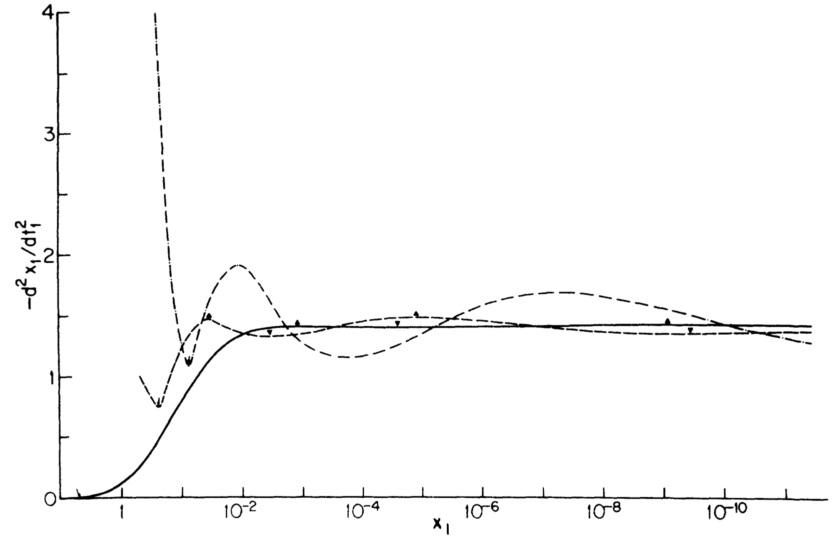

FIG. 3. The acceleration in the attractive case approaches $-\sqrt{2}$ as the particles reach the origin. Curves are shown for $d=10$ (solid curve), $d=1$ (dashed curve) and $d=0.5$ (dot-dashed curve). The small triangles indicate minima (▼) and maxima (▲) in the $d=10$ and $d=1$ curves. The small arrows indicate the position at which, due to retarded effects, the motion of the other particle is first felt.

$$\gamma_1{}^{-3}=\left[\frac{2a}{(n-1)!}\,s_1{}^{n-1}\right]^{3/2}+\cdots\ . \tag{6d}$$

Sutstitution into Eq. (4) gives directly

$$\frac{as_1{}^{n-2}}{(n-2)!}=4\left[\frac{2a}{(n-1)!}\right]^{1/2}\left(\frac{2n!}{a}\right)^{-(n+1)/n}$$
$$\times\ s_1{}^{3(n-1)/2-(n+1)/n}\ . \tag{7}$$

Equating powers of s_1 gives

$$n=2\ , \tag{8}$$

$$a=\sqrt{2}\ . \tag{9}$$

Figure 3 shows that indeed the acceleration does approach $-\sqrt{2}$ as $x_1\to+0$.

B. Repulsive Case

Like-charged particles are initially moving directly at each other from large separation with energies (kinetic plus rest) $\gamma_1(-\infty)$ and velocities $dx_1(-\infty)/dt_1$. The numerical calculation is started with the particles at various separations large enough ($d\simeq 200$) that the results show no dependence on small changes in d. A plot of $x_1(t_1)$ is presented in Fig. 4. In contrast to the half-retarded plus half-advanced case treated by Andersen and von Baeyer,[5] it is clear here that $x_1(t_1)\neq x_1(-t_1)$.

Also in contrast to the Andersen and von Baeyer results, we have no problems with stability of solutions and can investigate collisions in the high-energy region. We find no minimum for

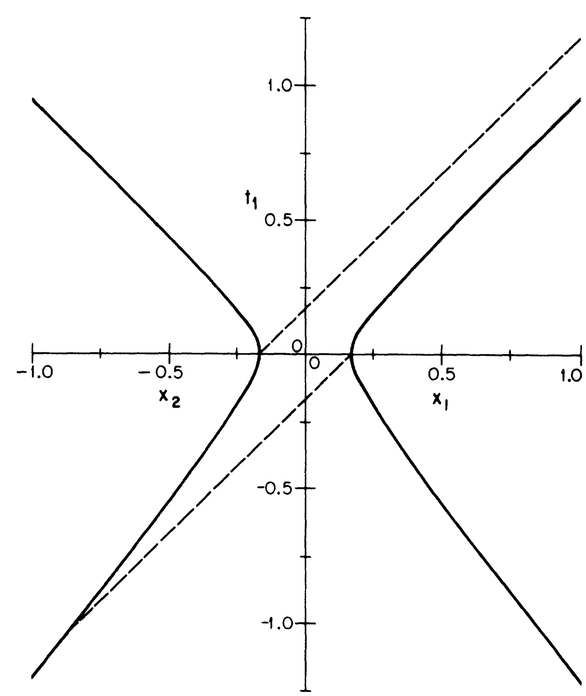

FIG. 4. The position in the repulsive case for an initial velocity of -0.8. The dashed lines show the relation between present (right-hand side) and retarded (left-hand side) positions. Note the asymmetry of the curves about $t_1=0$.

$2x_1(0)$, the distance of closest approach; instead, as shown in Fig. 5, $x_1(0)$ decreases monotonically with increasing initial energies and shows no indication of nonmonotonic behavior over energies investigated, $\gamma_1 \lesssim 25$, and over separations down to $2x_1(0) \simeq 10^{-5}$.

The most striking feature of our results for the repulsive case is that, after collision, the particle energies γ_1 are appreciably higher than before (see Fig. 5): $\gamma_1(\infty) > \gamma_1(-\infty)$. Since there is no radiation reaction in our formulism, we expected the energies to be the same before and after collision, as found by Andersen and von Baeyer in their calculation with half-retarded plus half-advanced fields. On closer examination however, it does appear understandable that the particles gain energy: Since each particle "sees" the other at its retarded position, the maximum repulsion between the two particles occurs after the particles have turned around and are traveling outward. This is shown

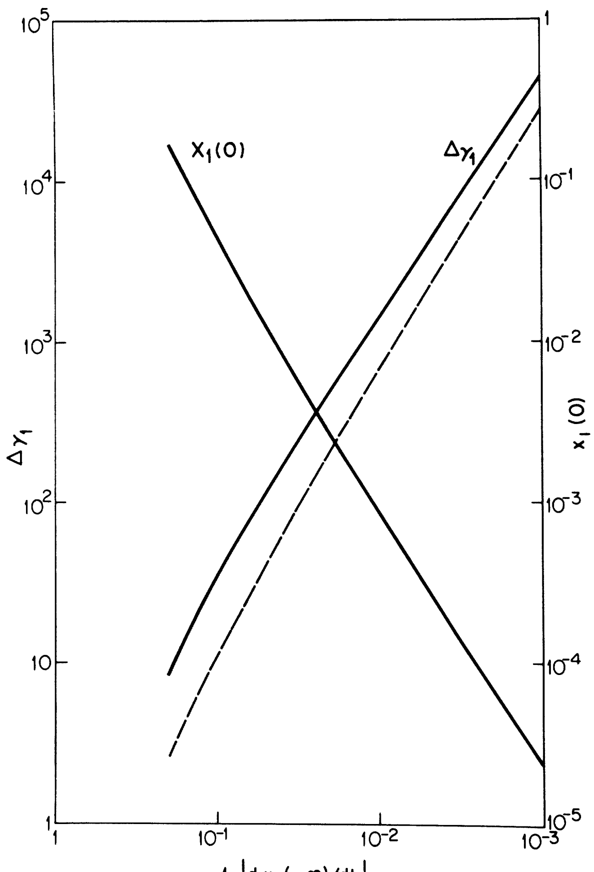

FIG. 5. The increase in particle energy during collision and the distance of closest approach to the origin as a function of initial velocity for the repulsive case. The dashed curve is the analytic high-energy approximation [Eq. (11)].

in Fig. 4, which is for the relatively low initial velocity $-dx_1(-\infty)/dt_1 = 0.8$. When one particle is at its turning point $x_1(0)$, its interaction with the other particle is due to the other particle's position and motion at the retarded time, connected in Fig. 4 to $x_1(0)$ by a dashed line. The motion of one of the particles at $t=0$ affects the other particle much later, at a time well off the diagram in Fig. 4.

The equation of motion is, however, consistent with energy conservation [Eq. (2)], so that the energy in the fields [R.H.S. of Eq. (2)] must decrease during the collision and approach a constant level lower than the original one. This is no longer paradoxical when we recall that since no self-interaction occurs, the electromagnetic-interaction field energy is not necessarily positive definite.

For $t \to \pm\infty$, we can estimate the field energy. By differentiating the Liénard-Wiechert potentials, one obtains[10] for rectilinear motion on the x axis

$$\vec{E}^{(1)} = \left\{ \left[\frac{-\hat{R}_1 - \hat{x}\, dx_1/dt_1}{\gamma_1^2 R_1^2} - \frac{d^2x_1}{dt_1^2} \frac{\hat{x} - \hat{R}_1 \xi}{R_1} \right] \kappa^{-3} \right\}_{\text{ret}},$$
$$\vec{B}^{(1)} = -\hat{R}_1 \times \vec{E}^{(1)}, \tag{10}$$

where $\hat{x}$ is a unit vector along the x axis, $\vec{R}_1$ is the vector from the point at which the field is determined to particle 1, $\hat{R}_1$ is a unit vector in the direction of $\vec{R}_1$ and has the x component $\xi \equiv \hat{R}_1 \cdot \hat{x}$, and $\kappa = 1 + \xi dx_1/dt_1$. The first term on the R.H.S. of Eq. (10) is the Lorentz-transformed static field. The second term is the radiation field.

At $t = -\infty$ there has been no acceleration, and consequently, only the Lorentz-transformed static field exists. The interaction term thus has the form of an integral over all space of an expression varying as $R_1^{-2} R_2^{-2}$. This interaction vanishes as $|\vec{R}_1 - \vec{R}_2|^{-1}$. At $t = +\infty$, there exists an additional radiation field. Now the particle acceleration is appreciable only over a small interval Δt of time near $t=0$. As a result, the radiation field is confined to a spherical shell of thickness $\simeq \Delta t$ and radius $\simeq t$, approximately concentric with $x_1(0)$. Furthermore, it is seen to be zero along the x axis. Since the volume of the shell increases as R_1^{+2} whereas the overlap of the Lorentz-transformed static field of particle 2 with the radiation field of particle 1 decreases as $R_2^{-2} R_1^{-1}$ or faster, this interaction also vanishes in the limit $t \to \infty$. Similar arguments can be made for the magnetic fields as well.

The only contribution to the electromagnetic-interaction field energy which survives in the limit $t \to \infty$ is due to the overlap of the radiation fields. The interaction volume is the overlap of the two expanding shells and is sketched for a number of times in Fig. 6. This volume is approximately $\pi(R\Delta t)^2/x_1(0)$ for $t \gg \Delta t$. The approximate values of $\vec{E}^{(1)} \cdot \vec{E}^{(2)}$ and $\vec{B}^{(1)} \cdot \vec{B}^{(2)}$ at large t are easily found by evaluating Eq. (10) in a plane perpendicular to the x axis and cutting the axis at x_1:

$$\vec{E}^{(1)} \cdot \vec{E}^{(2)} \simeq \vec{B}^{(1)} \cdot \vec{B}^{(2)} \simeq -R^{-2}[d^2x_1(0)/dt_1^{\,2}]^2 .$$

The fields $\vec{E}_1$ and $\vec{E}_2$ are in opposite directions; hence, the minus sign. The same holds for $\vec{B}_1$ and $\vec{B}_2$. In the limit of high particle energies, the interaction energy is therefore roughly given by

$$\frac{1}{4\pi}\int d^3R\,[\vec{E}^{(1)} \cdot \vec{E}^{(2)} + \vec{B}^{(1)} \cdot \vec{B}^{(2)}] \simeq \frac{-[\Delta t\, d^2x_1(0)/dt_1^{\,2}]^2}{2x_1(0)} . \tag{11}$$

This expression has been evaluated by setting the full width at half height of the $d^2x_1(t_1)/dt_1^{\,2}$ curve equal to Δt. Results shown in Fig. 5 are seen to be in quite satisfactory agreement (considering the rough nature of the calculation) with

$$-\Delta\gamma_1 \equiv -\gamma_1(\infty) + \gamma_1(-\infty).$$

V. SUMMARY AND CONCLUSIONS

The dynamics of two charged particles of equal mass moving along a straight line in the model with interaction via retarded fields and with no radiation reaction have been investigated by numerical calculation. New and unexpected results have been found for both attractive and repulsive cases and these have been explained analytically. In the attractive case, the acceleration is found to approach $-d^2x_1/dt_1^{\,2} \to \sqrt{2}$ as $x_1 \to 0$. In the repulsive case, the particles gain a substantial amount of kinetic energy during the collision, even though the total (field plus particle) energy is conserved. The latter result appears unphysical and provides an argument for *not* omitting radiation reaction.

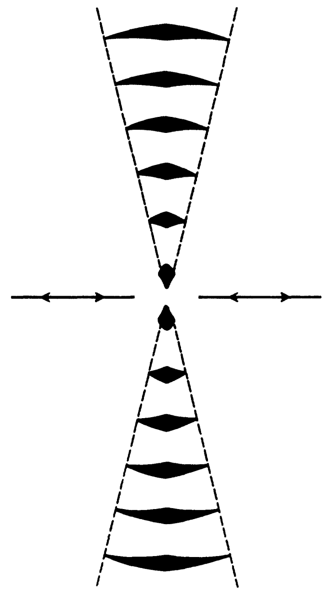

FIG. 6. The interaction radiation pattern resulting from the collision of particles of like charges. The negative-energy pulse is shown at successive times as it moves outward from the origin.

In this connection, we remark that we have calculated the energy radiated by the accelerating particle by integrating the Larmor power term[10] and find that in all cases, much more energy is radiated than the energy gained by the particle. Consequently, a proper inclusion of radiation reaction would cause the particles to lose rather than gain energy. The effect of the negative interaction energy which we have found would still be present, but it would be overshadowed by the effects of radiation reaction.

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Nonuniqueness of Canonical Field Quantization in Riemannian Space-Time*

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We point out and discuss an ambiguity which arises in the quantum theory of fields when the background metric is not explicitly Minkowskian–in other words, when an external gravitational field, real or apparent, is present. A general theory of a canonical neutral scalar field in a static universe, including the construction of a Fock space, is presented. It is applied to a portion of two-dimensional flat space-time equipped with a non-Cartesian space-time coordinate system with respect to which the metric is nonetheless static. The resulting particle interpretation of the field is shown to be different from the standard one in special-relativistic free-field theory. The ambiguity frustrates an attempt to define uniquely the energy-momentum tensor by the usual method of normal ordering. We discuss various suggestions for (1) distinguishing a unique correct quantization in a given physical situation, or (2) reinterpreting seemingly inequivalent theories as physically equivalent. In passing it is shown that the vacuum state and the energy density of a free field in a box with periodic boundary conditions differ from those associated with a region of the same size in infinite space; this result should be of interest outside the gravitational context.

I. INTRODUCTION

In recent years generalizations of elementary quantum field theory to Riemannian space-times[1] have begun to be applied seriously to various cosmological and astrophysical problems.[2–5] In this work it is necessary to proceed beyond the establishment of field equations and commutation relations[6] to the construction of some framework of *observables* and *quantum states*, and to give this apparatus a physical interpretation. This is traditionally done in terms of a Hilbert space of quantum-state vectors in which the fields are realized as operators.[7] The most common strategy in the Riemannian context is to choose (if possible) a coordinate system in which the field equation can be solved by separation of variables and to quantize the resulting "normal mode" structure in close analogy to the standard quantization of a free field in flat space. In the case of a static metric (e.g., Ref. 3) one is thus led to what appears to be a unique theory, which we outline in Sec. II A. In time-dependent problems (e.g., Refs. 2, 4, 5) the situation is less clear, since there is not an unambiguous division of the solutions of the field equation into positive- and negative-frequency parts; it has been suggested that the concept of *particle* loses some of its physical significance in such situations.[8–10]

Such constructions are not manifestly generally covariant. If a given space-time admits two or more of them, there is no guarantee that they will agree physically; if not, of course, at most one of them can be correct. In particular, any procedure which purports to apply to all Riemannian metrics of a certain form (e.g., static metrics) must yield physically sensible results when the metric considered is that of *ordinary flat space* equipped with a curvilinear coordinate system. One might hope to use this principle as a criterion for the correctness of the theories, or as a guide to the choice of the correct ansatz in the cases where ambiguities remain.

In this paper a neutral scalar field in a patch of two-dimensional Minkowski space is quantized according to the "unique" prescription for static metrics referred to above. It is shown that the resulting notions of *particles* and *vacuum state* are completely different from those of the standard Fock representation. This ambiguity affects the definition of the energy-momentum tensor, the