PHYSICAL REVIEW D VOLUME 7, NUMBER 1 1 JANUARY 1973

Dispersion Sum Rules for $\Sigma\Lambda$ Transition Magnetic Moment

S. Mallik
Saha Institute of Nuclear Physics, Calcutta-9, India
(Received 6 June 1972)

We consider sum rules for the process $\pi\Lambda \to \pi\Sigma$ obtained by equating the fixed-t and the fixed-u dispersion relations for the invariant amplitudes at a suitable point. The sum rules relate the transition magnetic moment for $\Sigma \to \Lambda + \gamma$ to the contributions from s-channel singularities. The results obtained are in agreement with the predictions of SU(3) and SU(6) symmetry.

It has been shown[1] that many of the higher-symmetry relations among the trilinear hadron couplings emerge from dispersion relations in a well-defined dynamical limit. The derivation starts from the sum rules obtained by equating at threshold the unsubtracted dispersion relations for the appropriate elastic amplitudes in the forward and the backward directions. However, these sum rules cannot be easily extended to (two-body) inelastic processes, owing to the complication caused by the presence of kinematic branch points in the dispersed variable.

In this note we analyze an analogous type of sum rules for the process $\pi^+\Lambda \to \pi^+\Sigma^0$. These sum rules are obtained[2] by equating the fixed-t and the fixed-u dispersion relations for the invariant amplitudes at a suitable point (s_0, t_0, u_0), called the matching point. These are[3]

$$\frac{A_t(s_0, t_0, u_0)}{4\pi(s_0-u_0)} = \frac{1}{\pi}\int_{(m_\Lambda+\mu)^2}^{\infty} ds' \,\frac{\mathrm{Im}[A(s', t_0, u') - A(s', t', u_0)]/4\pi}{(s'-s_0)(u'-u_0)}\,, \tag{1}$$

$$\frac{B_t(s_0, t_0, u_0)}{4\pi} = -\tfrac{4}{3}\alpha(1-\alpha)\frac{g_r^2}{4\pi}\frac{1}{m_\Sigma^2-u_0} + \frac{1}{\pi}\int_{(m_\Lambda+\mu)^2}^{\infty} ds'\left(\frac{\mathrm{Im}[B(s', t_0, u') - B(s', t', u_0)]/4\pi}{s'-s_0} + \frac{\mathrm{Im}B(s', t_0, u')/4\pi}{u'-u_0}\right). \tag{2}$$

Here A_t and B_t stand for the t-channel contribution. The Σ pole is written on the assumption of SU(3) symmetry for the meson-baryon (pseudoscalar) coupling, and the F/D ratio of the two types of couplings is specified by the parameter $\alpha = F/(F+D)$.

The t-channel contribution near threshold is assumed to be given entirely by the ρ-meson pole. This assumption has been extensively tested[4] in the sum rules for other pion-baryon processes. The form of the $\rho\Sigma\Lambda$ coupling is obtained from the hypothesis of universality of the ρ-meson coupling.[5] ρ can then couple to Σ and Λ through the induced magnetic moment $\kappa_{\Sigma\Lambda}$ only. The ρ-meson-exchange contributions to the amplitudes are then found to be

$$\frac{A_t(s,t,u)}{4\pi(s-u)} = \frac{g_{\rho\pi\pi}^2}{4\pi}\frac{\kappa_{\Sigma\Lambda}}{2\bar{m}}\frac{1}{m_\rho^2-t}\,, \tag{3}$$

$$\frac{B_t(s,t,u)}{4\pi} = -\frac{g_{\rho\pi\pi}^2}{4\pi}\frac{2\kappa_{\Sigma\Lambda}}{m_\rho^2-t}\,. \tag{4}$$

$g_{\rho\pi\pi}^2/4\pi$ as evaluated from the observed decay width of the ρ meson is

$$g_{\rho\pi\pi}^2/4\pi = 2.43 \pm 0.4\,. \tag{5}$$

In determining the matching point (s_0, t_0, u_0) in the sum rules we have to keep in mind the restrictions imposed by requiring (i) the validity of unsubtracted dispersion relations, (ii) the reliability of resonance saturation of sum rules in the narrow-width approximation to which we take resort in evaluating the dispersion integrals, and (iii) the convergence of partial-wave expansion of the imaginary part of the amplitudes in dispersion integrals. The asymptotic behavior of the amplitudes as predicted by usual Regge asymptotics is well known and allows unsubtracted dispersion relations to be written for $t_0 \lesssim 30\mu^2$ and $u_0 \lesssim 72\mu^2$, respectively. The narrow-width approximation for the resonances makes it desirable to evaluate the sum rules at a point not too close to any of the resonances. We thus prefer to take s_0 not above $\pi\Sigma$ threshold.

The convergence of the partial-wave expansion of the imaginary part of the amplitudes is determined by the nearest singularity in the $\cos\theta$ plane which, assuming Mandelstam representation, may be obtained from the boundary of the (s, t) spectral function. This boundary is given by

$$(t-16\mu^2)[s-(m_\Lambda+\mu)^2][s-(m_\Lambda-\mu)^2]-8\mu^2[s(m_\Lambda{}^2-m_\Sigma{}^2+8\mu^2)+(m_\Sigma{}^2-m_\Lambda{}^2)(m_\Lambda{}^2-\mu^2)],\qquad (m_\Lambda+\mu)^2<s<s_1 \tag{6}$$

$$(t-4\mu^2)[s-(m_\Sigma+2\mu)^2][s-(m_\Sigma-2\mu)^2]+4\mu^2[2s(m_\Lambda{}^2-m_\Sigma{}^2-2\mu^2)+(m_\Sigma{}^2-2m_\Lambda{}^2-2\mu^2)(m_\Sigma{}^2+2\mu^2)+(m_\Sigma{}^2-4\mu^2)^2],\qquad s_1<s<\infty \tag{7}$$

where $s_1\simeq 118\mu^2$. The partial-wave series converges in a region given by

$$4\mu^2>t_0>\max_{(m_\Lambda+\mu)^2<s<\infty}[-t(s)-s+\Sigma-(m_\Lambda{}^2-\mu^2)(m_\Sigma{}^2-\mu^2)/s]\simeq -25.5\mu^2, \tag{8}$$

$$4\mu^2>u_0>\max_{(m_\Lambda+\mu)^2<s<\infty}[-t(s)+\Sigma-s]\simeq 23.0\mu^2, \tag{9}$$

for the fixed-t and fixed-u dispersion relations, respectively. $t(s)$ is given by Eqs. (6) and (7). It is clear that, strictly speaking, Mandelstam analyticity does not allow the expansion of the imaginary part of the amplitudes in the fixed-u dispersion relations for any value of u_0. However, we may note that the branch points corresponding to the exchange of two pions in the t channel are weak and may be ignored, in consistency with our approximation for the t-channel singularities that only the ρ meson contributes significantly in this channel. Also, since the dispersion integrals in the sum rules are evaluated in resonance approximation, we may require the convergence of the partial-wave expansion in the resonance region only, i.e., s' (the integration variable in the dispersion integrals) up to $\sim 200\mu^2$. It is then clear that the restriction on partial-wave expansion in t_0 and u_0 may be greatly relaxed. (An idea of the range of t_0 and u_0 resulting from neglecting the spectral function in the region $4\mu^2<t<16\mu^2$ and confining s' to $\sim 200\mu^2$ may be obtained from the dashed curves CC' and DD' in Fig. 1.) We may now take the matching point at $s_0=(m_\Sigma+\mu)^2=90.7\mu^2$, $t_0=10\mu^2$, and $u_0=37.5\mu^2$, which satisfy all the requirements stated above.

In evaluating the contribution of resonances to the sum rules in the narrow-width approximation, we take the parameters of the resonances, in particular the widths and the branching ratios, from experiment.[6] The process considered being inelastic, this procedure is to be supplemented with a determination of the signs of the contribution of resonance exchanges arising from the relative phase at the two vertices. We determine the relative phases by assuming SU(3) symmetry.[7] The result of resonance saturation of the sum rules is shown in Table I. In this connection we point out two uncertainties from which these evaluations suffer. First, in view of the fact[8] that the SU(3) predictions for the couplings with octets and decouplets of higher resonances, except for the $\frac{3}{2}^+$ decouplet, are in rough agreement with experiment, it is not certain that the unbroken coupling constants will bear the same phase relationship as the actual (broken) coupling constants obtained by introducing adequate symmetry-breaking interaction. Secondly, the experimental situation re-

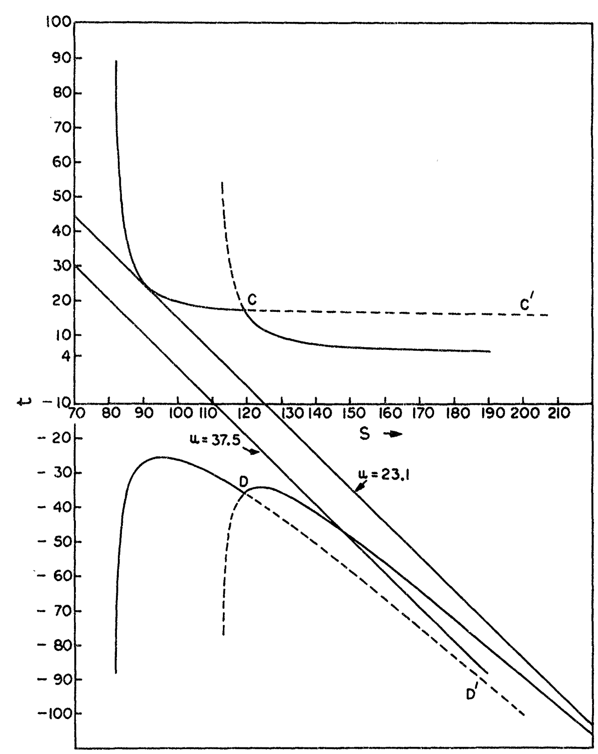

FIG. 1. Singularities determining the ranges of t_0 and u_0 for the fixed-t and the fixed-u dispersion relations for $\pi\Lambda\to\pi\Sigma$ scattering. The upper and the lower curves represent plots of the boundary of the spectral function and the right-hand side of the inequality (8). (Energies are in pion-mass units.)

TABLE I. Resonance contributions to the sum rules in units of $\mu=\hbar=c=1$.

J^P	Resonance (mass in MeV)	Contributions to sum rules A amplitude	B amplitude
$\frac{3}{2}^+$	(1383)	0.172 ± 0.029	-0.063 ± 0.011
$\frac{3}{2}^-$	(1670)	$0.037\pm$?	$-0.081\pm$?
$\frac{1}{2}^-$	(1750)	0	0
$\frac{5}{2}^-$	(1765)	$0.008\pm$?	$-0.018\pm$?
$\frac{5}{2}^+$	(1910)	0	-0.002
$\frac{7}{2}^+$	(2030)	0.009 ± 0.003	0.002
Total resonance contribution		0.226 ± 0.029	-0.162 ± 0.011
Σ pole contribution		0	-0.229
Total (right-hand side)		0.226 ± 0.029	-0.391 ± 0.011

garding the resonance structure around the region of $\Sigma(1670)$ is uncertain[6, 9]; more resonances may be discovered in this region on better resolution. We take the width and the branching ratios of $\Sigma(1670)$ derived from a formation experiment. We obtain the following values for $\kappa_{\Sigma\Lambda}$ from our sum rules:

$$\kappa_{\Sigma\Lambda}=1.85\pm0.38 \quad (A \text{ sum rule}), \tag{10}$$

$$\kappa_{\Sigma\Lambda}=1.60\pm0.27 \quad (B \text{ sum rule}), \tag{11}$$

in good agreement with the SU(3) prediction, viz.,

$$\kappa_{\Sigma\Lambda}=-\tfrac{1}{2}\sqrt{3}\;\kappa_n=1.66\,. \tag{12}$$

Reasonable variation in the matching point does not significantly alter the values obtained above.

It is interesting to consider the case of degenerate Σ and Λ mass. The kinematic complication which led us to consider the present sum rules instead of those obtained by equating the forward and backward dispersion relations now disappears, and we wish to investigate whether any symmetry result emerges in this degeneracy limit. There is one sum rule of the latter type which is particularly convenient in investigating dynamically the results of higher symmetry, viz., that for the helicity-nonflip amplitude, since it has the distinctive feature of suppressing the contribution of even partial waves. It is therefore a good approximation to retain in this sum rule only the contribution of Σ and $\Sigma(1383)$ in the dispersion integral. Thus, following the method of Ref. 1, we equate the dispersion relations for the amplitude

$$D(\omega_L)=\frac{1}{4\pi}[A(\omega_L)+\omega_L B(\omega_L)] \tag{13}$$

in the forward direction and

$$F(\omega^2)=\frac{1}{4\pi}\left[\frac{E}{m\,\omega}A(\omega^2)+B(\omega^2)\right] \tag{14}$$

in the backward direction to obtain the sum rule as

$$-\frac{g_{\rho\pi\pi}^{\;2}}{4\pi}\frac{\kappa_{\Sigma\Lambda}}{2m^2}=-\frac{4}{\sqrt{3}}\alpha(1-\alpha)\frac{g_r^{\;2}}{4\pi m^2}+\frac{4}{3\sqrt{3}}\frac{g^{*2}}{4\pi M^2}\frac{m}{M}\,, \tag{15}$$

where g^* is the $NN^*\pi$ coupling constant, ω (ω_L) is the c.m. (lab) pion energy, and M is the mass of $\Sigma(1383)$. Consider now the limit in which the induced ρ coupling tends to zero. Then, with $\alpha=\frac{2}{5}$ as given by SU(6) symmetry, we obtain the result

$$\frac{18}{25}\frac{g_r^{\;2}}{4\pi}=\frac{g^{*2}}{4\pi}\,, \tag{16}$$

known from SU(6) symmetry.[10] Indeed, the sum rule, Eq. (15), may be considered together with the three other sum rules obtained in Ref. 1 by considering amplitudes for πN, $\pi\Sigma$, and $\pi\Xi$ scatterings. All these consistently yield SU(6)-symmetry results in the same limit as above.

I am grateful to Professor H. Banerjee for suggesting the investigation and thank him and Professor B. Dutta-Roy for helpful discussions.

[1]H. Banerjee and B. Dutta-Roy, Phys. Rev. D 2, 2414 (1970).

[2]D. H. Lyth, Rev. Mod. Phys. 37, 709 (1965).

[3]Our notation and convention are those of J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963). The difference in kinematics for the inelastic process appears only in the formulas relating the invariant amplitudes A and B to the spin-nonflip and -flip amplitudes f_1 and f_2; the nucleon mass m is to be replaced by $\bar{m}=\frac{1}{2}(m_\Sigma+m_\Lambda)$, and $(E\pm m)$ by $[(E_\Lambda\pm m_\Lambda)(E_\Sigma\pm m_\Sigma)]^{1/2}$.

[4]H. Banerjee, S. Mallik, and B. Dutta-Roy, Nuovo Cimento 66, 475 (1970).

[5]J. J. Sakurai, Ann. Phys. (N.Y.) 1, 11 (1960).

[6]Particle Data Group, Rev. Mod. Phys. 43, S1 (1971).

[7]D. E. Plane, P. Baillon, C. Bricman, M. Ferro-Luzzi, J. Meyer, E. Pagiola, N. Schmitz, E. Burkhardt, H. Filthuth, E. Kluge, H. Oberlack, R. Barloutaud, P. Granet, J. P. Porte, and J. Prevost, Nucl. Phys. B22, 93 (1970).

[8]J. Meyer, in *Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany,*

1967, edited by H. Filthuth (North-Holland, Amsterdam, 1968), p. 117.

[9]D. H. Miller, in *Hyperon Resonances–70,* Proceedings of the Duke Conference on Hyperon Resonances, 1970, edited by E. C. Fowler (Moore, Durham, N. C., 1970), p. 229.

[10]F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

PHYSICAL REVIEW D VOLUME 7, NUMBER 1 1 JANUARY 1973

Erratum

Nonleptonic Decays of Hyperons and Current Algebra, Francis C. P. Chan [Phys. Rev. 171, 1543 (1968)]. Page 1547, Table II, line 6 should read:

Amplitude	ETC	Baryon	Total
$P(\Xi_-^-)\times 10^7$	0.16	−0.90	−0.74